# Sensorless Exact Input-Output Linearization Control of the Induction Machine, Based on Parallel Stator Resistance and Speed MRAS Observer, with a Flux Sliding Mode Observer

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**Abstract:** This paper presents an exact input-output linearization control scheme for rotor speed and rotor flux control of induction motor drive. In this scheme the motor model, is described in the fixed stator frame, and it is linearized, using exact input-output linearization technique, allowing a decoupled control of rotor flux and torque. To avoid the use of mechanical sensor, the rotor speed estimation is made by an observer using a specific MRAS (Model reference adaptive system) technique; this observer is designed to perform simultaneous estimation of stator resistance and rotor speed. In order to estimate rotor flux, a sliding mode observer is proposed in this paper. Simulation results are realized and presented to validate and to prove the effectiveness of the proposed Sensorless control.

Keywords: Input-output linearization, Induction Motor Drive, MRAS observer, Sliding Mode.

# 1. Introduction

Induction motors are widely used in industry especially in variable speed applications. An induction motor is simple in operation, rugged, maintenance free and generally less expensive than other machines.

However, its model is complicated for various reasons. The dynamic behavior of the motor is described by a fifth-order highly coupled and nonlinear dynamical system, the rotor electric variables (fluxes and currents) are practically not measured; and some of its physical parameters may vary significantly while operating the motor (stator and mainly rotor resistance, due to heating, magnetizing induction due to saturation);

Those difficulties have whetted the curiosity of scientists and researcher in laboratories, in the last few years, Evidenced by the growing number of publications that discuss the subject. Different control strategies were developed, like Field Oriented Control (FOC) proposed by Blaschke [1], and Direct Torque Control (DTC) have been extensively reported and discussed in the literature [1]-[4] to achieve a decoupled control of induction motor.

In vector control the torque and flux are decoupled by a suitable decoupling network. Then the flux component and the torque are controlled independently and respectively by stator direct-axis current and stator quadratic-axis current to control the induction motor (IM) as a separately excited DC motor.

A disadvantage of the FOC is that the method assumes that the magnitude of the rotor flux is regulated to a constant value. Therefore, the rotor speed is only asymptotically decoupled from the rotor flux. To improve the I.M control performances others technique were conceived like the sliding mode control, it is characterized by simplicity of design and attractive robustness properties. Its major drawback is the chattering phenomenon [5]-[8].

By contrast, the passivity based control [9,10], doesn't cancel all the nonlinearity but ensure system stability, but its experimental implementation is still difficult.

Backstepping control approach [11]-[15], is more recent. Its present form is due to Krstic, Kanellakopoulos and Kokotovic. This control technique offers good performance in both steady state and transient operations.

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Among technique that we find in the literature, there is the exact input-output linearization technique [16]-[23] used in this work. The technique of input-output linearization based on the differential geometry allows by a diffeomorphic transformation and a state feedback to uncouple and linearize the model put under canonical and then controlled using linear control techniques. This technique has the advantage of being able to separately control flux and torque even in mode of variation of flux. This method cancels the nonlinear terms in the plant model which fails when the physical parameters varies.

These control techniques can not guarantee good performances without the use of suitable state observer. Among the observation technique used, there are the sliding mode techniques used in this work to estimate flux, and MRAS technique, used, in particular, in Sensorless IM drives at the first time by Schauder (1992). MRAS is interesting since it leads to relatively easy to implement system with high speed adaptation [24]-[27].

In this work we propose a new structure of MRAS Observer, allowing simultaneous observation of stator resistance and rotor speed.

This article is organized into three main sections, in the first one we design the exact inputoutput linearization control, in the second one we present the flux sliding mode observer, the last one we present the design of the parallel MRAS observer of stator resistance and rotor speed.

Simulation results, given at the end, illustrate the good performance of this combination of control method and observation techniques.

#### 2. Input-Output Linearization of the Induction Machine

In order to reduce the complexity of the three phase model and then simplify the control design, an equivalent two phase representation is chosen. Under the assumptions of linearity of the magnetic circuit and neglecting iron losses, a two phase IM model in the fixed stator, reference frame ( $\alpha$ ,  $\beta$ ) can be described as:

$$\begin{cases} \frac{d\Phi_{r\alpha}}{dt} = \lambda_r . (L_m . i_{s\alpha} - \Phi_{r\alpha}) - p . \Omega . \Phi_{r\beta} \\ \frac{d\Phi_{r\beta}}{dt} = \lambda_r . (L_m . i_{s\beta} - \Phi_{r\beta}) + p . \Omega . \Phi_{r\alpha} \\ \frac{di_{s\alpha}}{dt} = -\gamma . i_{s\alpha} + k . \lambda_r . \Phi_{r\alpha} + k . p . \Omega . \Phi_{r\beta} + \delta . v_{s\alpha} \\ \frac{di_{s\beta}}{dt} = -\gamma . i_{s\beta} - k . p . \Omega . \Phi_{r\alpha} + k . \lambda_r . \Phi_{r\beta} + \delta . v_{s\beta} \\ \frac{d\Omega}{dt} = \frac{\mu}{J} . \left( \Phi_{r\alpha} . i_{s\beta} - \Phi_{r\beta} . i_{s\alpha} \right) - \frac{f}{J} . \Omega - \frac{T_L}{J} \end{cases}$$

