Nonlocal Timoshenko Beam for Vibrations of Magnetically Affected Inclined Single-Walled Carbon Nanotubes as Nanofluidic Conveyors

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This work displays both longitudinal and transverse vibrations of magnetically affected inclined single-walled carbon nanotubes for conveying fluid flow. By employing an equivalent continuum structure on the basis of the nonlocal Timoshenko beam model as well as plug-like model for the nanofluidic flow inside the pore, the nonlocal governing equations are obtained accounting for nonlocality, frictionless nature of the inside wall, the Knudsen number, and full longitudinal and transverse interactions of the fluid flow with the single-walled carbon nanotubes. By implementing Galerkin-based assumed mode method, the equations of motion are discretized appropriately and then solved for the unknown dynamical deformation fields. The roles of nanofluidic flow velocity, small-scale parameter, inclination angle of single-walled carbon nanotube, and magnetic field strength on maximum values of longitudinal and transverse displacements are explained and discussed. The results show that the maximum dynamic deflection of the inclined nanotube would lessen by increase of the magnetic field strength. This fact is also more apparent for higher levels of fluid flow velocity. Additionally, variation of the longitudinal magnetic field has a trivial influence on the variation of longitudinal displacement. The predicted results reveal that application of the longitudinal magnetic field could be used as an efficient methodology to reduce the lateral vibrations of single-walled carbon nanotubes as nanofluidic conveyors.

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1. Introduction

Realizing and controlling of vibrations of carbon nanotubes (CNTs) and carbon pipes for conveying fluid flow is currently a subject of huge interest among scientists of various disciplines. Actually, these tiny pore-like structures have been recognized as efficient tools for fast transport of waters [1–4]. It is mainly attributed to the frictionless nature and smoothness of the CNT surface. By increase of the flow velocity, the lateral stiffness of nanostructure would be generally reduced due to the centrifugal effect of the fluid flow. Additionally, the maximum dynamic deformations and stresses within the nanostructure would increase with the velocity of the moving inside flow because of a combination of lateral, Coriolis, and centrifugal acceleration effects. As a result, if the stiffness of CNT-elements could be improved efficiently, their vibrations would be also controlled. It implies that the capability of the nanostructure as well as nanosystems with CNTs would be enhanced from mechanical points of view.

There exist several experimental works that display enhancement of mechanical properties of CNT-based composites by application of magnetic fields [5, 6]. This fact is the chief motivation of this study. Here we are focusing in developing a mathematical model to investigate vibrations of fluid-conveying single-walled carbon annotates (SWCNTs) by taking their full advantage through application of magnetic field. Previous investigations have been confirmed this fact theoretically that the transverse dynamic behavior of CNTs would be enhanced by exploiting a longitudinal magnetic field [7–13]; however, the role of magnetic field on vibrations of SWCNTs as nanofluidic conveyors has not been explored yet.

At the small scale, vibrations of each atom rely on vibrations of its neighboring atoms. This truth cannot be displayed by the classical elasticity theory (CET). To improve this deficiency, such a theory is elegantly generalized by Eringen [14–17] through the so-called nonlocal continuum theory (NCT). From elasticity points of view, the main feature of this theory is that the stress of each point does not only depend on the state of stress of that point but also on the state of stresses at its neighboring points. To date, there exist a large body of researches on nonlocal dynamics of nanoscale structures (i.e., rod-like, beam-like, plate-like, and shell-like nanostructures) including free vibration [18–24], forced vibrations [25–27], and nonlinear vibrations [28–31]. Further, vibrations of fluids-conveying SWCNTs has been widely researched by the NCT of Eringen from various aspects including free transverse vibration and dynamic instability [32–36] as well as longitudinal and transverse forced vibrations [37, 38]. In most of these works, the dynamical bending response of nanofluid-conveying CNTs — the most crucial deformation of such loaded nanostructures — has been modeled using nonlocal beam theories.

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In the present work, nonlocal vibrations of magnetically affected inclined SWCNTs conveying nanofluidic flow are of high interest. By employing nonlocal Timoshenko beam model for SWCNTs and plug-like model for moving inside nanofluidic flow, the governing equations pertinent to longitudinal and transverse vibrations of SWCNTs are obtained. To evaluate the elastodynamic fields of interest, the Galerkin-based assumed mode method is implemented and the resulted set of ordinary differential equations is solved using Newmark-β approach. The roles of influential factors on both longitudinal and transverse dynamic displacements are scrutinized and discussed.

