Determination of the Modulus of Elasticity by Bending Tests of Specimens with Nonuniform Cross Section - Supplementary Material

M. Gebhardt [1,2], *H. Steinke* [2], *V. Slowik* [1,*]

[1] Institute of Experimental Mechanics, Leipzig University of Applied Sciences, Karl-Liebknecht-Straße 132, 04277 Leipzig, Germany
[2] Institute of Anatomy, Leipzig University, Liebigstraße 13, 04103 Leipzig, Germany
[*] Corresponding author

Table of contents

1. Additional information on the experiments

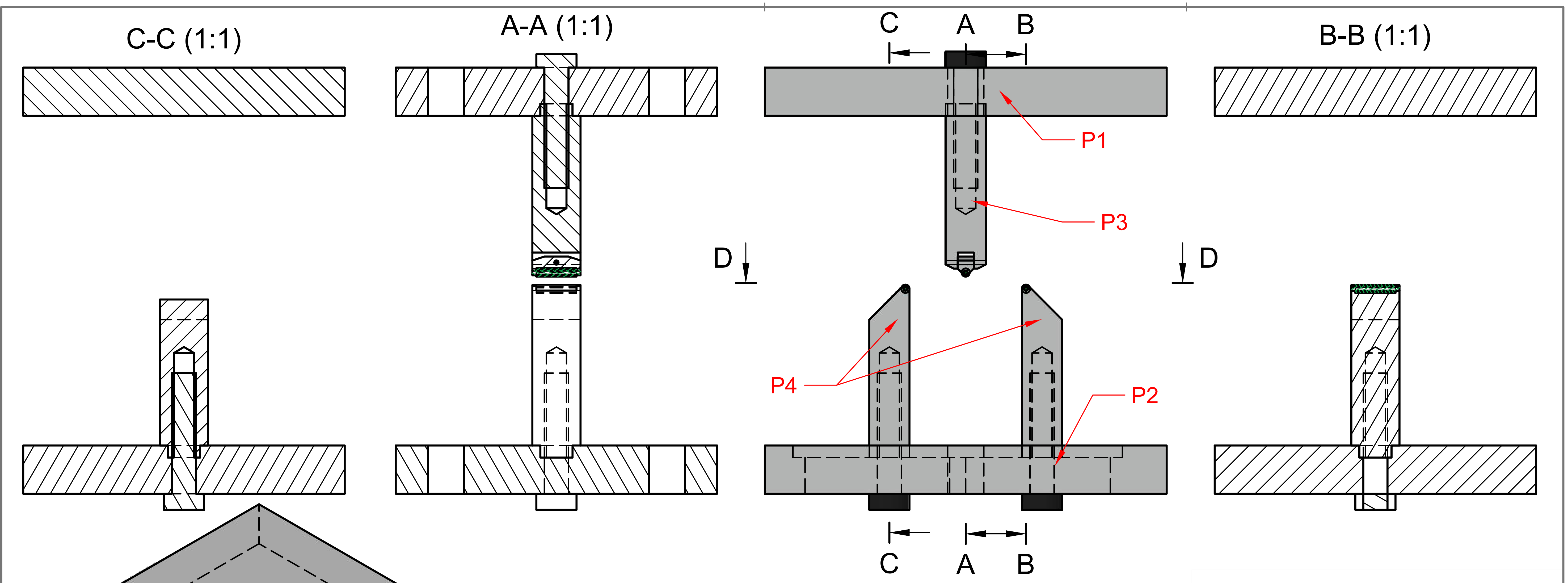
1.1. Test procedure

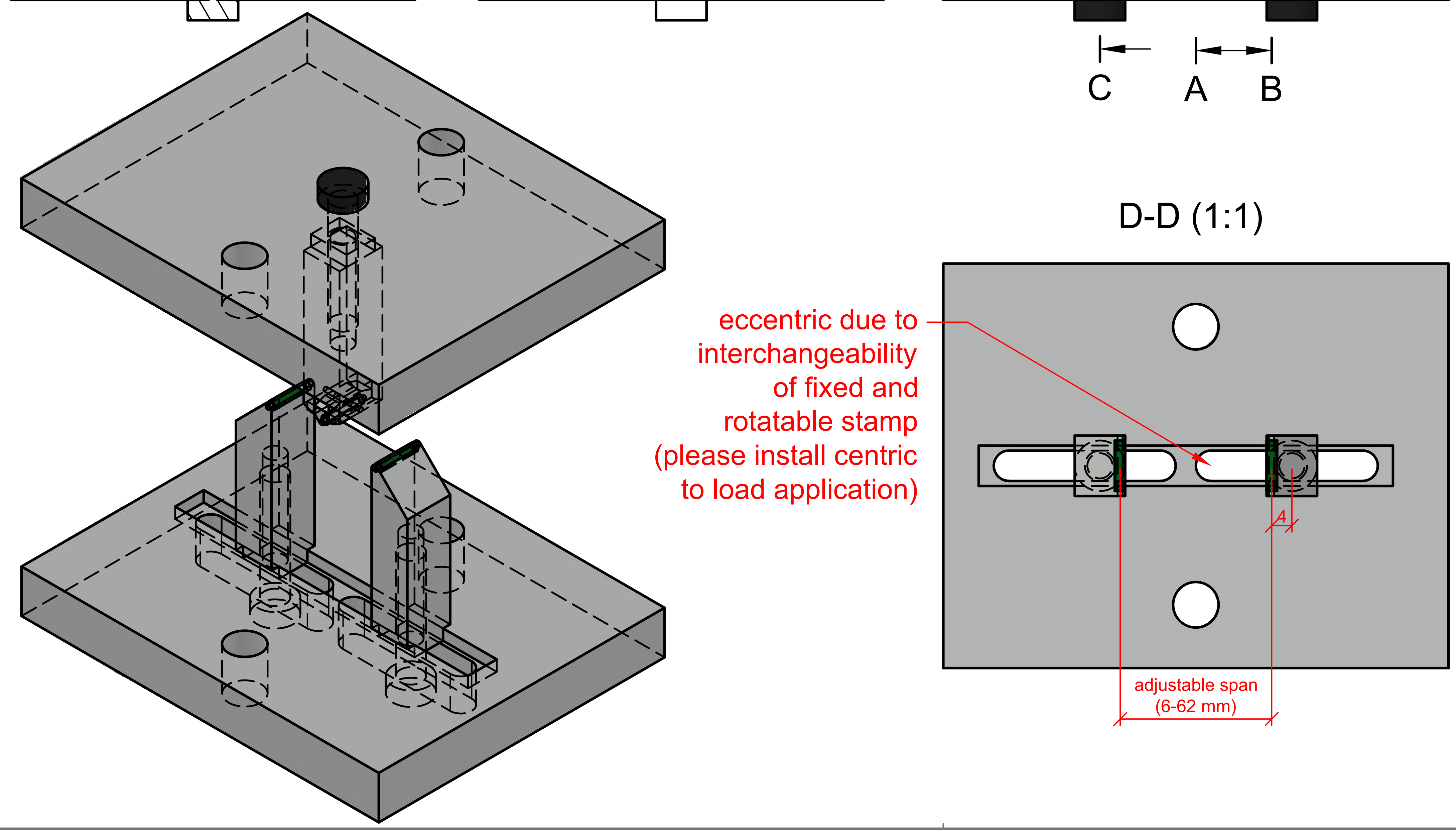
- Displacement controlled testing with:
  $\dot{w} = (\dot{\varepsilon} \times s^2)/(6 \times t)$, $\dot{\varepsilon} = 0.005\ \mathrm{s}^{-1}$, $s = 20\ \mathrm{mm}$, $t =$ thickness in midspan
- Maximum force (safety threshold) $F_{max,sec} = 250\ \mathrm{N}$
- Load until failure, or decrease of force by $0.3 \times F_{max}$

1.2. Measurement

- Conventional:
  - Data acquisition system: QuantumX (100 Hz sampling rate, no filters; MX840B, Hottinger Brüel & Kjaer, Darmstadt, Germany)
  - Displacement: LVDT W10K, Hottinger Brüel & Kjaer, Darmstadt, Germany)
  - Force: internal load cell of testing machine and additional 20 kg-load cell (S40S-G3-0020, Bosche, Damme, Germany)
- Optical:
  - Measuring method: Digital Image Correlation (DIC; 4 Hz sampling rate; Q400 DCM 12.0, Limess, Krefeld, Germany; Istra4D, Dantec Dynamics, Skovlunde, Denmark) with 50 mm lens (VS-5018H1, VS Technology, Tokyo, Japan) and spacer rings (5 + 10 mm), object distance ~ 250 mm, camera distance ~ 164 mm
  - Lighting: two area LED lights (T120, Shenzhen Neewer Technology, Shenzhen, China) at 5600 K
  - Calibration: Istra4D calibration target (2 mm marked white glass [GL-02-WMB_9x9])
  - Marking:
    - Indirect: Speckle pattern with measurement marks laser printed on weather-resistant labels (AST-Speckle-Pattern_Labels_V03-2.pdf on L4775, Avery Zweckform, Oberlaindhaim, Germany)
    - Direct:
      - Opaque white priming (Eberhard Faber, Stein, Germany) with brush (GH30414, da Vinci Künstlerpinselfabrik DEFET, Nuremberg, Germany)
      - Black toner dust (2220D, Ricoh, Tokyo, Japan)
      - Pollinator can with flat filter (one layer of URMA, 1-2-3 Filter, Reinbek, Germany)
      - Draught-proof chamber
- Synchronization: Analog trigger (5 V batterie with switch)

1.3. Drawings of the test device





Parts

No.	Qty.	Description	Drawing sub no.
P1	1	Base plate top	*-001
P2	1	Base plate bottom	*-002
P3	1	Stamp rotatable	*-003
P4	2	Stamp fixed	*-004
P5	3	DIN 7984 M6-1.0 x 30 A2	-

Scale:	Format:
-	A3

Material:
Stainless steel V2

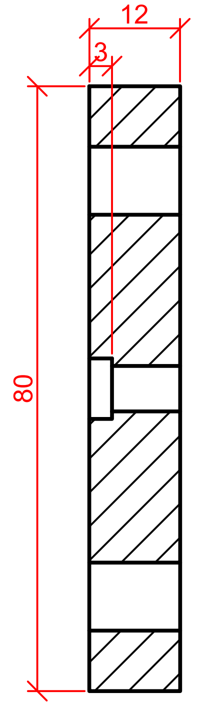
Description:
Three-point bending test setup for cortical bone
Overview