Where:

$$\sigma = 1 - \frac{L_m^2}{L_r L_s} ; \quad k = \frac{L_m}{\sigma L_s L_r} ; \quad T_r = \frac{L_r}{R_r} ; \quad \lambda_r = \frac{R_r}{L_r} ; \quad \gamma = \frac{1}{\sigma L_s} \left( R_s + \frac{R_r L_m^2}{L_r^2} \right) \quad ; \quad \mu = p \frac{L_m}{L_r} ; \quad \beta = \frac{1}{\sigma L_s}$$

We saw that the dynamic equations of the induction machine are non-linear, what make the control difficult to conceive. However, with some transformations, the nonlinear system can be converted into the corresponding linear system. Feedback linearization is one of the approaches for the nonlinear control design [18], [19]. The fundamental idea is to apply linear control techniques for the nonlinear system. It has been used to solve a lot of practical control

#### Sensorless Exact Input-Output Linearization Control of the Induction

problems in industry by transforming a nonlinear system dynamics into a fully or partly linear one. The simplest form of feedback linearization is to cancel the nonlinearities of a nonlinear system so that the closed-loop dynamics is in a linear form.

The structure of the proposed I.M control is represented below, in Figure 1. The induction motor model (1) is put in the following form:

$$\begin{cases} \dot{x} = f(x) + g(x) \\ y = h(x) \end{cases}$$

Where:

$$\begin{aligned} x &= \left[ \Phi_{r\alpha}, \Phi_{r\beta}, i_{s\alpha}, i_{s\beta}, \Omega \right]^{T} \\ f(x) &= \begin{bmatrix} -\lambda_{r} \cdot \Phi_{r\alpha} - p \cdot \Omega \cdot \Phi_{r\beta} + \lambda_{r} \cdot L_{m} \cdot i_{s\alpha} \\ p \cdot \Omega \cdot \Phi_{r\alpha} - \lambda_{r} \cdot \Phi_{r\beta} + \lambda_{r} \cdot L_{m} \cdot i_{s\beta} \\ k \cdot \lambda_{r} \cdot \Phi_{r\alpha} + k \cdot p \cdot \Omega \cdot \Phi_{r\beta} - \gamma \cdot i_{s\alpha} \\ -k \cdot p \cdot \Omega \cdot \Phi_{r\alpha} + k \cdot \lambda_{r} \cdot \Phi_{r\beta} - \gamma \cdot i_{s\beta} \\ \frac{\mu}{J} \left( \Phi_{r\alpha} \cdot i_{s\beta} - \Phi_{r\beta} \cdot i_{s\alpha} \right) - \frac{f}{J} \cdot \Omega - \frac{T_{L}}{J} \\ g(x) &= \begin{bmatrix} 0 & 0 & \delta \cdot u_{\alpha} & 0 \\ 0 & 0 & \delta \cdot u_{\alpha} & 0 & 0 \end{bmatrix}^{T} \end{aligned}$$

Having two control variables  $u_{\alpha}$  and  $u_{\beta}$ , it is possible to decompose the model into two independent systems and then control separately two outputs. We choose as the outputs, the electromagnetic torque Te and the rotor flux modulus  $\Phi_r^2$ .

$$h(x) = \begin{bmatrix} T_e \\ \Phi_r^2 \end{bmatrix} = \begin{bmatrix} \mu (\Phi_{r\alpha} i_{s\beta} - \Phi_{r\beta} i_{s\alpha}) \\ \Phi_{r\alpha}^2 + \Phi_{r\beta}^2 \end{bmatrix}$$

In order to be able to impose arbitrary dynamics on every output  $y_1 = T_e$  and  $y_2 = \Phi_r^2$ , we mast find a differential relation linear between the output  $y_1$  and  $y_2$  and input of command  $u_{\alpha}$  and  $u_{\beta}$ , it is necessary to find a return of state, in a way that the system in closed buckle is linear and decoupled. Thus, it is necessary to derive the output function  $h_1(x)$  and  $h_2(x)$  respectively  $r_1$  and  $r_2$  (corresponding relative degrees) time, until create differential equations where intervene the commands  $(u_{\alpha}, u_{\beta})$ .