2. Development of a nonlocal continuum-based model

2.1. Description of the nanomechanical model

Consider an inclined fluid-conveying SWCNT with inclination angle α that is subjected to a longitudinal magnetic field of strength H_x, and a plug-like fluid starts to move from the bottom support such that the location of its front would be $x_f = ct$ where c denotes the mean velocity of the fluid flow. The SWCNT is simulated by a beam-like element whose density, cross-sectional area, moment inertia, and elastic modulus are denoted by ρ_b, A_b, I_b, and E_b, respectively. This is commonly called equivalent continuum structure (ECS). The ECS is a hollow cylinder whose mean radius, r_m, and length, l_b, are exactly similar to those of the parent nanotube and the thickness of its wall, t_b, is about 0.34 nm. Herein, we are concerned with examining longitudinal and transverse vibrations of SWCNTs conveying fluid flow in the presence of longitudinal magnetic field. To this end, a nonlocal continuum-based model will be presented in the next part and the nonlocal elastic fields of the magnetically affected SWCNT used for conveying fluid flow are numerically calculated.

2.2. Basic formulations

Using the Timoshenko beam model [39, 40], the kinetic energy for vibrated SWCNT, $T(t)$, the elastic strain energy, $U(t)$, and the work done by both interactional forces exerted on the inside wall and the Lorentz magnetic force on the whole body of the ECS, $W(t)$, are stated by

$$T(t) = \frac{1}{2}\int_0^{l_b} \rho_b \left(A_b\dot{u}^2 + A_b\dot{w}^2 + I_b\dot{\theta}^2\right) \mathrm{d}x, \tag{1a}$$

$$U(t) = \frac{1}{2}\int_0^{l_b} \Big(N_b^{nl}u_{,x} - \theta_{,x}M_b^{nl} + (w_{,x}-\theta)\,Q_b^{nl}\Big)\,\mathrm{d}x, \tag{1b}$$

$$W(t) = \int_0^{l_b} \Big(\left(f_t - f_n w_{,x}\right)u + \left(f_t w_{,x} + f_n + f_z\right)w\Big)\,\mathrm{d}x, \tag{1c}$$

where the density, cross-sectional area, and moment inertia of the fluid flow are ρ_f, A_f, and I_f, respectively, N_b^{nl}, Q_b^{nl}, and M_b^{nl} in order denote the nonlocal longitudinal force, nonlocal resultant shear force, and the nonlocal bending moment within the SWCNT, f_t and f_n represent the longitudinal and transverse interactional forces between the fluid flow and the SWCNT per unit length of the inclined ECS, $f_z = \eta A_b H_x^2 w_{,xx}$ displays the applied transverse Lorentz force per unit length of the inclined SWCNT, and η is the magnetic permeability of the SWCNT. Through exploitation of thr Hamilton principle, the longitudinal and transverse equations of motion of the magnetically affected SWCNT conveying fluid flow are obtained as

$$\rho_b A_b\ddot{u} - N_{b,x}^{nl} - f_t + f_n\theta = 0, \tag{2a}$$

$$\rho_b I_b\ddot{\theta} - Q_b^{nl} + M_{b,x}^{nl} = 0, \tag{2b}$$

$$\rho_b A_b\ddot{w} - Q_{b,x}^{nl} - f_n - f_t\theta = f_{z'}. \tag{2c}$$

By considering an infinitesimal element of the fluid flow inside the ECS modeled based on the Timoshenko beam theory, the following equations of motion are readily obtainable:

$$(PA_f)_{,x} + f_t - f_n\theta + \rho_f A_f\left(g_x - g_z\theta + \ddot{u} + 2c\dot{u}_{,x} + c^2u_{,xx}\right) = 0, \tag{3a}$$

$$(PA_f\theta)_{,x} + f_t\theta + f_n + \rho_f A_f\left(-g_z - g_x\theta + \ddot{w} + 2c\dot{w}_{,x} + c^2w_{,xx}\right) = 0, \tag{3b}$$

where P is the nanofluidic pressure, $g_x = g\sin\alpha$, $g_z = g\cos\alpha$, and g is the gravitational acceleration. By considering the slip boundary condition, the exerted longitudinal force on the inner wall of the ECS could be written as [37, 38]:

$$f_t = \eta_{f0}K_t\left(c - u_{,t}\right), \tag{4a}$$

$$K_t = \frac{8\pi\sigma_v\left(1-b\mathrm{Kn}\right)}{\left(1+a_f\mathrm{Kn}\right)\left(\sigma_v(1-b\mathrm{Kn}) + 4(2-\sigma_v)\mathrm{Kn}\right)}, \tag{4b}$$

where η_{f0} denotes the fluid bulk viscosity, σ_v is the coefficient of tangential moment accomodation, $a_f = \frac{2a_0}{\pi}\tan^{-1}(a_1\mathrm{Kn}^B)$ in which Kn is the Knudsen number, $a_0 = \frac{64b}{3\pi(b-4)}$, $b = -1$, $a_1 = 4$, and $B = 0.4$ [41].