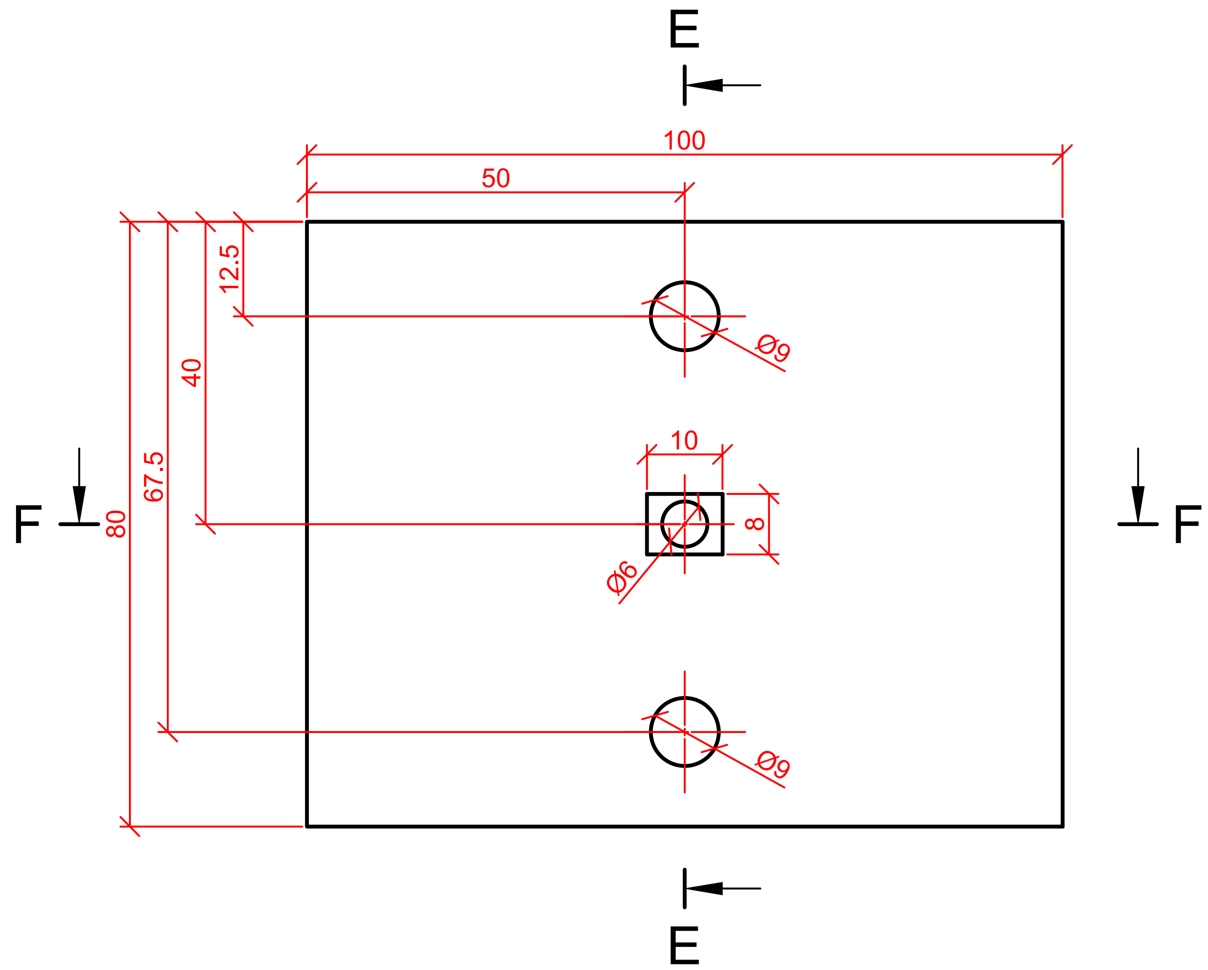
Drawing number:	Revision:	Original date:	Revision date:
PF004-004-000	4	27.02.2019	10.08.2022

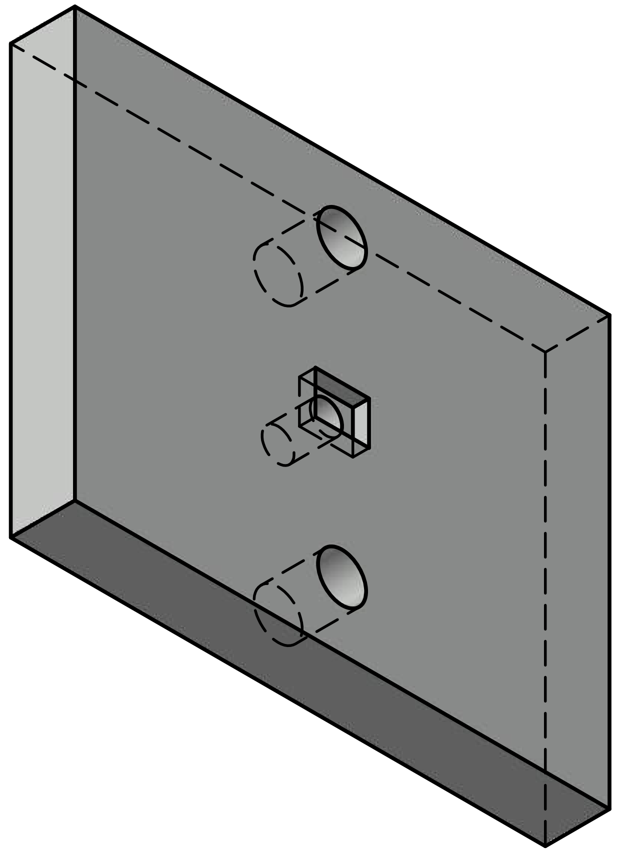
Drawer:
Marc Gebhardt

Organisation:
HTWK Leipzig University of Applied Sciences

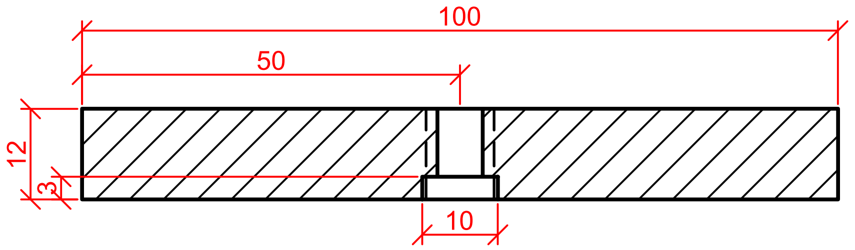
E-E (1:1)



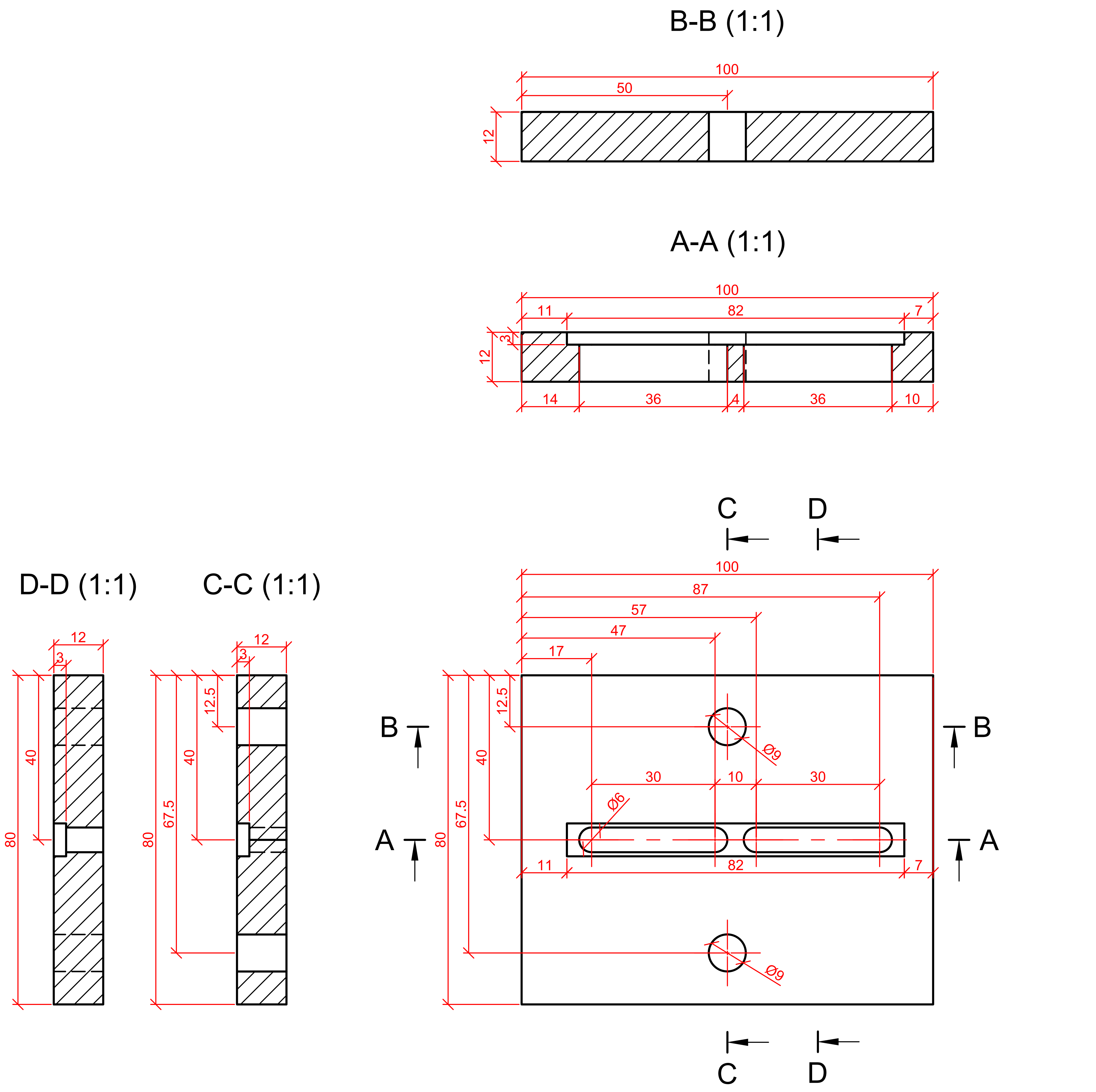


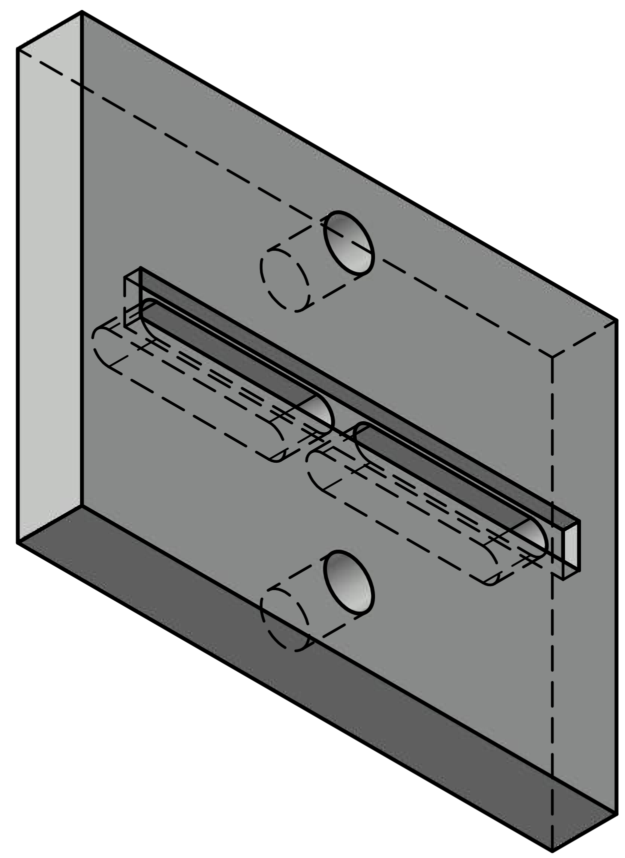


F-F (1:1)



Scale: -			Format: A3
Material: Stainless steel V2			
Description: Three-point bending test setup for cortical bone P1 - Base plate top			
Drawing number: PF004-004-001	Revision: 4	Original date: 27.02.2019	Revision date: 10.08.2022
Drawer: Marc Gebhardt		Organisation: HTWK Leipzig University of Applied Sciences	