This linearization operation is possible in case where the total relative degree  $r = r_1 + r_2$  is lower or equal to the order of the system n (r  $\leq$  n), so the system is controllable. By successive derivation we can write:

$$\begin{cases} \frac{dy_1^{r_1}}{dt} = L_f^{r_1} h_1(x) + \sum_{j=1}^2 L_{g_j} L_f^{r_1 - 1} h_1(x) u_j \\ \frac{dy_2^{r_2}}{dt} = L_f^{r_2} h_2(x) + \sum_{j=1}^2 L_{g_j} L_f^{r_2 - 1} h_2(x) u_j \end{cases}$$

 $L_f h$  is h lie derivation in direction of the vector field  $f_{(.)}$  such as:

$$\begin{cases} L_f h(x) = dh.f = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \\ f(x) = (f_1(x), f_2(x), \dots, f_n(x)) \end{cases}$$

And  $r = [r_1, r_2]$  is the relative degree which has to satisfy the following conditions:

Also let us define the matrix of decoupling:

$$D(x) = \begin{bmatrix} L_{g_1} L_f^{r_1 - 1} . h_1(x) & L_{g_2} L_f^{r_1 - 1} . h_1(x) \\ L_{g_1} L_f^{r_2 - 1} . h_2(x) & L_{g_2} L_f^{r_2 - 1} . h_2(x) \end{bmatrix}$$

We can then write the system of equation (3) under the following matrix form:



Figure 1. Structure of the nonlinear control (input-output linearization)

$$\begin{bmatrix} h_1^{(r_1)}(x) \\ h_2^{(r_2)}(x) \end{bmatrix} = \begin{bmatrix} L_f^{r_1} \cdot h_1 \\ L_f^{r_2} \cdot h_2 \end{bmatrix} + D(x) \begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}$$

# *A.* For the first output: torque Te

Consider the machine model defined by (2) and determine the order of derivative for which the first variable to be set (the electromagnetic torque) is explicitly affected by the controls  $u_{\alpha}$ and  $u_{\beta}$ .

By successive derivations, the relative degree  $r_1$  associated with the torque of the MAS is equal 1:

$$\frac{dy_1}{dt} = \frac{dT_e}{dt} = L_f h_1(x) + L_{g_1} L^0{}_f h_1(x) u_\alpha + L_{g_2} L^0{}_f h_1(x) u_\beta$$
$$L_f h_1(x) = \mu \left( \Phi'_{r\alpha} i_{s\beta} + \Phi_{r\alpha} i'_{s\beta} - \Phi'_{r\beta} i_{s\alpha} - \Phi_{r\beta} i'_{s\alpha} \right)$$

Then we get this expression:  $L_{f}h_{1}(x) = A_{1} \cdot \left( \Phi_{r\alpha} i_{s\beta} - \Phi_{r\beta} i_{s\alpha} \right) + A_{2} \cdot \left( \Phi_{r\alpha} \cdot i_{s\alpha} + \Phi_{r\beta} i_{s\beta} \right) + A_{3} \cdot \left( \Phi_{r\alpha}^{2} + \Phi_{r\beta}^{2} \right)$ 

Where:

$$\begin{cases} A_1 = -\mu . (\lambda_r + \gamma) \\ A_2 = -p . \Omega . \mu \\ A_3 = k . A_2 \end{cases}$$

And:

$$\begin{cases} L_{g_1} L^0{}_f h_1(x) = -p.k.\Phi_{r\beta} \\ L_{g_2} L^0{}_f h_1(x) = p.k.\Phi_{r\alpha} \end{cases}$$

B. For the second output: square of rotor flux  $\Phi_r^2$ .

In the same way as previously, the relative degree  $r_2$ , associated with the second output, is equal 2. Then after the second derivation we get:

$$\frac{d^2 y_2}{dt^2} = L^2 f h_2(x) + L_{g_1} L^1 f h_2(x) u_\alpha + L_{g_2} L^1 f h_2(x) u_\beta$$
  
Where:  
$$L_f h_2(x) = -2.\lambda_r \cdot \left( \Phi_{r\alpha}^2 + \Phi_{r\beta}^2 \right) + 2.L_m \cdot \lambda_r \cdot \left( \Phi_{r\alpha} \cdot i_{s\alpha} + \Phi_{r\beta} \cdot i_{s\beta} \right)$$
$$\Rightarrow L^2 f h_2(x) = B_1 \cdot \left( i_{s\alpha}^2 + i_{s\beta}^2 \right) + B_2 \cdot \left( \Phi_{r\alpha} \cdot i_{s\beta} - \Phi_{r\beta} \cdot i_{s\alpha} \right) + B_3 \cdot \left( \Phi_{r\alpha} \cdot i_{s\alpha} + \Phi_{r\beta} \cdot i_{s\beta} \right) + B_4 \cdot \left( \Phi_{r\alpha}^2 + \Phi_{r\beta}^2 \right)$$