Through introducing Eqs. (3a) and (3b) to Eqs. (2a)-(2c) in view of Eq. (4a) and assuming a plug-like nanofluid flow from the left end of the SWCNT to the right end, the governing equations of the inclined SWCNT conveying fluid flow under longitudinal magnetic field take the following form:

$$\rho_b A_b\ddot{u} - N_{b,x}^{nl} + \rho_f A_f g_z\theta(1-H(x-x_f)) = \eta_{f0}\Lambda_f^2 c(1-H(x-x_f)), \tag{5a}$$

$$\rho_b I_b\ddot{\theta} + \rho_f I_f\left(\ddot{\theta} + 2c\dot{\theta}_{,x} + c^2\theta_{,xx}\right)(1-H(x-x_f)) - Q_b^{nl} + M_{b,x}^{nl} = 0, \tag{5b}$$

$$\begin{aligned}&\rho_b A_b\ddot{w} - Q_{b,x}^{nl} - \eta A_b H_x^2 w_{,xx} + PA_f\theta_{,x} + \rho_f A_f\left(\ddot{w} + 2c\dot{w}_{,x} + c^2w_{,xx}\right)(1-H(x-x_f))\\ &- \left(2\rho_f A_f g_x + \eta_{f0}\Lambda_f c\right)\theta(1-H(x-x_f)) = \rho_f A_f g_z(1-H(x-x_f)),\end{aligned} \tag{5c}$$

where H is the Heaviside function. The nonlocal forces of the ECS modeled by the Timoshenko beam model are linked to their corresponding local forces as follows:

$$N_b^{nl} - (e_0a)^2 N_{b,xx}^{nl} = E_b A_b u_{,x}, \tag{6a}$$

$$Q_b^{nl} - (e_0a)^2 Q_{b,xx}^{nl} = k_s G_b A_b \left(w_{,x} - \theta\right), \tag{6b}$$

$$M_b^{nl} - (e_0a)^2 M_{b,xx}^{nl} = -E_b I_b \theta_{,x}, \tag{6c}$$

in which e_0a is the small-scale factor. By mixing Eqs. (5a)-(5c) and Eqs. (6a)-(6c) through considering the following dimensionless quantities:

$$\xi = \frac{x}{l_b},\ \overline{u} = \frac{u}{l_b},\ \overline{w} = \frac{w}{l_b},\ \mu = \frac{e_0a}{l_b},\ \overline{\theta} = \theta,$$
$$\tau = \frac{1}{l_b}\sqrt{\frac{k_sG_b}{\rho_b}}t,\ \eta = \frac{E_bI_b}{k_sG_bA_bl_b^2},\ \lambda = \frac{l_b}{r_b},$$
$$\overline{P} = \frac{PA_f}{k_sG_bA_b},\ \beta = \frac{c}{C_T},\ \overline{J}_f = \frac{\rho_fI_f}{\rho_bI_b},\ \xi_f = \beta\tau,$$
$$\gamma_m = \frac{\sqrt{g_m l_b}}{C_T};\ m = x \text{ or } z,\ \overline{H}_x = H_x\sqrt{\frac{\eta}{k_sG_b}}, \tag{7}$$
$$\overline{m}_f = \frac{\rho_fA_f}{\rho_bA_b},\ \overline{\eta}_{f0} = \frac{\eta_{f0}l_b}{C_L\rho_fA_f},\ \lambda_f = \frac{l_b}{r_f},\ \beta = \frac{c}{C_T},$$

where $r_f = \sqrt{I_f/A_f}$, $r_b = \sqrt{I_b/A_b}$, $C_T = \sqrt{k_sG_b/\rho_b}$, and $C_L = \sqrt{E_b/\rho_b}$, the dimensionless equations of motion of the inclined SWCNT acted upon by both longitudinal magnetic field and nanofluidic flow in terms of deformation fields take the following form:

$$\overline{u}_{,\tau\tau} - \mu^2\overline{u}_{,\tau\tau\xi\xi} + \overline{m}_f\overline{\eta}_{f0}\Lambda_f^2(C_L/C_T) \tag{8a}$$
$$\times\left(\overline{u}_{,\tau}(1-H(\xi-\xi_f)) - \mu^2\left(\overline{u}_{,\tau}(1-H(\xi-\xi_f))\right)_{,\xi\xi}\right)$$
$$+\overline{m}_f\gamma_z^2\left(\overline{\theta}(1-H(\xi-\xi_f)) - \mu^2\left(\overline{\theta}(1-H(\xi-\xi_f))\right)_{,\xi\xi}\right)$$
$$-(C_L/C_T)^2\overline{u}_{,\xi\xi} = \overline{m}_f\overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)$$
$$\times\beta\left((1-H(\xi-\xi_f)) - \mu^2(1-H(\xi-\xi_f))_{,\xi\xi}\right),$$