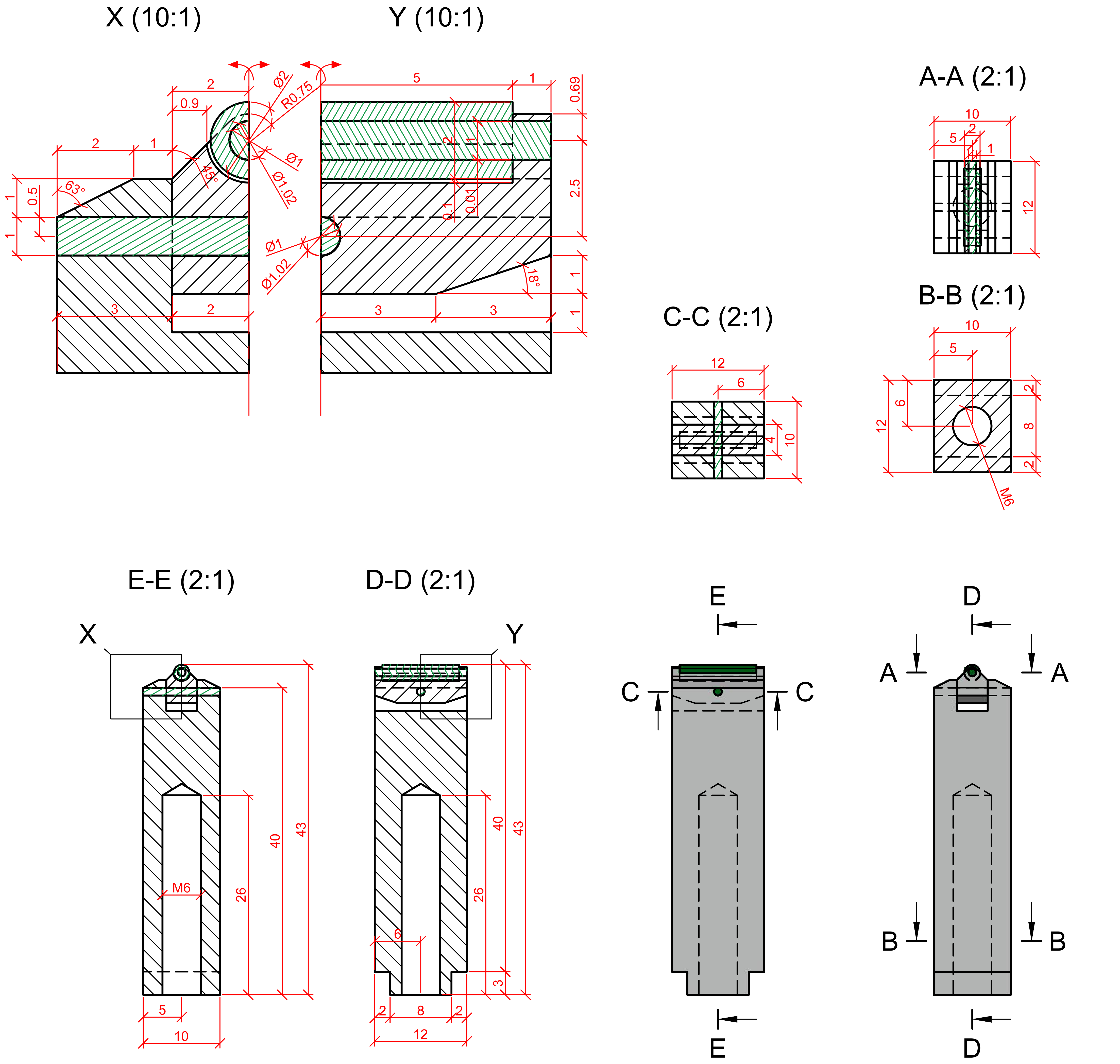


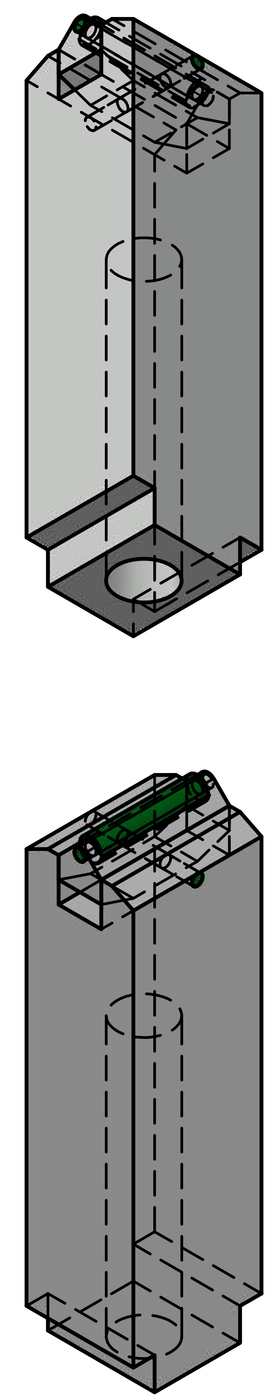


Scale: -	Format: A3
Material: Stainless steel V2	
Description: Three-point bending test setup for cortical bone P2 - Base plate bottom	

Drawing number:	Revision:	Original date:	Revision date:
PF004-004-002	4	27.02.2019	10.08.2022

Drawer:	Organisation:
Marc Gebhardt	HTWK Leipzig University of Applied Sciences





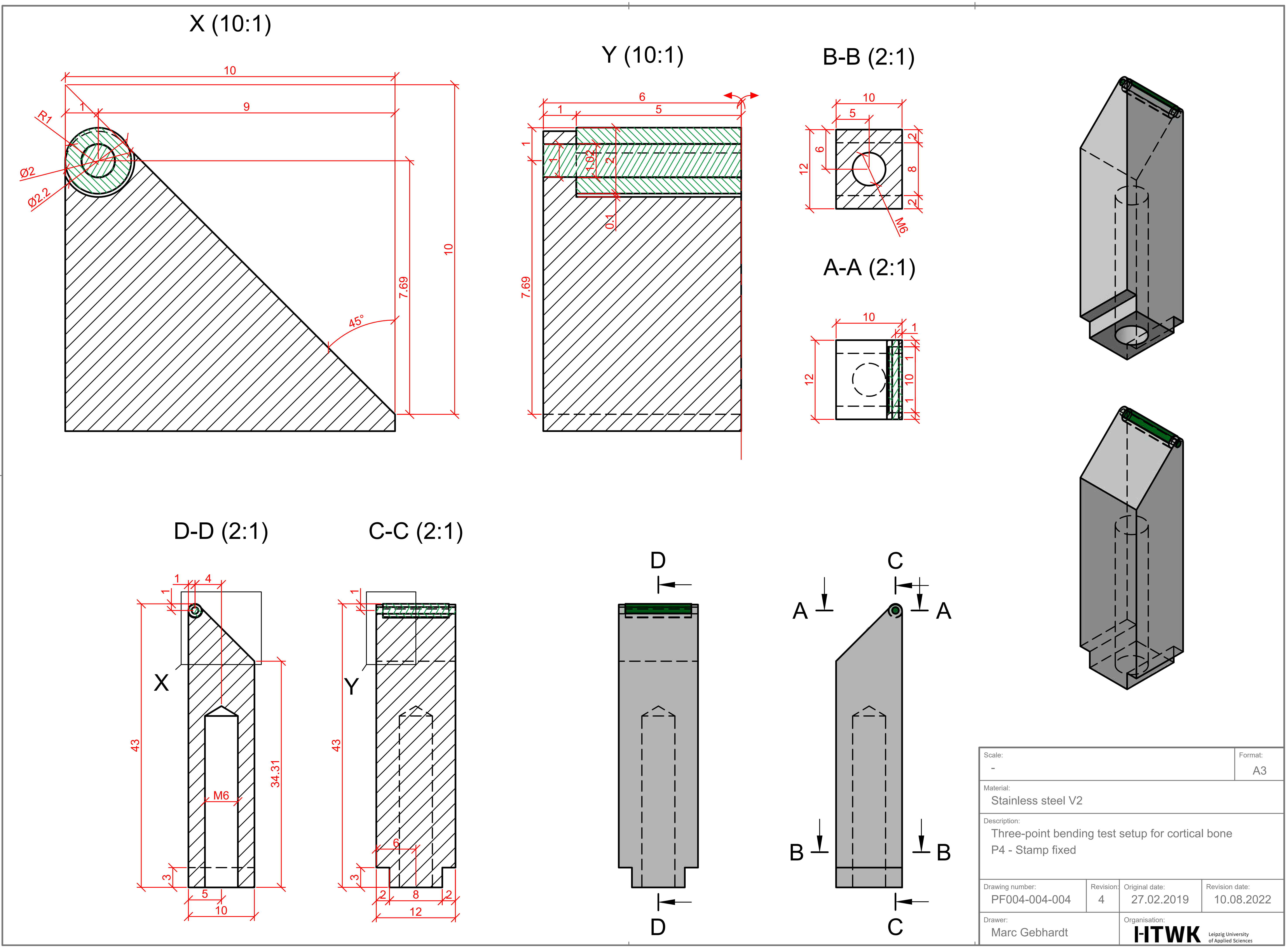
Scale:	Format:
-	A3

Material: Stainless steel V2

Description: Three-point bending test setup for cortical bone
P3 - Stamp rotatable

Drawing number:	Revision:	Original date:	Revision date:
PF004-004-003	4	27.02.2019	10.08.2022

Drawer:	Organisation:
Marc Gebhardt	HTWK Leipzig University of Applied Sciences



Scale:	Format:
-	A3

Material: Stainless steel V2

Description: Three-point bending test setup for cortical bone
P4 - Stamp fixed

Drawing number:	Revision:	Original date:	Revision date:
PF004-004-004	4	27.02.2019	10.08.2022