Where:

$$\begin{cases} B_1 = 2.(L_m.\lambda_r)^2 , \quad B_2 = 2.p.\Omega.L_m.\lambda_r \\ B_3 = -6.L_m.\lambda_r^2 - 2.\gamma.L_m.\lambda_r , \quad B_4 = 4.\lambda_r^2 + 2.k.L_m.\lambda_r^2 \\ \int L_{g_1}L^1fh_2(x) = 2.k.R_r.\Phi_{r\alpha} \\ L_{g_2}L^1fh_2(x) = 2.k.R_r.\Phi_{r\beta} \end{cases}$$

# C. Feedback law

The matrix form of the system equations (4) allows as deducting the return of nonlinear state  $u = \alpha(x) + \beta(x).v$  that give to the system a linear input/output behavior: By putting:

$$\begin{cases} \alpha(x) = -D^{-1}(x) \begin{bmatrix} L_f^{n} \cdot h_1 \\ L_f^{n} \cdot h_2 \end{bmatrix} \\ \beta(x) = D^{-1}(x) \end{cases}$$

The input of the resulting linear system is defined as follow:  $\begin{bmatrix} u \\ z \end{bmatrix} \begin{bmatrix} u \\ z \end{bmatrix} \begin{bmatrix} u \\ z \end{bmatrix} \begin{bmatrix} u \\ z \end{bmatrix}$ 

$$\begin{bmatrix} u_{\alpha} \\ u_{\beta} \end{bmatrix} = D^{-1}(x) \left( \begin{bmatrix} v_{\alpha} \\ v_{\beta} \end{bmatrix} - \begin{bmatrix} L_{f}^{r_{1}} \cdot h_{1} \\ L_{f}^{r_{2}} \cdot h_{2} \end{bmatrix} \right)$$

Where:

 $v_{\alpha}$  and  $v_{\beta}$  : are the new inputs controls.

$$D(x) = \begin{bmatrix} L_{g_1} L_f^{0} . h_1(x) & L_{g_2} L_f^{0} . h_1(x) \\ L_{g_1} L_f^{-1} . h_2(x) & L_{g_2} L_f^{-1} . h_2(x) \end{bmatrix}$$
$$= \begin{bmatrix} -p . k . \Phi_{r\beta} & p . k . \Phi_{r\alpha} \\ 2 . k . R_r . \Phi_{r\alpha} & 2 . k . R_r . \Phi_{r\beta} \end{bmatrix}$$

The commands  $u_{\alpha}$  and  $u_{\beta}$  can be determined if the decoupling matrix D(x) is not singular, thus D(x) is invertible if  $\Phi_{r\alpha}^2 + \Phi_{r\beta}^2 \neq 0$ .

Care must be taken to this singularity condition in the simulation as well as implementation of the control strategy. At the starting point of the simulation/implementation, the flux magnitude is zero. In case of digital implementation of the system, to avoid numerical saturation of the processor, this singularity can be avoided by putting a small positive value for the flux magnitude.

By applying this feedback law (5), a new dynamic model of the machine is obtained, which is represented by the both differential equations below:

$$\begin{bmatrix} h_1^{(r_1)}(x) \\ h_2^{(r_2)}(x) \end{bmatrix} = \begin{bmatrix} \frac{dT_e}{dt} \\ \frac{d^2(\Phi_r^2)}{dt^2} \end{bmatrix} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix}$$

D. Control law

Now, we define the control law, which allows automatic control of electromagnetic torque  $T_e$  to  $T_{e_{ref}}$  and rotor flux modulus  $\Phi_r^2$  to  $\Phi_{ref}^2$ , as follow:

$$\begin{cases} v_{\alpha} = -k_1 \cdot \left( T_e - T_{eref} \right) + \dot{T}_{eref} \\ v_{\beta} = -k_2 \cdot \left( \Phi^2_r - \Phi^2_{ref} \right) - k_3 \cdot \left( \dot{\Phi}^2_r - \dot{\Phi}^2_{ref} \right) + \ddot{\Phi}^2_{ref} \end{cases}$$

We define the variables errors as:

$$\begin{cases} e_1 = \left(T_e - T_{e_{ref}}\right) \\ e_2 = \left(\Phi^2_r - \Phi^2_{ref}\right) \end{cases}$$

So the dynamics of errors is determined by:

$$\begin{cases} \dot{e}_1 + k_1 \cdot e_1 = 0\\ \ddot{e}_2 + k_3 \cdot \dot{e}_2 + k_2 \cdot e_2 = 0 \end{cases}$$

The dynamics of the two errors will be stable if the roots of polynomials corresponding characteristics are placed in the left half-plane of the complex plane.