$$\overline{\theta}_{,\tau\tau} - \mu^2\overline{\theta}_{,\tau\tau\xi\xi} + \overline{J}_f\Big(\left(\overline{\theta}_{,\tau\tau} + 2\beta\overline{\theta}_{,\xi\tau} + \beta^2\overline{\theta}_{,\xi\xi}\right) \tag{8b}$$
$$\times(1-H(\xi-\xi_f)) - \mu^2\Big(\left(\overline{\theta}_{,\tau\tau} + 2\beta\overline{\theta}_{,\xi\tau} + \beta^2\overline{\theta}_{,\xi\xi}\right)$$
$$\times(1-H(\xi-\xi_f))\Big)_{,\xi\xi}\Big) - \lambda^2\overline{w}_{,\xi} + \lambda^2\left(\overline{\theta} - \eta\overline{\theta}_{,\xi\xi}\right) = 0,$$

$$\overline{w}_{,\tau\tau} - \mu^2\overline{w}_{,\tau\tau\xi\xi} - \overline{w}_{,\xi\xi} + \overline{\theta}_{,\xi} \tag{8c}$$
$$+\overline{m}_f\Big(\left(\overline{w}_{,\tau\tau} + 2\beta\overline{w}_{,\tau\xi} + \beta^2\overline{w}_{,\xi\xi}\right)(1-H(\xi-\xi_f))$$
$$-\mu^2\left(\left(\overline{w}_{,\tau\tau} + 2\beta\overline{w}_{,\tau\xi} + \beta^2\overline{w}_{,\xi\xi}\right)(1-H(\xi-\xi_f))\right)_{,\xi\xi}\Big)$$
$$-\overline{m}_f\left(2\gamma_x^2 + \overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\beta\right)\Big(\overline{\theta}(1-H(\xi-\xi_f))$$
$$-\mu^2\left(\overline{\theta}(1-H(\xi-\xi_f))\right)_{,\xi\xi}\Big) - \overline{H}_x^2(\overline{w}_{,\xi\xi} - \mu^2\overline{w}_{,\xi\xi\xi\xi})$$
$$+\overline{P}\overline{\theta}_{,\xi} - \mu^2\left(\overline{P}_{,\xi\xi}\overline{\theta}_{,\xi} + 2\overline{P}_{,\xi}\overline{\theta}_{,\xi\xi} + \overline{P}\overline{\theta}_{,\xi\xi\xi}\right) =$$
$$\overline{m}_f\gamma_z^2\left(1-H(\xi-\xi_f) - \mu^2\left(1-H(\xi-\xi_f)\right)_{,\xi\xi}\right).$$

Furthermore, the nonlocal axial force, shear force, and bending moment of the SWCNT in terms of the deformation fields are given by

$$N_b^{nl} = k_sG_bA_b\Big\{(C_L/C_T)^2\overline{u}_{,\xi} + \mu^2\big(\overline{u}_{,\tau\tau} + \overline{m}_f\left(\gamma_z\right)^2$$
$$\times\overline{\theta}\left(1-H(\xi-\xi_f)\right) - \overline{m}_f\overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\left(\beta-\overline{u}_{,\tau}\right)$$
$$\times(1-H(x-x_f))\big)_{,\xi}\Big\}, \tag{9a}$$

$$\overline{Q}_b^{nl} = k_sG_bA_b\Big\{\overline{w}_{,\xi} - \overline{\theta} + \mu^2\big(\overline{w}_{,\tau\tau} - \overline{H}_x^2\overline{w}_{,\xi\xi} + \overline{P}\overline{\theta}_{,\xi}$$
$$-\overline{m}_f\left(2\left(\gamma_x\right)^2 + \overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\beta\right)\overline{\theta}\left(1-H(\xi-\xi_f)\right)$$
$$+\overline{m}_f\left(\overline{w}_{,\tau\tau} + 2\beta\overline{w}_{,\tau\xi} + \left(\beta\right)^2\overline{w}_{,\xi\xi}\right)\left(1-H(\xi-\xi_f)\right)$$
$$-\overline{m}_f\left(\gamma_z\right)^2\left(1-H(\xi-\xi_f)\right)\big)_{,\xi}\Big\}, \tag{9b}$$