Drawer:	Organisation:
Marc Gebhardt	HTWK Leipzig University of Applied Sciences

2. Additional evaluation results

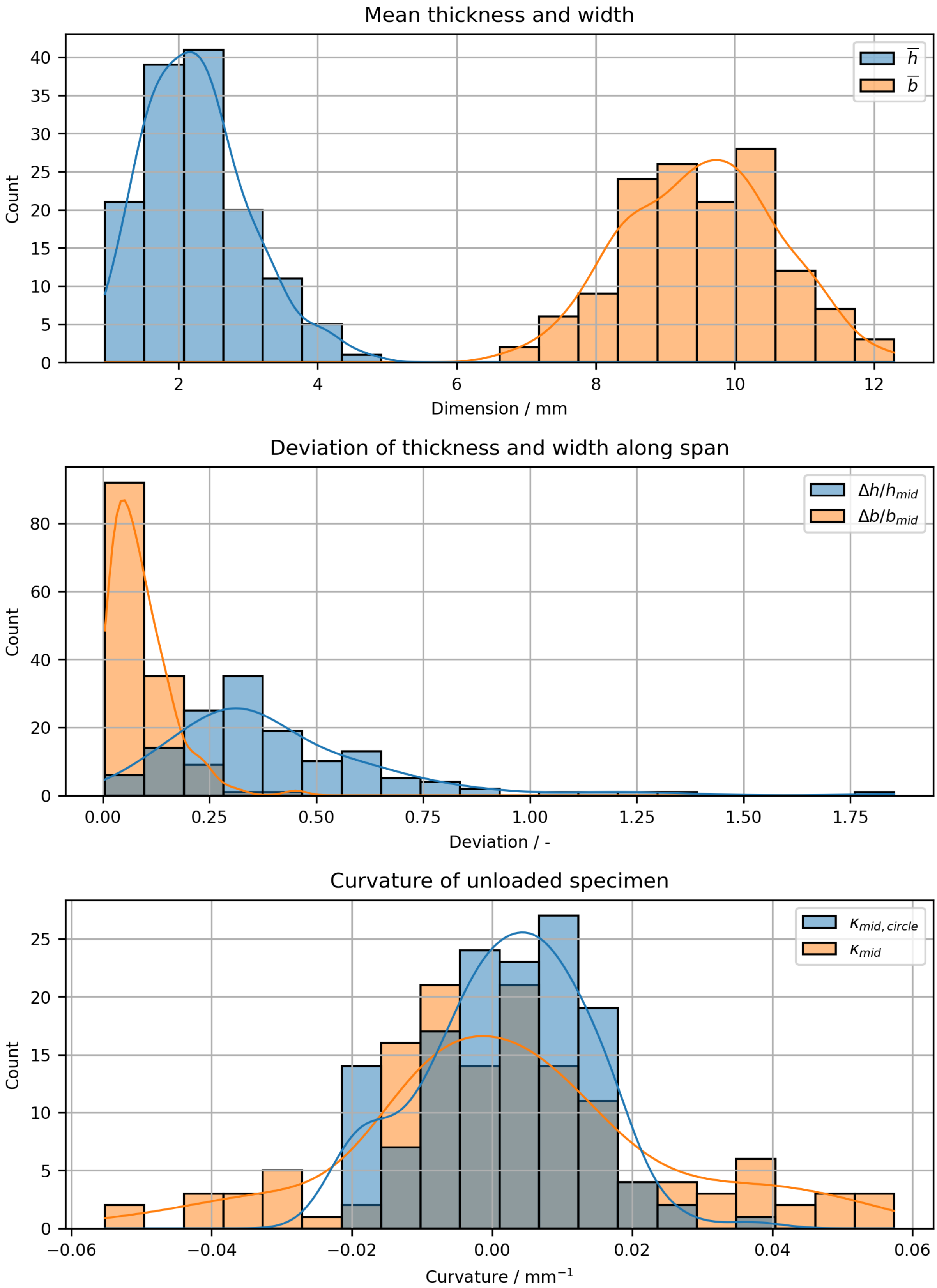


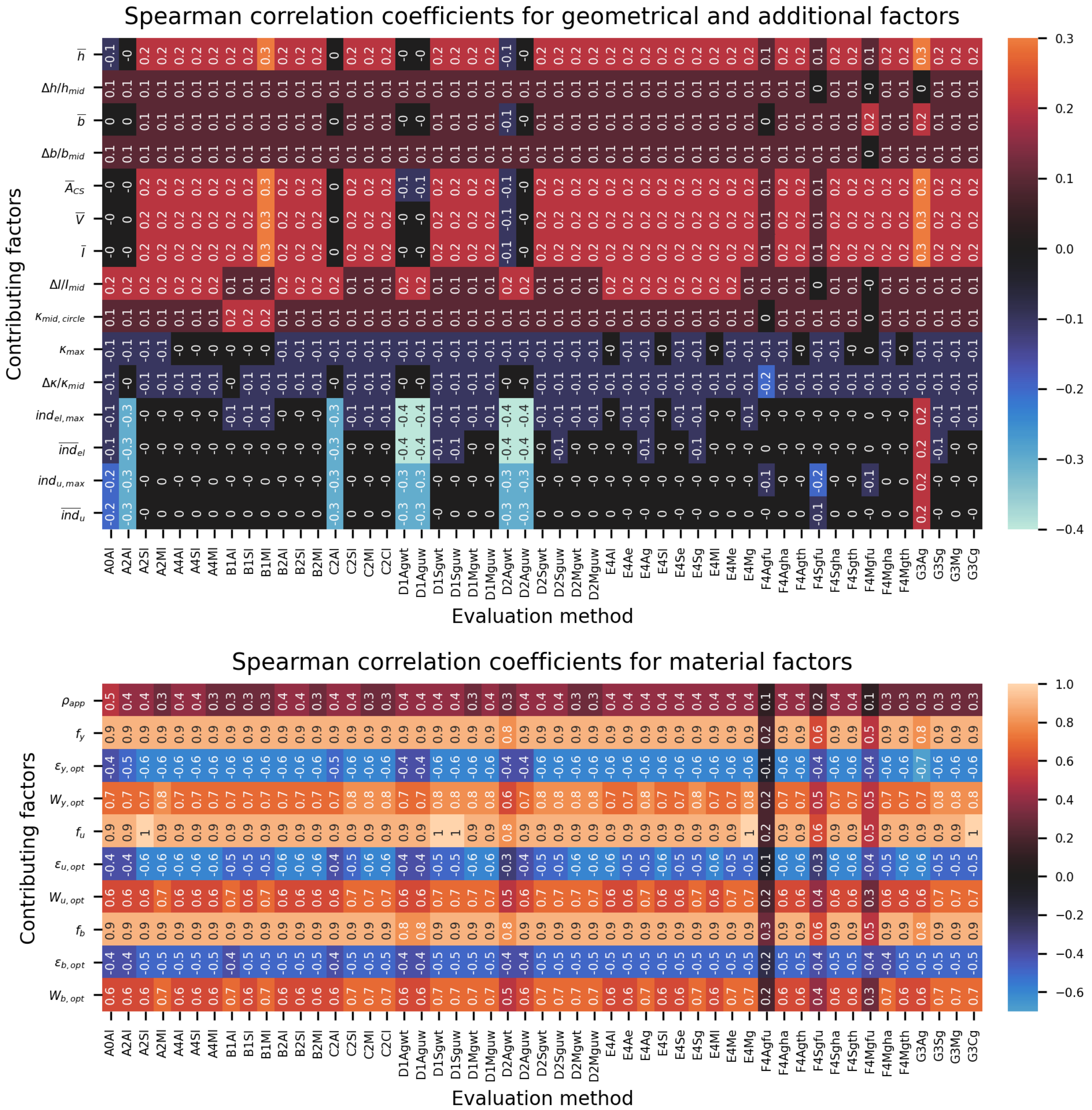
Figure 1: Distribution of geometrical properties ($\Delta h/h_{mid}$: difference between maximum and minimum height divided by the height at midspan, width accordingly, $\kappa_{mid,circle}$: curvature of an inscribed circle, κ_{mid}: midspan curvature of fitted function)

Table 1: Material properties

Parameter	Unit	Minimum	Maximum	Average	Median	Standard deviation	Coefficient of variation in [-]	Confidence interval minimum	Confidence interval maximum
Apparent density	[g/cm³]	0.986	1.866	1.463	1.459	0.187	0.128	1.430	1.492
Yield strength	[MPa]	1.449	95.754	22.645	17.382	16.916	0.747	19.749	25.306
Strain (yield)	[%]	0.404	3.839	1.646	1.608	0.462	0.281	1.572	1.726
External work (yield)	[mJ]	0.383	35.227	8.594	6.797	6.858	0.798	7.471	9.689
Ultimate strength	[MPa]	1.954	106.147	28.197	23.001	19.727	0.700	24.837	31.212
Strain (ultimate)	[%]	0.680	13.812	3.940	3.542	1.779	0.451	3.671	4.246
External work (ultimate)	[mJ]	2.902	244.207	31.538	23.254	28.602	0.907	27.036	36.229
Fracture strength	[MPa]	1.447	94.716	22.947	18.634	16.870	0.735	20.005	25.574
Strain (fracture)	[%]	0.680	15.165	5.483	5.153	2.288	0.417	5.242	5.665
External work (fracture)	[mJ]	2.902	273.483	48.112	38.481	40.013	0.832	42.186	50.812

Table 2: Statistical evaluation for selected methods (modulus of elasticity determined incrementally in refined range, with exclusion of statistical outliers on the specimen level)

Method	Modulus of elasticity in [MPa]								Coefficient of variation of evaluation range in [%]							
	Minimum	Maximum	Average	Median	Standard deviation	Coefficient of variation in [-]	Confidence interval minimum	Confidence interval maximum	Minimum	Maximum	Average	Median	Standard deviation	Coefficient of variation in [-]	Confidence interval minimum	Confidence interval maximum
A0Al	29	4333	882	631	781	0.885	753	1006	2.8	74.4	11.8	10.4	7.2	0.607	10.7	13.1
A2Ml	45	631862	7931	1312	56987	7.186	1570	18702	1.9	156.5	12.8	10.1	17.0	1.334	10.5	16.0
A4Ml	40	469439	6723	1257	44064	6.554	1566	15099	1.9	168.9	15.5	10.7	22.0	1.421	12.2	19.4
B1Ml	35	8337	1944	1473	1683	0.866	1693	2208	2.6	46.8	12.7	12.4	6.1	0.483	11.8	13.8
B2Ml	41	6669	1610	1205	1388	0.862	1385	1815	1.4	43.5	11.1	10.3	4.9	0.443	10.3	12.0
C2Ml	47	6335	1708	1281	1425	0.834	1475	1926	5.0	26.3	10.6	9.9	3.2	0.303	10.0	11.2
C2Cl	45	6357	1721	1293	1440	0.836	1488	1942	4.7	25.7	10.6	9.8	3.4	0.322	10.0	11.3
D1Mguw	48	6495	1738	1342	1431	0.823	1502	1960	4.5	26.4	10.7	9.7	4.0	0.372	10.1	11.4
D1Mgwt	47	6454	1739	1308	1444	0.830	1504	1963	4.2	25.8	10.3	9.6	3.4	0.328	9.8	11.0
D2Mguw	50	6543	1786	1349	1471	0.824	1543	2012	2.2	26.4	10.5	9.8	3.4	0.323	9.9	11.1
D2Mgwt	48	6455	1748	1318	1448	0.829	1511	1971	4.1	26.3	10.5	9.9	3.3	0.313	10.0	11.2
E4Ml	42	7737	1747	1285	1540	0.882	1492	1982	1.3	113.2	12.1	10.4	10.5	0.862	10.7	14.0
E4Me	41	6110	1366	961	1256	0.920	1168	1565	2.5	49.7	13.0	11.8	6.4	0.491	12.0	14.1
E4Mg	42	5940	1486	1085	1279	0.861	1277	1681	2.4	31.4	11.7	11.0	4.8	0.408	10.9	12.5
F4Mgha	44	26540	2086	1406	2725	1.306	1671	2585	2.3	1216.2	21.9	9.9	103.9	4.749	11.1	40.5
F4Mgth	41	6854	1715	1285	1451	0.846	1469	1945	1.4	104.6	12.8	10.3	12.4	0.975	10.9	14.8
G3Mg	45	6063	1574	1171	1351	0.858	1355	1782	2.9	28.1	11.6	11.1	4.5	0.390	10.9	12.4
G3Cg	43	5826	1504	1095	1286	0.855	1296	1702	3.0	26.9	11.6	10.9	4.5	0.389	10.8	12.3



Symbols

h - Thickness
b - Width
A_{CS} - Cross-section area
V - Volume
I - Second moment of inertia
κ - Geometrical curvature
ind - Support indentation
ρ_{app} - Apparent density
f - Strength
ε - Strain
W - External work

$\overline{X}$ - Arithmetic average
ΔX - Difference between max. and min.
X_{mid} - Midspan position
X_{circle} - Approximated as circle
X_{el} - Elastic range
X_y - Yield
X_u - Ultimate
X_b - Break
X_{opt} - Determination via DIC

Figure 2: Influences on the incrementally in refined range determined moduli of elasticity.

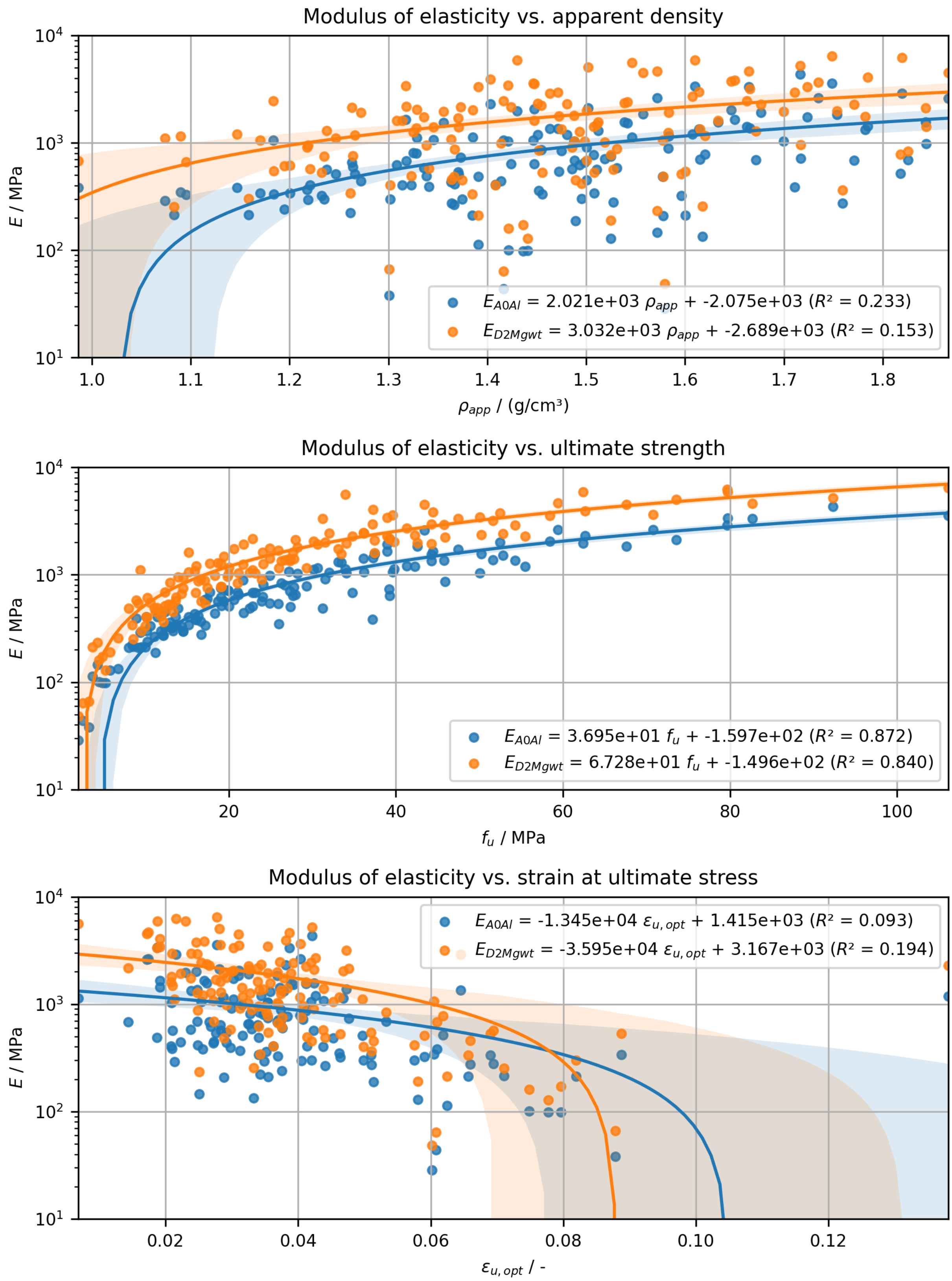


Figure 3: Correlation diagrams with linear regressions for moduli of elasticity incrementally in refined range determined by Methods A0Al and D2Mgwt; confidence intervals determined by bootstrapping for a confidence level of 95 %.