#### E. Speed loop PI corrector Determination

Speed loop is the outer loop and the torque is the inner loop of the first decoupled subsystem, where  $h_1(x) = T_e$  is the output and  $v_{\alpha}$  is the input. So the torque loop must be faster than the speed. In this case the block diagram representing this structure is as follows:

Sensorless Exact Input-Output Linearization Control of the Induction



Figure 2. Block-scheme of the speed loop

According to this scheme, we can determine the transfer function of the speed loop as below:

$$\frac{\Omega(s)}{\Omega_{ref}(s)} = \frac{1}{1 + \left(\frac{k_p + f}{k_i}\right)s + \left(\frac{J}{k_i}\right)s^2}$$

and then calculate the suitable P.I parameters  $(k_i, k_p)$ .

## 3. Rotor Flux Sliding Mode Observer (SMO)

A. Induction machine model

In this section we use a model of asynchronous machine formulated in a stationary stator reference frame  $(\alpha, \beta)$ , as showing below:

$$\begin{cases} \dot{I}_s = -\gamma . I_s + k . A . \Phi_r + \delta . V_s \\ \dot{\Phi}_r = L_m . \lambda_r . I_s - A . \Phi_r \end{cases}$$

With:

$$I_{s} = [i_{s\alpha} \ i_{s\beta}]^{T};$$
  

$$\Phi_{r} = [\Phi_{r\alpha} \ \Phi_{r\beta}]^{T}$$
  

$$A = \begin{bmatrix} \lambda_{r} & \omega \\ -\omega & \lambda_{r} \end{bmatrix};$$
  

$$V_{s} = [v_{s\alpha} \ v_{s\beta}]^{T}$$

B. Rotor flux SMO model

The equations representing the observer model are given below:

$$\begin{split} \hat{I}_s &= -\gamma \cdot \hat{I}_s + k \cdot A \cdot \hat{\Phi}_r + \delta \cdot V_s + D_i \cdot u_s \\ \dot{\hat{\Phi}}_r &= L_m \cdot \lambda_r \cdot \hat{I}_s - A \cdot \hat{\Phi}_r + D_\varphi \cdot u_s \end{split}$$

The dynamics of the estimation error is expressed by the following equations:

$$\begin{cases} \dot{\widetilde{I}}_{s} = -\gamma \cdot \widetilde{I}_{s} + k \cdot A \cdot \widetilde{\Phi}_{r} - D_{i} \cdot u_{s} \\ \dot{\widetilde{\Phi}}_{r} = L_{m} \cdot \lambda_{r} \cdot \widetilde{I}_{s} - A \cdot \widetilde{\Phi}_{r} - D_{\varphi} \cdot u_{s} \end{cases}$$

Where:

- $\widetilde{I}_s = I_s \hat{I}_s$ : is the stator current estimation error.
- $\tilde{\Phi}_r = \Phi_r \hat{\Phi}_r$ : is the rotor flux estimation error.
- $u_s = [sign(S_1) \ sign(S_2)]^T$
- $S = [s_1 \ s_2]^T = \eta.(\widetilde{I}_s)$ : is the sliding mode surface.
- $D_{\varphi}$ ,  $D_i$ ,  $\eta$ : are the matrix (2x2) that we will determine later.

C. Design of the rotor flux SMO We consider the following Lyapunov candidate function: -1 c .S

$$V = \frac{1}{2} . S^T$$

Its time derivative is the following:

 $\dot{V} = S^T \dot{S}$ 

We suppose that  $\frac{d\eta}{dt} = 0$ , thus we obtain:

$$\begin{split} \dot{V} &= S^T \, \eta . \dot{\widetilde{I}}_s \\ \dot{V} &= S^T . \eta . \Big( - \gamma . \widetilde{I}_s + k . A . \widetilde{\Phi}_r \Big) - S^T . \eta . D_i . u_s \end{split}$$

In order to have  $\dot{V}$  negative definite and satisfy the condition of attractiveness, we must have:

$$S^{T}.\eta.(-\gamma.\widetilde{I}_{s}+k.A.\widetilde{\Phi}_{r}) < S^{T}.\eta.D_{i}.u_{s}$$

If we put:

$$\eta.D_i = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$$

Then we obtain the condition below:

 $|\mu_1||S_1| + |\mu_2||S_2| > S^T \cdot \eta \cdot \left(-\gamma \cdot \widetilde{I}_s + k \cdot A \cdot \widetilde{\Phi}_r\right)$ 