$$\overline{M}_b^{nl} = k_sG_bA_bl_b\Big\{-\eta\overline{\theta}_{,\xi} + \mu^2\big(\overline{w}_{,\tau\tau} - \overline{H}_x^2\overline{w}_{,\xi\xi} + \overline{P}\overline{\theta}_{,\xi}$$
$$-\overline{m}_f\left(2\left(\gamma_x\right)^2 + \overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\left(\beta-\overline{u}_{,\tau}\right)\right)$$
$$\times\overline{\theta}\left(1-H(\xi-\xi_f)\right) + \overline{m}_f\left(\overline{w}_{,\tau\tau} + 2\beta\overline{w}_{,\tau\xi} + \left(\beta\right)^2\overline{w}_{,\xi\xi}\right)$$
$$\times\left(1-H(\xi-\xi_f)\right) - \lambda^{-2}\overline{\theta}_{,\tau\tau\xi}$$
$$+\lambda^{-2}\overline{J}_f\left(\overline{\theta}_{,\tau\tau} + 2\beta\overline{\theta}_{,\tau\xi} + \left(\beta\right)^2\overline{\theta}_{,\xi\xi}\right)\left(1-H(\xi-\xi_f)\right)$$
$$-\overline{m}_f\left(\gamma_z\right)^2\left(1-H(\xi-\xi_f)\right)\big)\Big\}. \tag{9c}$$

2.3. Numerical analysis via Galerkin-based assumed mode method

Seeking analytical solutions for the developed nonlocal governing equations in the previous part is not an easy task. It is chiefly related to the existing coupling between longitudinal and transverse equations of motion. In this section, the Galerkin approach in conjunction with assumed mode method is employed. To this end, both sides of Eqs. (8a)–(8c) are multiplied by $\delta\overline{u}$, $\delta\overline{\theta}$, and $\delta\overline{w}$, respectively, and then the resulting expressions are integrated over [0,1]. As a result,

$$\int_0^1\Big\{\delta\overline{u}\overline{u}_{,\tau\tau} + \mu^2\delta\overline{u}_{,\xi}\overline{u}_{,\xi\tau\tau} + \overline{m}_f\gamma_z^2\left(\delta\overline{u} - \mu^2\delta\overline{u}_{,\xi\xi}\right)$$
$$\times\overline{\theta}(1-H(x-x_f)) + (C_L/C_T)^2\delta\overline{u}_{,\xi}\overline{u}_{,\xi}$$
$$-\overline{m}_f\overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\left(\beta-\overline{u}_{,\tau}\right)\left(\delta\overline{u} - \mu^2\delta\overline{u}_{,\xi\xi}\right)$$
$$\times(1-H(\xi-\xi_f)) + \lambda^{-2}\left(\delta\overline{\theta}\overline{\theta}_{,\tau\tau} + \mu^2\delta\overline{\theta}_{,\xi}\overline{\theta}_{,\tau\tau\xi}\right)$$
$$+\lambda^{-2}\overline{J}_f\left(\delta\overline{\theta} - \mu^2\delta\overline{\theta}_{,\xi\xi}\right)\left(\overline{\theta}_{,\tau\tau} + 2\beta\overline{\theta}_{,\tau\xi} + \beta^2\overline{\theta}_{,\xi\xi}\right)$$
$$\times\left(1-H(\xi-\xi_f)\right) - \delta\overline{\theta}\left(\overline{w}_{,\xi} - \overline{\theta}\right) + \eta\delta\overline{\theta}_{,\xi}\overline{\theta}_{,\xi} + \delta\overline{w}\overline{w}_{,\tau\tau}$$
$$+\mu^2\delta\overline{w}_{,\xi}\overline{w}_{,\xi} + \delta\overline{w}_{,\xi}\left(\overline{w}_{,\xi} - \overline{\theta}\right) + \overline{m}_f\left(\delta\overline{w} - \mu^2\delta\overline{w}_{,\xi\xi}\right)$$
$$\times\left(\overline{w}_{,\tau\tau} + 2\beta\overline{w}_{,\xi\tau} + \left(\beta\right)^2\overline{w}_{,\xi\xi}\right)\left(1-H(\xi-\xi_f)\right)$$
$$+\overline{H}_x^2\left(\delta\overline{w}_{,\xi}\overline{w}_{,\xi} + \mu^2\delta\overline{w}_{,\xi\xi}\overline{w}_{,\xi\xi}\right) + \mu^2\overline{P}_{,\xi}\delta\overline{w}_{,\xi}\overline{\theta}_{,\xi}$$
$$+\overline{P}\left(\delta\overline{w}\overline{\theta}_{,\xi} + \mu^2\delta\overline{w}_{,\xi}\overline{\theta}_{,\xi\xi}\right) - \overline{m}_f(2\gamma_x^2$$
$$+\overline{\eta}_{f0}\Lambda_f^2(C_L/C_T)\beta)\left(\delta\overline{w} - \mu^2\delta\overline{w}_{,\xi\xi}\right)\overline{\theta}\left(1-H(\xi-\xi_f)\right)$$
$$-\overline{m}_f\gamma_z^2\left(\delta\overline{w} - \mu^2\delta\overline{w}_{,\xi\xi}\right)\left(1-H(\xi-\xi_f)\right)\Big\}\mathrm{d}\xi = 0. \tag{10}$$