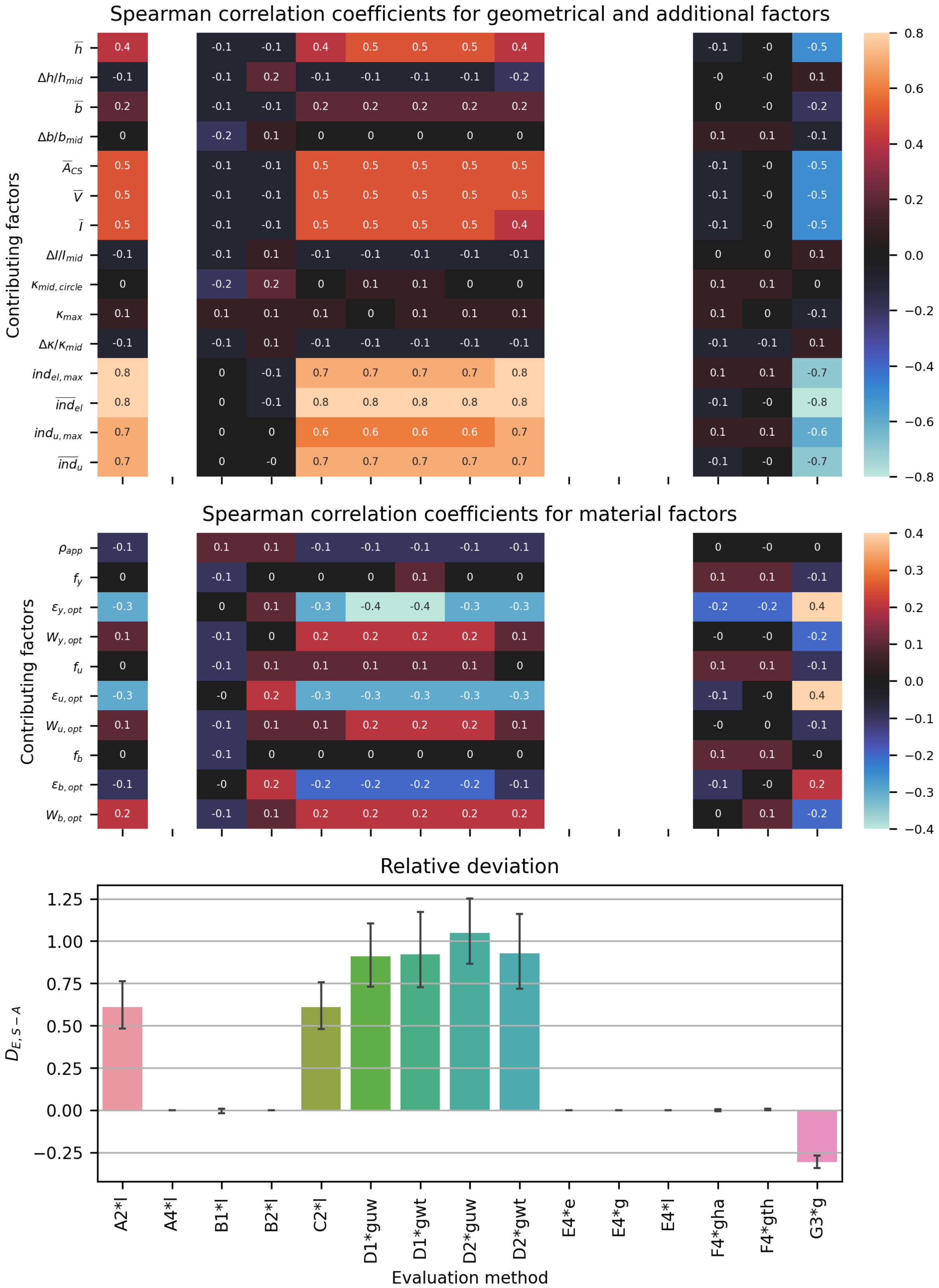


Figure 4: Influences on the moduli of elasticity determined without (**A*) and with (**S*) elimination of the support indentation, confidence intervals determined by bootstrapping for a confidence level of 95 %.

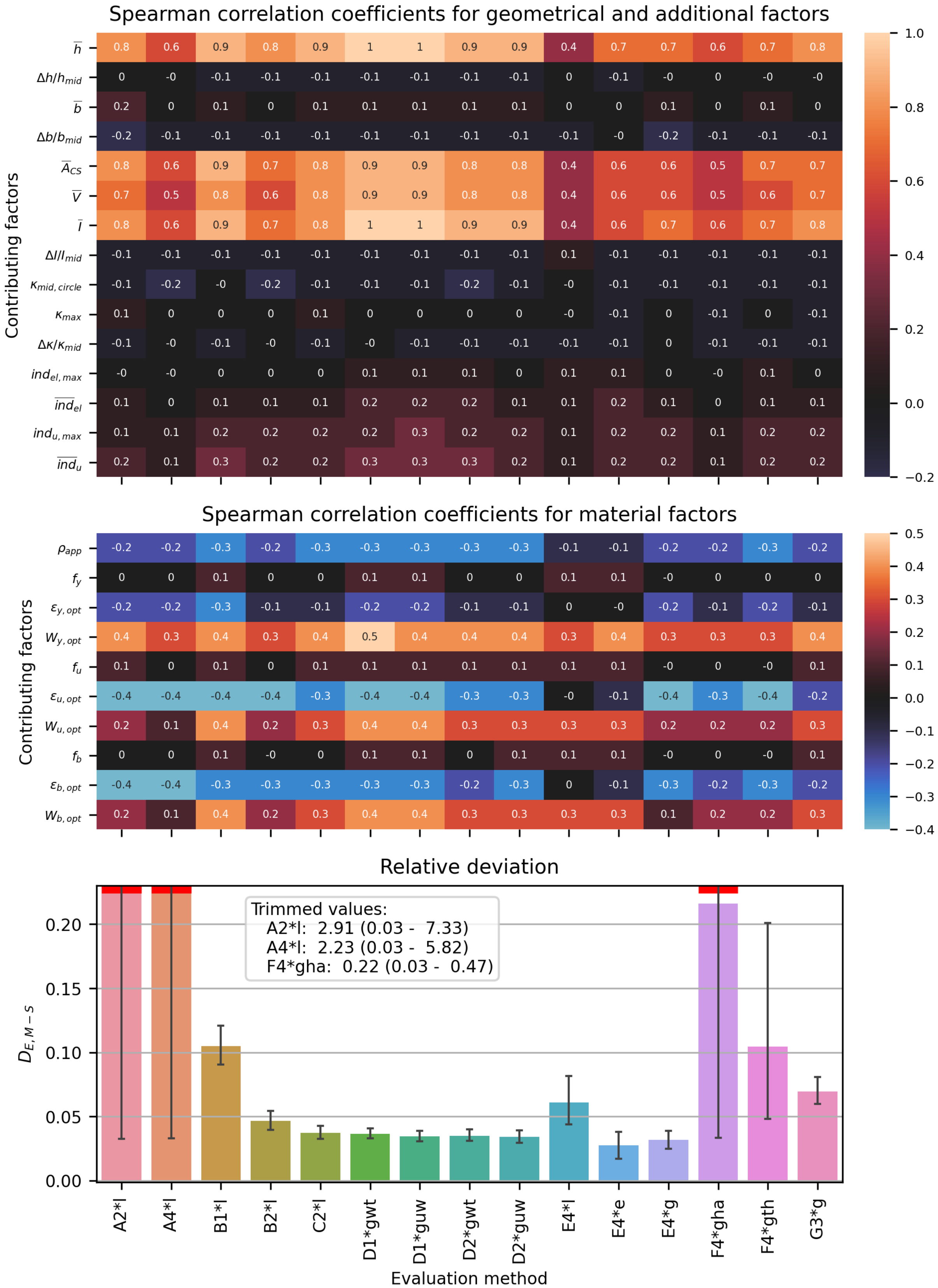


Figure 5: Influences on the moduli of elasticity determined without (**S*) and with (**M*) elimination of the shear deformation, with support indentation eliminated in both cases, confidence intervals determined by bootstrapping for a confidence level of 95 %.

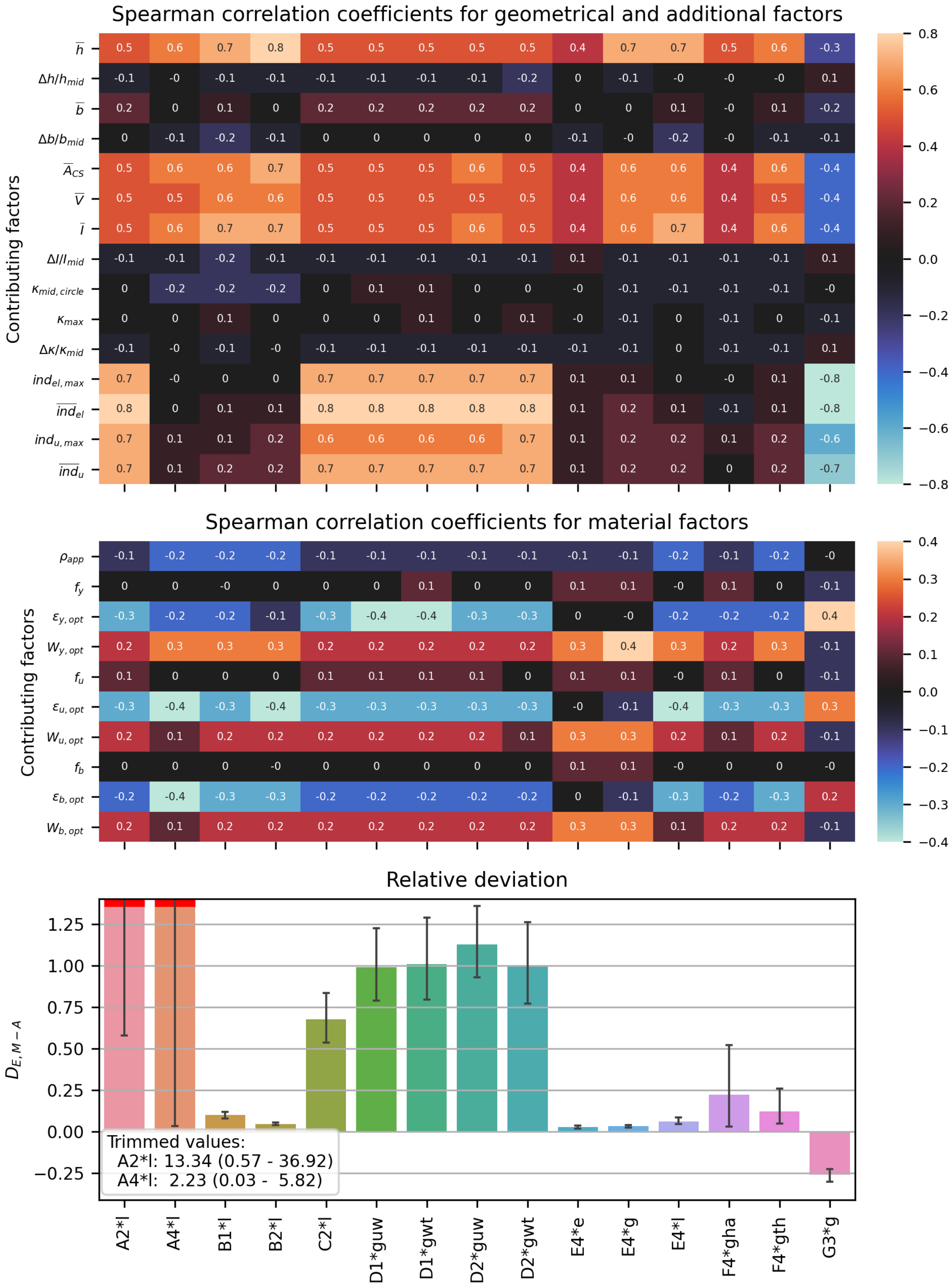


Figure 6: Influences on the moduli of elasticity determined without (**A*) and with (**M*) elimination of the shear deformation, as well as the support indentation, confidence intervals determined by bootstrapping for a confidence level of 95 %.