When the sliding mode will be reached, the switching surface will verify:

 $\widetilde{I}_s = \widetilde{I}_s = 0$ 

Therefore we obtain:

$$\Rightarrow u_s = D_i^{-1}.k.A.\widetilde{\Phi}_i$$

By putting this equation in (18) we get this equation:

$$\widetilde{\Phi}_r = - \left( A + D_{\varphi} . D_i^{-1} . k . A \right) \widetilde{\Phi}_r$$

We put:

$$A + D_{\varphi} \cdot D_i^{-1} \cdot k \cdot A = \mathbf{P}$$
$$\Rightarrow \dot{\widetilde{\Phi}}_r = -\mathbf{P} \cdot \widetilde{\Phi}_r$$

In order to have an exponential convergence we choose P under the following form:

$$\mathbf{P} = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix}$$

Where p1 and p2 are a positives constants. Then we obtain de following equation:

$$D_{\varphi} = (\mathbf{P} - A) \cdot A^{-1} \cdot k^{-1} \cdot D_i$$

Now if we put:

$$\eta = A^{-1}.k$$

So we finally find the following equations:

$$D_i = k.A \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \quad ; \quad D_{\varphi} = (\mathbf{P} - A) \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}$$

Finally the condition of attractiveness becomes:

$$\mu_1 |S_1| + \mu_2 |S_2| > S^T \widetilde{\Phi}_r$$

To complete the design of the observer, it now suffices to choose properly the observer parameters. The  $(p_1, p_2)$  parameters must be chosen to determine the dynamics of observer convergence and the  $(\mu_1, \mu_2)$  parameters must be chosen to satisfy the condition of attractiveness and stability of the observer.

#### 4. Parallel Stator Resistance and Rotor Speed MRAS Observer

The simultaneous estimation of rotor speed and the stator resistance is realized in this section by using a technique based on a MRAS structure composed of three models and two adaptation mechanisms providing the estimate of the rotor speed and stator resistance.

The structure of this observer is given in Figureure 3, it shows that stator resistance is estimated using a voltage model adjustable by it, which provided an estimate of the stator current. The stator current estimation error will be used by an adaptive mechanism to estimate the value of stator resistance.

The speed estimation is performed in parallel with that of stator resistance; it is based on an adaptation mechanism that uses the error between the estimated flux components supplied by a current model, adjustable by rotor speed estimation, and the flux components provided by the flux sliding mode observer.

The following Figure represents the observation technique structure.



Figure 3. Parallel MRAS Observer structure

By using the dynamic model of the asynchronous machine, formulated in a stator reference frame  $(\alpha, \beta)$ , and by using measurements of the stator currents and voltages, we build two estimators one for stator currents estimation the others for rotor flux estimation.

A. Adjustable Current Model

Consider the two first equation of induction machine model (1):

$$\begin{cases} \frac{d\hat{\Phi}_{r\alpha I}}{dt} = \lambda_r . (L_m . i_{s\alpha} - \hat{\Phi}_{r\alpha I}) & -\hat{\omega} . \hat{\Phi}_{r\beta I} \\ \frac{d\hat{\Phi}_{r\beta I}}{dt} = \lambda_r . (L_m . i_{s\beta} - \hat{\Phi}_{r\beta I}) & + \hat{\omega} . \hat{\Phi}_{r\alpha I} \end{cases}$$

This model is adjustable by rotor speed, it provide the flux estimation by using measured stator current and estimated rotor speed.

## B. Adjustable Voltage Model

We can use the following voltage model as adjustable model for stator resistance estimation:

$$\begin{cases} \frac{d\hat{i}_{s\alpha}}{dt} = \delta \left[ v_{s\alpha} - L_m \cdot \lambda_r \cdot \frac{d\hat{\Phi}_{r\alpha I}}{dt} - \hat{R}_s \cdot \hat{i}_{s\alpha} \right] \\ \frac{d\hat{i}_{s\beta}}{dt} = \delta \left[ v_{s\beta} - L_m \cdot \lambda_r \cdot \frac{d\hat{\Phi}_{r\beta I}}{dt} - \hat{R}_s \cdot \hat{i}_{s\beta} \right] \end{cases}$$

In this model we use the measured voltage and the estimated flux provided by the adjustable current model, to estimate stator current.