The unknown dimensionless deformation fields can be expressed in terms of admissible mode shapes as follows:

$$\overline{u}(\xi,\tau)=\sum_{i=1}^{N_m}\phi_i^u(\xi)\overline{u}_i(\tau),\ \overline{\theta}(\xi,\tau)=\sum_{i=1}^{N_m}\phi_i^\theta(\xi)\overline{\theta}_i(\tau),$$
$$\overline{w}(\xi,\tau)=\sum_{i=1}^{N_m}\phi_i^w(\xi)\overline{w}_i(\tau), \tag{11}$$

where N_m is the number of vibrational modes, $\overline{u}_i(\tau)$, $\overline{\theta}_i(\tau)$ and $\overline{w}_i(\tau)$ denote the time-dependent parameters associated with the i-th mode of longitudinal displacement, i-th mode of angle of deflection, and i-th mode of deflection, respectively. By introducing Eq. (11) to Eq. (10), it is obtainable

$$\begin{bmatrix}\overline{M}_b^{uu} & \overline{M}_b^{u\theta} & \overline{M}_b^{uw}\\ \overline{M}_b^{\theta u} & \overline{M}_b^{\theta\theta} & \overline{M}_b^{\theta w}\\ \overline{M}_b^{wu} & \overline{M}_b^{w\theta} & \overline{M}_b^{ww}\end{bmatrix}\begin{Bmatrix}\overline{u}_{,\tau\tau}\\ \overline{\Theta}_{,\tau\tau}\\ \overline{w}_{,\tau\tau}\end{Bmatrix}+\begin{bmatrix}\overline{C}_b^{uu} & \overline{C}_b^{u\theta} & \overline{C}_b^{uw}\\ \overline{C}_b^{\theta u} & \overline{C}_b^{\theta\theta} & \overline{C}_b^{\theta w}\\ \overline{C}_b^{wu} & \overline{C}_b^{w\theta} & \overline{C}_b^{ww}\end{bmatrix}\begin{Bmatrix}\overline{u}_{,\tau}\\ \overline{\Theta}_{,\tau}\\ \overline{w}_{,\tau}\end{Bmatrix}+\begin{bmatrix}\overline{K}_b^{uu} & \overline{K}_b^{u\theta} & \overline{K}_b^{uw}\\ \overline{K}_b^{\theta u} & \overline{K}_b^{\theta\theta} & \overline{K}_b^{\theta w}\\ \overline{K}_b^{wu} & \overline{K}_b^{w\theta} & \overline{K}_b^{ww}\end{bmatrix}\begin{Bmatrix}\overline{u}\\ \overline{\Theta}\\ \overline{w}\end{Bmatrix}=\begin{Bmatrix}\overline{f}_b^u\\ \overline{f}_b^\theta\\ \overline{f}_b^w\end{Bmatrix}, \tag{12}$$

where the force vectors and corresponding matrices are easily calculated. To solve the set of ordinary differential equations of Eq. (12) in the time domain, we employ generalized Newmark-β method [42].

3. Results and discussion

The geometrical and mechanical properties of the ECS are taken into account as follows: $E_b = 10^{12}$ Pa, $\rho_b = 2300$ kg/m^3, $\nu = 0.2$, and $r_m = 2$ nm. The inside moving nanofluid flow has the following properties: $\rho_f = 1000$ kg/m^3, $\eta_{f0} = 1.002\times10^{-3}$ Pa s, $\sigma_v = 0.8$, and l_f=0.3 nm. To more facilitate in vibrational analysis of the problem at hand, we consider the following dimensionless and normalized parameters: $\overline{H}_x^* = H_x\sqrt{\frac{\eta A_b l_b^2}{E_b I_b}}$, $u_N(\xi,\tau) = 8\overline{u}(\xi,\tau)/(\overline{m}_f\overline{\eta}_{f0}\Lambda_f^2\beta)$, $w_N(\xi,\tau) = 384\overline{w}(\xi,\tau)E_bI_b/(5\rho_fA_fgl_b^3)$, $N_{bN}(\xi,\tau) = 2\overline{N}_b(\xi,\tau)/(E_bA_b\overline{m}_f\overline{\eta}_{f0}\Lambda_f^2\beta)$, $M_{bN}(\xi,\tau) = 8\overline{M}_b(\xi,\tau)/(\rho_fA_fgl_b^2)$, and $c_N = c\lambda/(\pi\sqrt{E_b/\rho_b})$. Throughout this paper, the nanotube is assumed to be initially at rest and the results are presented for SWCNTs with simple and immovable ends in which its boundary conditions read exactly: $\overline{u}(0,\tau)=0$, $\overline{u}(1,\tau)=0$, $\overline{w}(0,\tau)=0$, $\overline{w}(1,\tau)=0$, and $\overline{\theta}_{,\xi}(0,\tau)=0$, $\overline{\theta}_{,\xi}(1,\tau)=0$. For these conditions, we exploit the following mode shape functions: $\phi_i^u(\xi)=\sin(i\pi\xi)$, $\phi_i^\theta(\xi)=\cos(i\pi\xi)$, $\phi_i^w(\xi)=\sin(i\pi\xi)$.