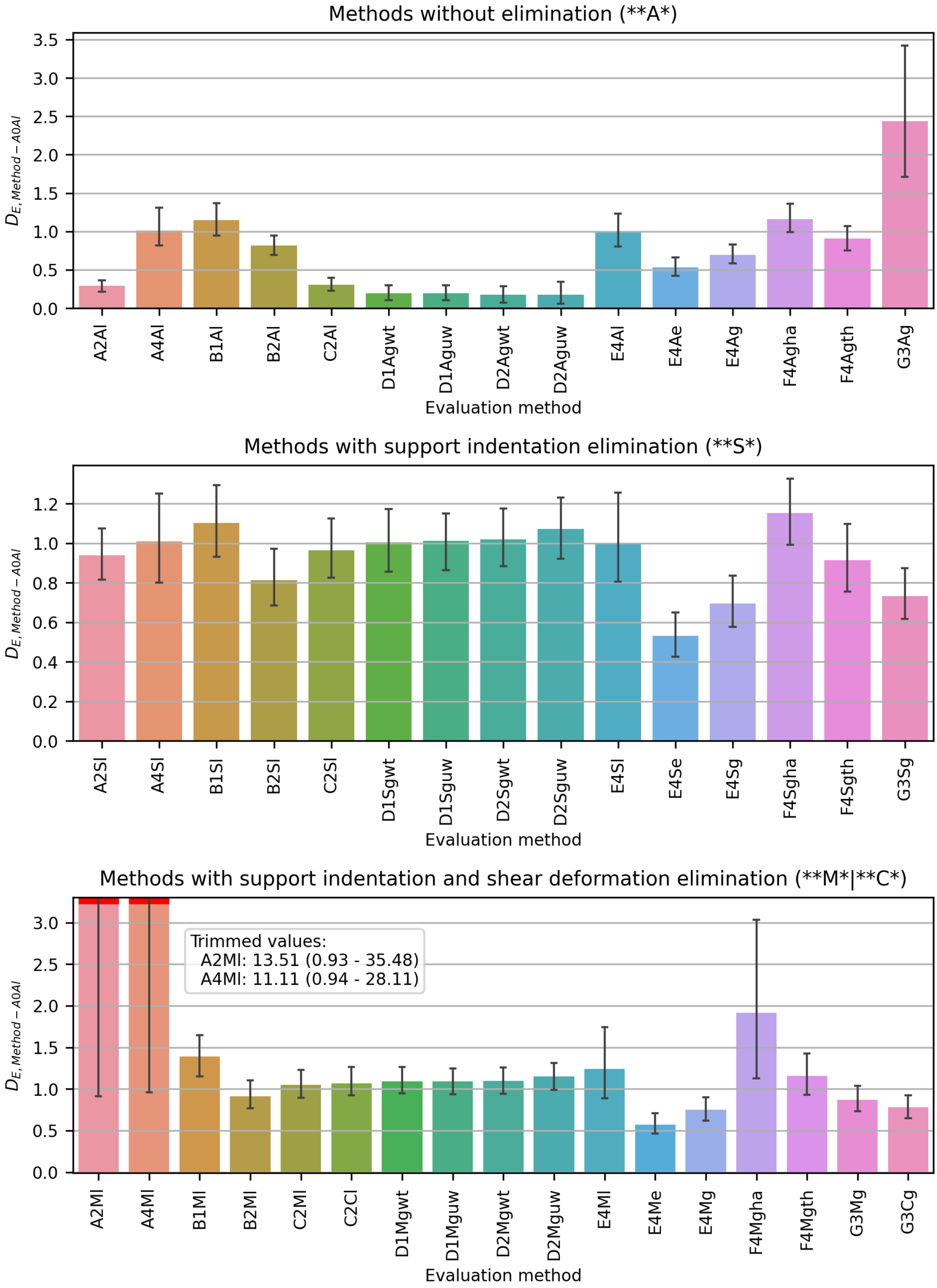


Figure 7: Relative deviation of the moduli of elasticity from the conventionally determined one (Method A0Al), confidence intervals determined by bootstrapping for a confidence level of 95 %.

3. Verification tests

3.1. Specimens

- Two rectangular beams ($hxbxl$ ~ 2x8x36 mm³) sawed from Polymethyl methacrylate (PMMA) of unknown age
- Five test specimens with different geometry, shown in Figure 8. Manufactured by Fused Deposition Modeling (FDM) with eSun Polylactic acid (PLA) (Creality Ender 3, d_{Nozzle} = 0.4 mm, t_{Layer} = 0.2 mm, T_{Print} = 200 °C, T_{Base} = 60 °C).

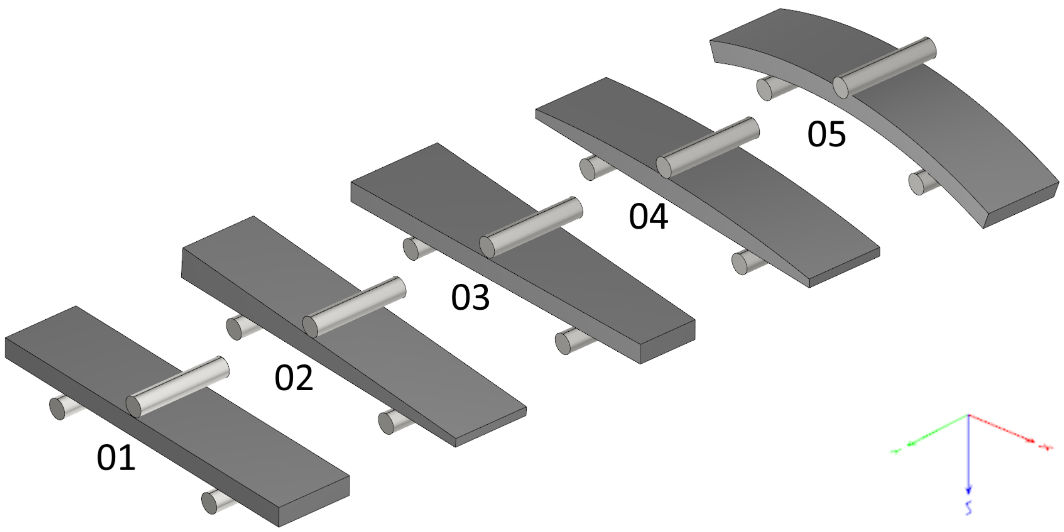


Figure 8: Shapes of the specimens made of Polylactic acid (PLA) for the verification tests schematically shown in test position A; length of the specimens 36 mm (from left to right: 01-uniform rectangular cross-section with the reference dimensions 2 × 8 mm² (height × width); 02-linearly varying height, at the supports 2.6 mm and 1.4 mm; 03-linearly varying width, at the supports 9 mm and 7 mm; 04-quadratic parabola for the height, 1.8 mm at the supports and 2.2 mm at midspan; 05-curved beam with uniform cross-section (reference dimensions), constant radius of curvature 50.5 mm).

3.2. Test procedure

- Elastic loading with strain limitation to 2 %
- Testing in original position (A), as well as 180° rotated about y-axis (B, see Figure 8)

3.3. Measurement

- Optical: three cameras (middle camera with 15 mm offset), 5 Hz sampling rate
- Marking - direct:
    - STEEIRO Airbrush Kit with 0.8 mm nozzle
    - Priming with Vallejo Premium Airbrush Color – White 63.001, 4 bar air pressure, 2.5 rotations of air screw, 100 mm distance
    - Marking with Vallejo Premium Airbrush Color – Black 62.020, 2 bar air pressure, 1.0 rotation of air screw, 100 mm distance, 3-5 times

3.4. Assumptions

- Poisson's ratio:
    - Polymethyl methacrylate (PMMA): 0.375
    - Polylactic acid (PLA): 0.340