We define the following estimation errors, respectively of speed, stator resistance, rotor flux and stator current:

$$\begin{cases} \vec{\omega} = \omega - \vec{\omega} \\ \vec{R}_s = R_s - \hat{R}_s \end{cases}$$
$$\begin{cases} \vec{\Phi}_{r\alpha} = \Phi_{r\alpha} - \hat{\Phi}_{r\alpha l} \\ \vec{\Phi}_{r\beta} = \Phi_{r\beta} - \hat{\Phi}_{r\beta l} \end{cases}$$
$$\begin{cases} \vec{i}_{s\alpha} = i_{s\alpha} - \hat{i}_{s\alpha} \\ \vec{i}_{s\beta} = i_{s\beta} - \hat{i}_{s\beta} \end{cases}$$

After deriving errors, we obtain the errors dynamic equations as follow:

$$\begin{cases} \dot{\tilde{\Phi}}_{r\alpha} = -\lambda_r \cdot \tilde{\Phi}_{r\alpha} - \omega \cdot \Phi_{r\beta} + \hat{\omega} \cdot \hat{\Phi}_{r\beta I} \\ \dot{\tilde{\Phi}}_{r\beta} = -\lambda_r \cdot \tilde{\Phi}_{r\beta} + \omega \cdot \Phi_{r\alpha} - \hat{\omega} \cdot \hat{\Phi}_{r\alpha I} \\ \dot{\tilde{i}}_{s\alpha} = -\delta \cdot (R_s \cdot i_{s\alpha} - \hat{R}_s \cdot \hat{i}_{s\alpha}) \\ \dot{\tilde{i}}_{s\beta} = -\delta \cdot (R_s \cdot i_{s\beta} - \hat{R}_s \cdot \hat{i}_{s\beta}) \end{cases}$$

# Sensorless Exact Input-Output Linearization Control of the Induction

By considering relation (11, 12 and 13), we can rewrite (14) as below:

$$\begin{cases} \dot{\tilde{\Phi}}_{r\alpha} = -\lambda_r . \tilde{\Phi}_{r\alpha} - \tilde{\omega} . \Phi_{r\beta} - \hat{\omega} . \tilde{\Phi}_{r\beta} \\ \dot{\tilde{\Phi}}_{r\beta} = -\lambda_r . \tilde{\Phi}_{r\beta} + \tilde{\omega} . \Phi_{r\alpha} + \hat{\omega} . \tilde{\Phi}_{r\alpha} \\ \dot{\tilde{i}}_{s\alpha} = -\delta . (\hat{R}_s . \tilde{i}_{s\alpha} + \tilde{R}_s . i_{s\alpha}) \\ \dot{\tilde{i}}_{s\beta} = -\delta . (\hat{R}_s . \tilde{i}_{s\beta} + \tilde{R}_s . i_{s\beta}) \end{cases}$$

C. Stability Analysis

In order to determine the observer stability condition, and then determine the adaptation mechanism that gives us the speed estimation, let us consider the following LCF:

$$V = \frac{1}{2}.\tilde{i}_{s\alpha}^{2} + \frac{1}{2}.\tilde{i}_{s\beta}^{2} + \frac{\tilde{R}_{s}^{2}}{2} + \frac{1}{2}.\tilde{\Phi}_{r\alpha}^{2} + \frac{1}{2}.\tilde{\Phi}_{r\beta}^{2} + \frac{\tilde{\omega}^{2}}{2}$$

The LCF derivative is as below:

$$\begin{split} \dot{V} &= -\delta.\hat{R}_{s}.\left(\tilde{i}_{s\alpha}^{2} + \tilde{i}_{s\beta}^{2}\right) - \delta.\left(i_{s\alpha}.\tilde{i}_{s\alpha} + i_{s\beta}.\tilde{i}_{s\beta}\right).\tilde{R}_{s} + \tilde{R}_{s}.\tilde{R}_{s} \\ &- \lambda_{r}.\left(\tilde{\Phi}_{r\alpha}^{2} + \tilde{\Phi}_{r\beta}^{2}\right) + \left(\Phi_{r\alpha}.\tilde{\Phi}_{r\beta} - \Phi_{r\beta}.\tilde{\Phi}_{r\alpha}\right).\tilde{\omega} + \dot{\tilde{\omega}}.\tilde{\omega} \\ \dot{V} &\leq \tilde{R}_{s} \left[ -\delta.(i_{s\alpha}.\tilde{i}_{s\alpha} + i_{s\beta}.\tilde{i}_{s\beta}) - \dot{\hat{R}} \right] + \tilde{\omega}.\left[\Phi_{r\alpha}.\tilde{\Phi}_{r\beta} - \Phi_{r\beta}.\tilde{\Phi}_{r\alpha} - \dot{\tilde{\omega}} \right] \end{split}$$

In order to make  $\dot{V}$  to be negative definite, we can for example force the second term to be null, then we can write:

$$\begin{cases} \dot{\hat{R}} = -\delta.(i_{s\alpha}.\tilde{i}_{s\alpha} + i_{s\beta}.\tilde{i}_{s\beta}) \\ \dot{\hat{\omega}} = (\Phi_{r\alpha}.\tilde{\Phi}_{r\beta} - \Phi_{r\beta}.\tilde{\Phi}_{r\alpha}) \end{cases}$$