In Fig. 1a–c, the time history plots of normalized longitudinal and transverse displacements as well as normalized nonlocal axial force and bending moment of the midspan point of the fluid-conveying SWCNT are provided. The plotted results are given for three levels of magnetic field strength (i.e., $\overline{H}_x^*$ = 0.1, 1, and 2) and three values of the small-scale parameter (i.e., e_0a=0, 0.5, and 1 nm) for the case of $\alpha = 0$, $c_N = 0.25$, and $\lambda = 15$. As it is seen, displacements and nonlocal axial force would slightly grow by increase of the small-scale parameter. Concerning the nonlocal bending moment, the nonlocal bending moment would sharply drop as the moving fluid flow passes the midspan point. Such a fact is mainly attributed to the nonlocality effect and the plug-like assumption of the nanofluid flow. As the nanofluid's front leaves the right end of the SWCNT, the free vibration would start. More important is the decrease of both deflection and nonlocal bending moment as the strength of the magnetic field increases. However, variation of the magnetic field strength has fairly no influence on the variation of the longitudinal displacement and the nonlocal axial force. The main reason behind the first fact is the incorporation of the magnetic field strength into the transverse stiffness of the nanostructure.

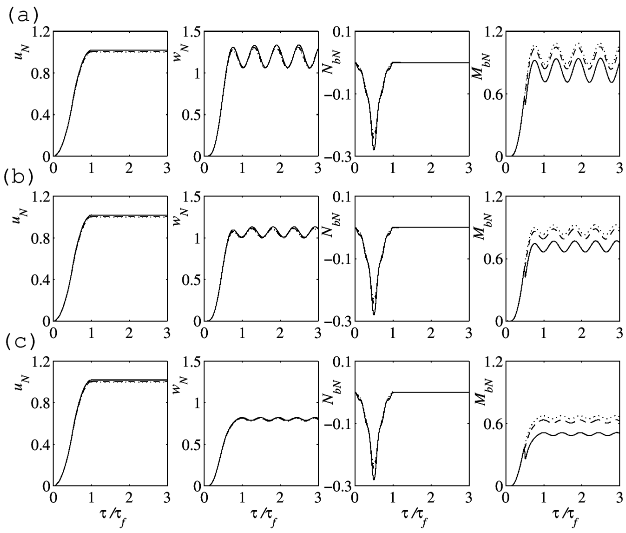

Fig. 1. Normalized longitudinal and transverse displacements, axial force, and bending moment of the midspan point of the fluid-conveying SWCNT as a function of normalized time: (a) $\overline{H}_x^* = 0.1$, (b) $\overline{H}_x^* = 1$, (c) $\overline{H}_x^* = 2$; (($\cdots$) $e_0a = 0$, ($-\ -$) $e_0a = 0.5$, (—) $e_0a = 1$ nm; $\alpha = 0$, $\lambda = 15$, $c_N = 0.25$).

To explain the role of magnetic field on the resulting crucial elastic fields within the nanotube, the plots of the maximum dynamic amplitude factors of deflection and the nonlocal bending moment, MDAF$_w$ = $\max\{w_N(\xi,\tau)\}$ and MDAF$_M$ = $\max\{M_{bN}(\xi,\tau)\}$, in terms of dimensionless strength of magnetic field have been demonstrated in Fig. 2a–c. The results are given for a SWCNT conveying nanofluidic flow of velocities c_N = 0.1, 0.2, and 0.3 in the cases of $\alpha = 10°$, $40°$, and $70°$. According to the plotted results, the maximum deflection and nonlocal bending moment would reduce by application of the longitudinal magnetic field. For nanotubes with lower inclination angles which are acted upon by nanofluidic flows of higher velocities, the influence of the magnetic field on the reduction of maximum elastic fields is more apparent.