3.5. Results

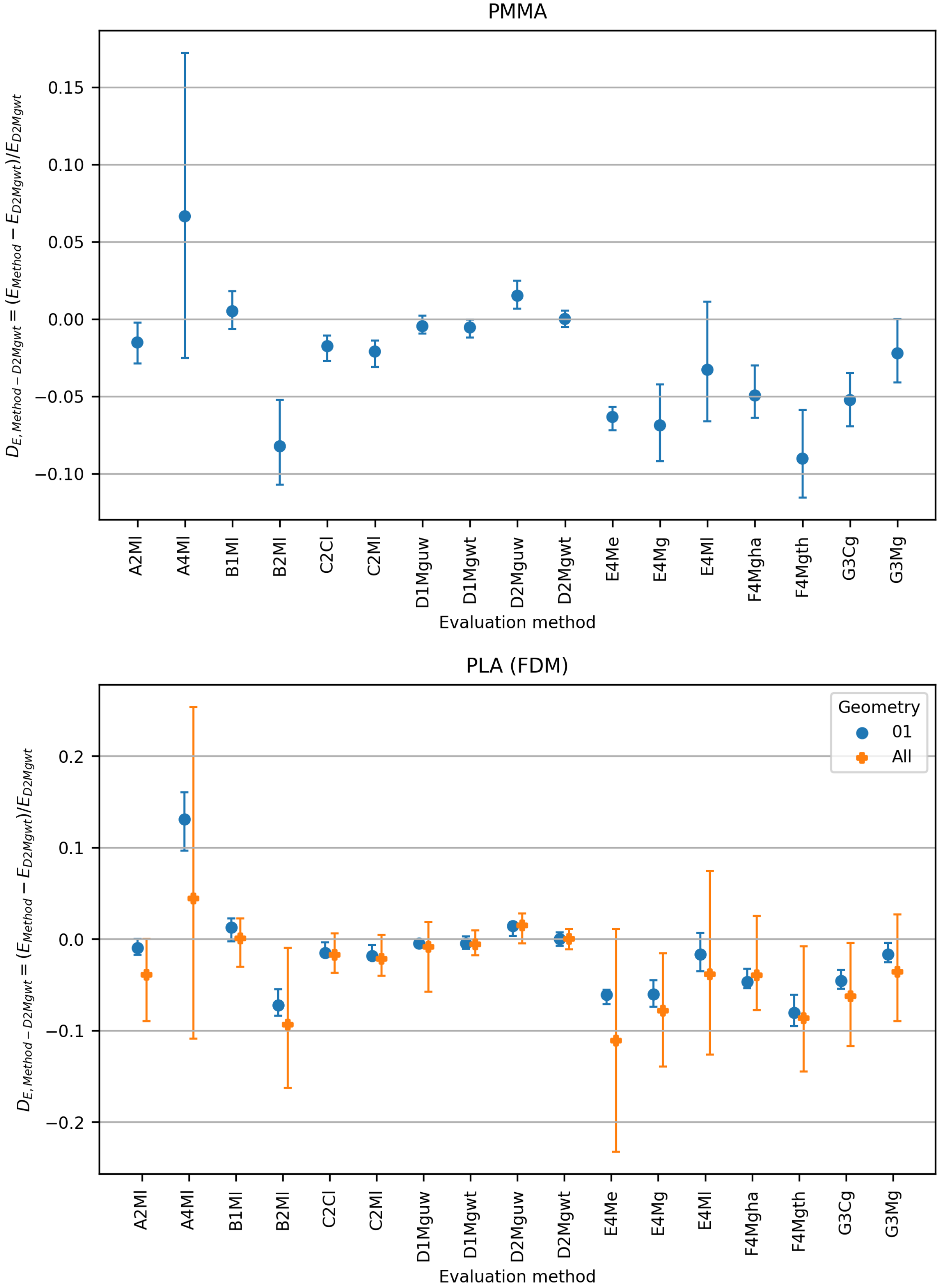
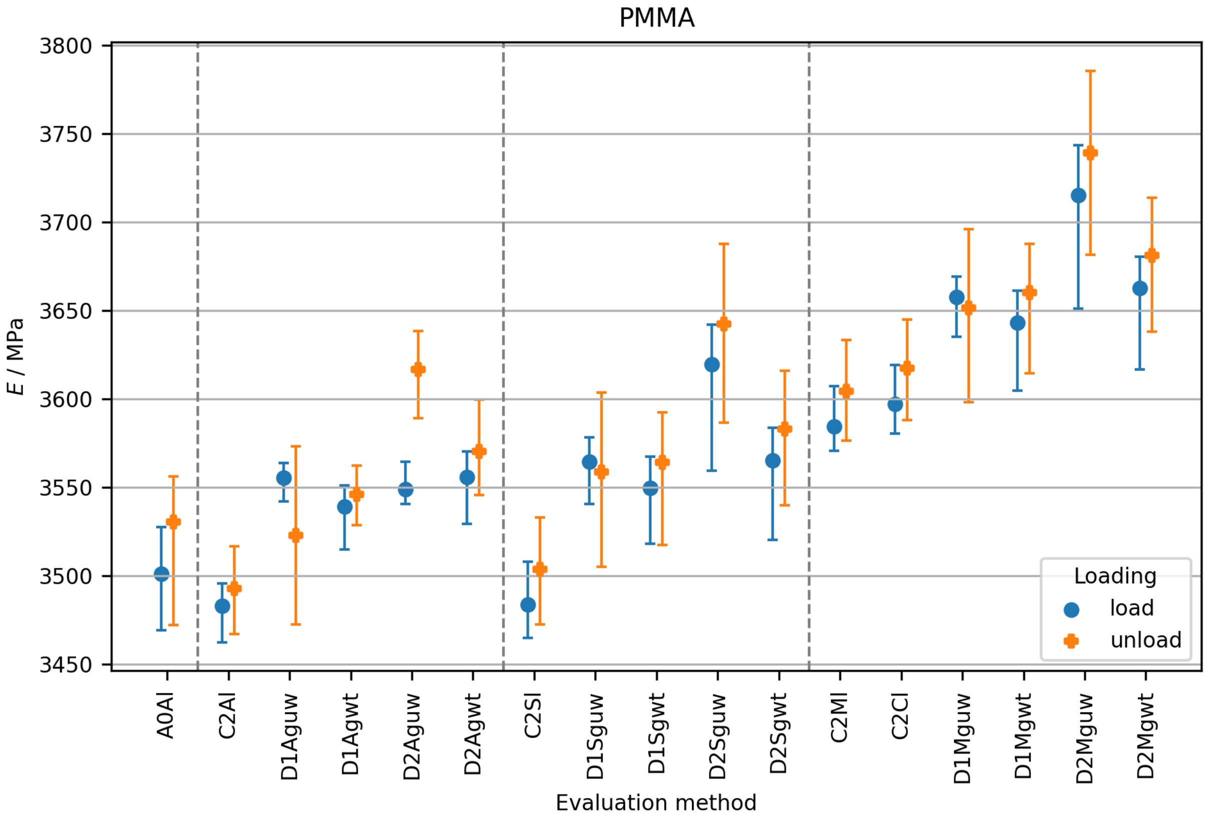


Figure 9: Verification tests – Relative deviation of the evaluation methods eliminating all falsifying influences (**M*) or correctly considering the shear deformation (**C*) from the preferred Method D2Mgwt; mean values with error bar showing minimum and maximum values.



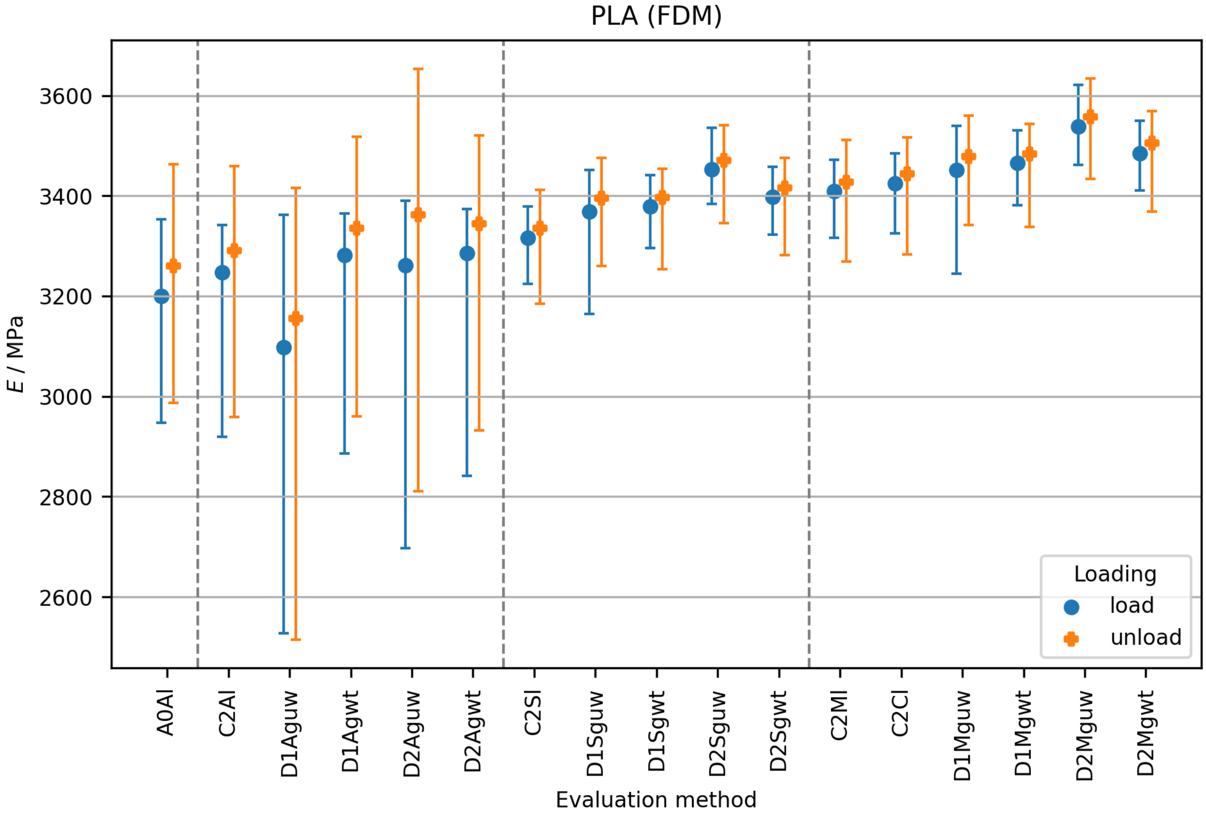
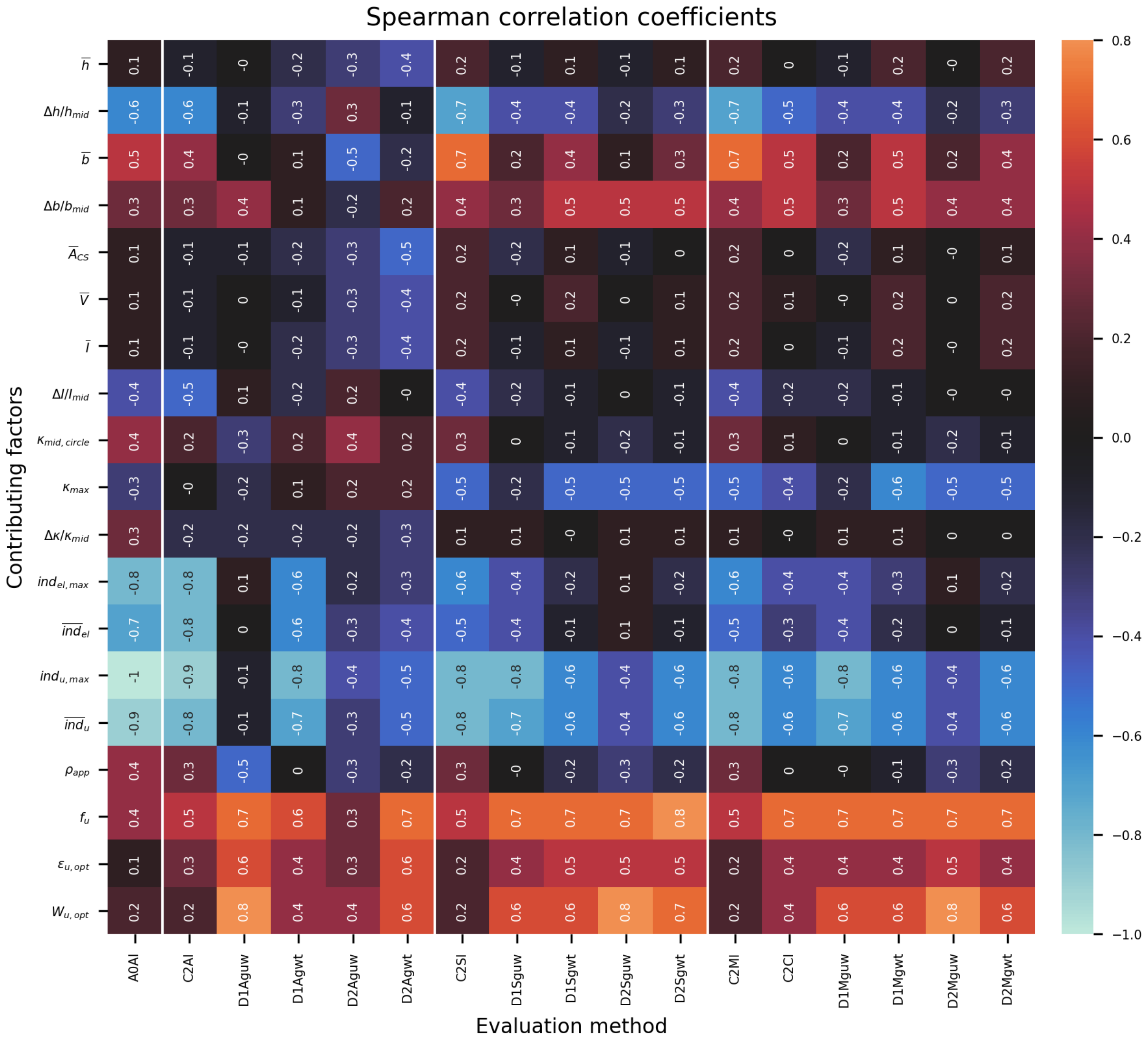


Figure 10: Verification tests - Selected moduli of elasticity for different materials and loading situations; mean values with error bar showing minimum and maximum values.



Symbols

h - Thickness
b - Width
A_{CS} - Cross-section area
V - Volume
I - Second moment of inertia
κ - Geometrical curvature
ind - Support indentation
ρ_{app} - Apparent density
f - Strength
ε - Strain
W - External work

$\overline{X}$ - Arithmetic average
ΔX - Difference between max. and min.
X_{mid} - Midspan position
X_{circle} - Approximated as circle
X_{el} - Elastic range
X_y - Yield
X_u - Ultimate
X_{opt} - Determination via DIC

Figure 11: Verification tests - Influences on modulus of elasticity obtained by selected evaluation methods for PLA specimens, in loading range only.