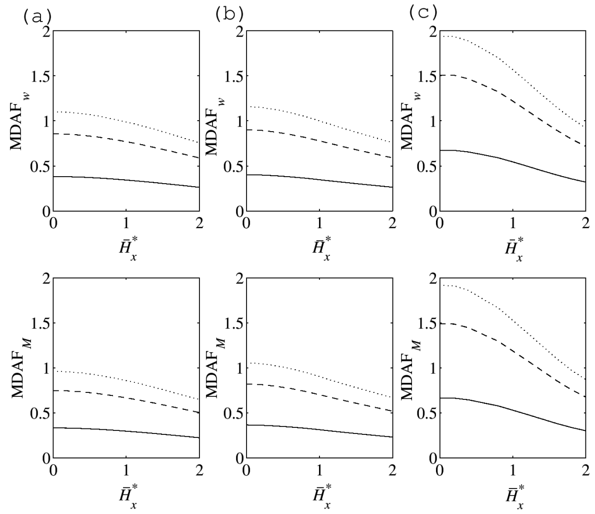

Fig. 2. Maximum dynamic amplitude factors of deflection and bending moment in terms of $\overline{H}_x^*$ for different values of fluid flow velocity and inclination angle: (a) $c_N = 0.1$, (b) $c_N = 0.2$, (c) $c_N = 0.3$; ($(\cdots)$ $\alpha = 10°$, $(-\ -)$ $\alpha = 40°$, (—) $\alpha = 70°$; λ=20, $e_0a = 1$ nm).

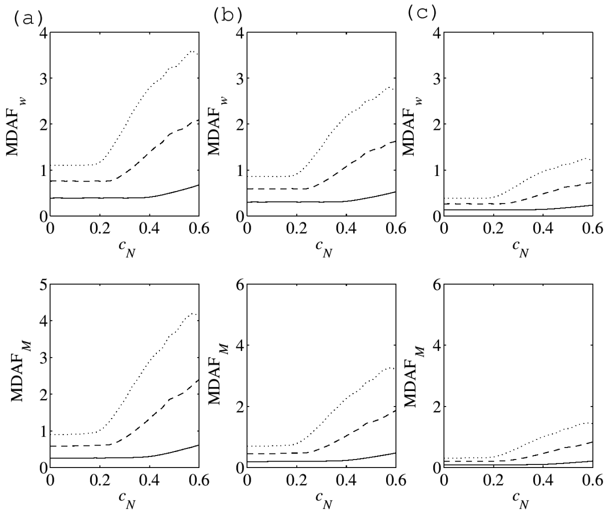

Fig. 3. Maximum dynamic amplitude factors of deflection and nonlocal bending moment in terms of c_N for different values of magnetic field strength and inclination angle: (a) $\overline{H}_x^* = 0.1$, (b) $\overline{H}_x^* = 2$, (c) $\overline{H}_x^* = 4$; ($(\cdots)$ $\alpha = 10°$, $(-\ -)$ $\alpha = 40°$, (—) $\alpha = 70°$; $\lambda = 20$, $e_0a = 1$ nm).

Figure 3a–c displays normalized maximum dynamic amplitude factors of deflection and bending moment of the fluid-conveying SWCNT as a function of normalized velocity of the nanofluidic flow. The results have been provided for three levels of magnetic field strength (i.e., $\overline{H}_x^* = 0.1$, 2, and 4) and three inclination angles (i.e., $\alpha = 10°$, 40°, and 70°). For the considered range of the moving inside fluid flow, the maximum deflections and bending moments would generally grow by increase of the velocity of fluid flow. This fact is more obvious for nanotubes with lower inclination angles. For all given levels of the fluid flow velocity, the maximum values of aforementioned elastic fields would reduce as the inclination angle decreases. Generally, the maximum values of deflections and bending moments of the SWCNT conveying nanofluids would reduce as the strength of the applied magnetic field on the nanostructure would increase. This fact holds true for all considered levels of inclination angle and fluid velocity.

4. Conclusions

Longitudinal and transverse vibrations of magnetically affected fluid-conveying SWCNTs with inclination were addressed. The equations of motion were gained using plug-like nanofluidic flow model in conjunction with the nonlocal Timoshenko beam theory. The assumed mode method was exploited for spatial discretization of deformation fields and their corresponding time-dependent parameters were evaluated based on the generalized Newmark-β time-discretization approach. The effects of crucial influential factors on maximum elastic fields were then explained and discussed in some detail. The obtained results suggest that exertion of longitudinal magnetic field can be effectively used to suppress transverse vibrations of inclined nanotubes as nanofluidic conveyors.

For more accurate prediction of elasto-dynamic fields of nanostructures, higher-order shear deformable beam theories [29, 38, 42–44] could be implemented in the context of nonlocal elasticity theory of Eringen. Additionally, vibrations of ensembles of SWCNTs or even multi-walled carbon nanotubes conveying fluid flow in the presence of longitudinal magnetic fields have not been explored yet. These are hot topics in the realm of nanomechanical science that could be researched by interested investigators in the near future.

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