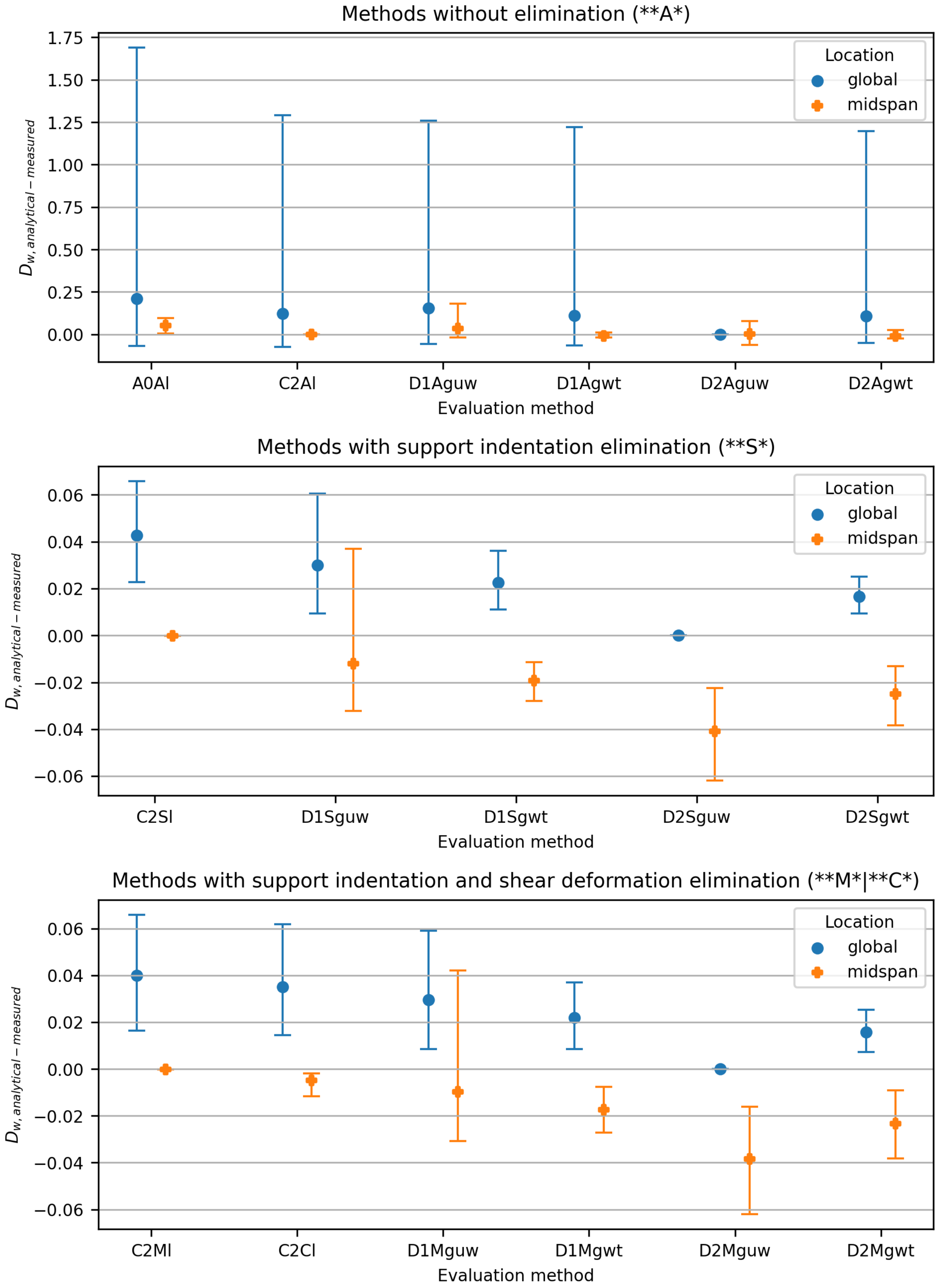


Figure 12: Verification tests - Relative deviation of the analytical elastic curve from the measured curve, where the analytical elastic curve was calculated for the modulus of elasticity obtained by different evaluation methods in loading range for the PLA specimens; mean values with error bar showing minimum and maximum values.

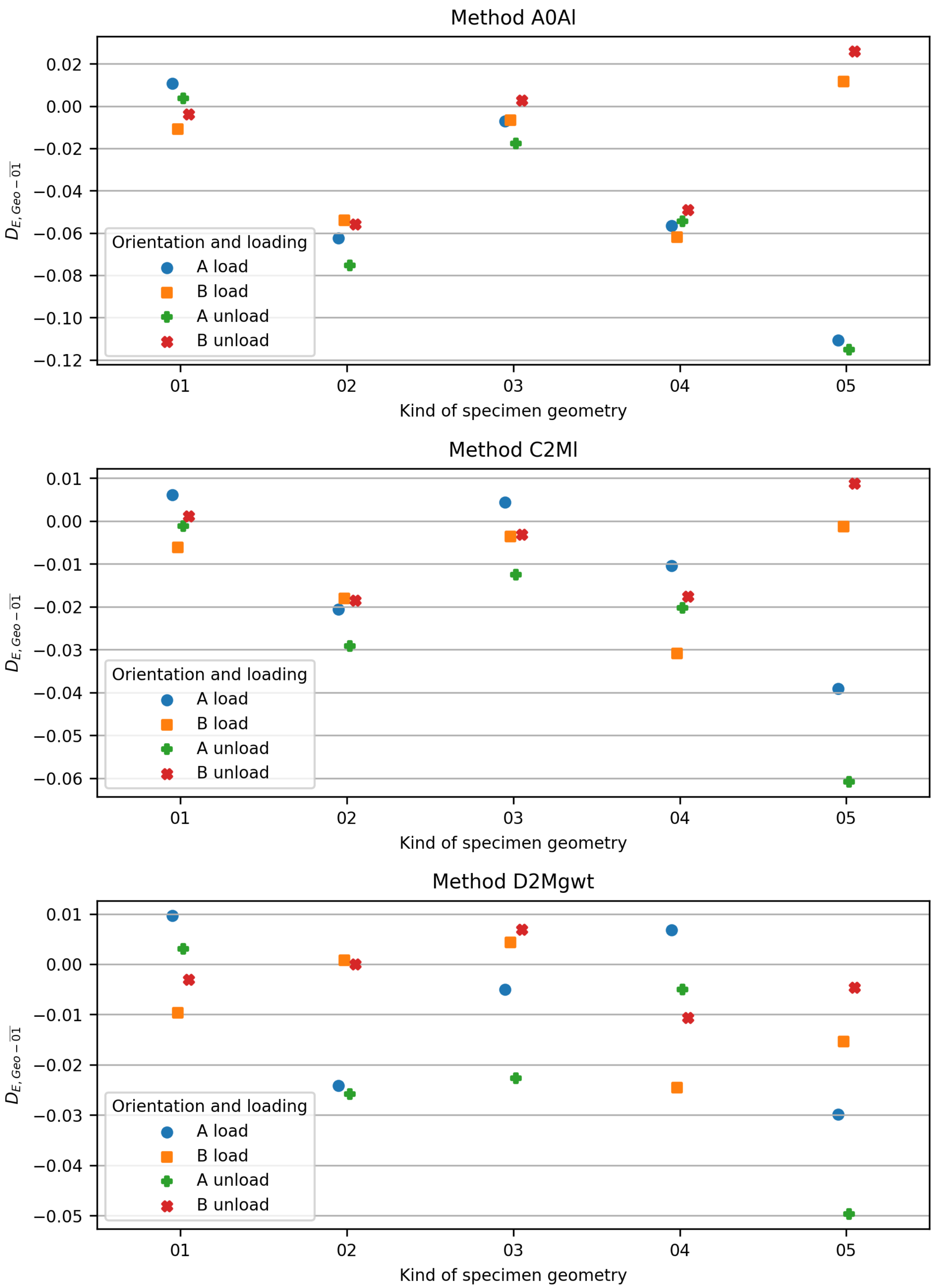


Figure 13: Verification tests - Relative deviation of the moduli of elasticity for the PLA specimen geometries from the mean values for the rectangular cuboid beam (01).

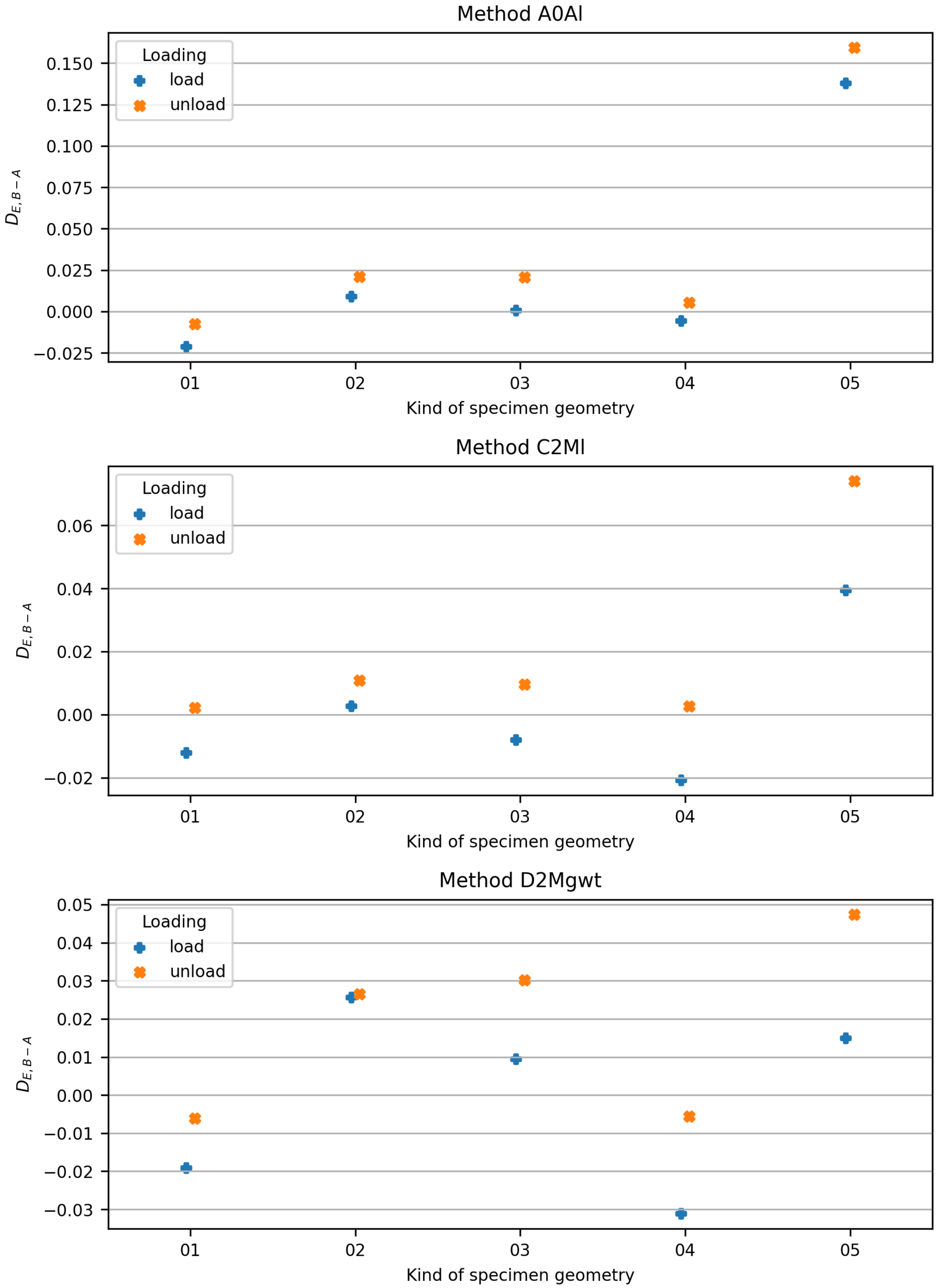


Figure 14: Verification tests - Relative deviation of the moduli of elasticity for the PLA specimen geometries in test position B from the one in test position A.