We have:

$$\dot{\widetilde{\omega}} = -\dot{\widetilde{\omega}}$$
 and  $\dot{\widetilde{R}}_s = -\dot{\widehat{R}}_s$ 

Then the corresponding adaptive law, that ensures the stability of the MRAS observer, is as follow:

$$\hat{\omega} = \int (\Phi_{r\alpha} \cdot \widetilde{\Phi}_{r\beta} - \Phi_{r\beta} \cdot \widetilde{\Phi}_{r\alpha}) . dt$$
$$\hat{R} = -\int \delta . (i_{s\alpha} \cdot \widetilde{i}_{s\alpha} + i_{s\beta} \cdot \widetilde{i}_{s\beta}) . dt$$

In order to decrease response temp of estimation and ensure a null steady error, we use PI controller as follow:

$$\hat{\omega} = kp_1(\Phi_{r\alpha}.\widetilde{\Phi}_{r\beta} - \Phi_{r\beta}.\widetilde{\Phi}_{r\alpha}) + ki_1 \cdot \int (\Phi_{r\alpha}.\widetilde{\Phi}_{r\beta} - \Phi_{r\beta}.\widetilde{\Phi}_{r\alpha}).dt$$
$$\hat{R} = -kp_2.(i_{s\alpha}.\widetilde{i}_{s\alpha} + i_{s\beta}.\widetilde{i}_{s\beta}) - ki_2.\int (i_{s\alpha}.\widetilde{i}_{s\alpha} + i_{s\beta}.\widetilde{i}_{s\beta}).dt$$

Where  $(kp_1, kp_2)$  and  $(ki_1, ki_2)$  are the proportional and integral positive gain, choosing to have good dynamic performance of the observer.

The last variable we need to know is the electromagnetic torque, which we choose to estimate using the following expression:

$$\hat{T}_e = \mu \left( \hat{\Phi}_{r\alpha} \, i_{s\beta} - \hat{\Phi}_{r\beta} \, i_{s\alpha} \right)$$

#### 5. Simulation Results

Using Matlab/Simulink, the proposed controller technique and observer performances have been verified by the following simulation results.

In order to ensure that the references are differentiable, and then the control law (5) to be realizable, in these simulations the references steps of speed and Flux are filtered, by a second-order low-pass filter with cut-off pulsation equal to 500 rd/s.

The Figure 4 shows us the simulation responses of the system commanded for a speed reference steps Figure 4a, in order to allows the observer to achieve the actual value of stator resistance, the first step is given at 0.5 s from 0 to 100 rad/s, the second one is from 100 rad/s to 150 rad/s at 1.5 s the last one is down from 150 rad/s to 50 rad/s at 2.5 s. Figure 4a shows that, the speed response is good, which presents a small responses time 0.3 s. The induction machine is loaded by 10 N.m at 0.5 s as showing in Figure 4b. We can see in Figure 4b that the torque has a good response. Figure 4b, Figure 4c and Figure 4d show that the currents, voltages and torque respect the physical limits of the induction machine. Figure 4e shows that the decoupling between the torque and the flux is correct, and that the flux and speed estimation error are very low as showing in Figure 4f and Figure 4g. Figure 4h shows the evolution of the ratio of the estimated stator resistance and the nominal value of the stator resistance, in this simulation we suppose that stator resistance doesn't varies. We can see that the estimation response is good.

The Figure 5 shows us the simulation responses of the asynchronous machine commanded for 100 rad/s of speed Figure 5a, and for a load torque steps Figure 5b, the first step is given at 0.5 s from 0 to 10 N.m, the second one is from 10 N.m to 20 N.m at 1.5 s, and the third one is down from 20 N.m to 5 N.m at 2.5 s. We can observe in Figure 5b that the motor torque follows the load torque. Also, we notice that stator currents and stator voltage still under maximum limits values of the induction machine as showing in Figure 5c and Figure 5d. Simulation results show that the speed and the flux in Figure 5e are not influenced by torque variation. We can see in Figure 5f, Figure 5g and Figure 5h that the estimation errors, of speed, rotor flux and stator resistance are very weak.

The Figure 6 shows the simulation responses of the system commanded for 100 rad/s in speed (Figure 6.a) and 10N.m in load torque (Figure 6.b) with a filtered steps variation of 70 % of the nominal stator resistance ( $R_{sn} = 2.2\Omega$ ) coming at 1.5s and another one given at 2.5s from 70% to 30% of the nominal stator resistance. We can see that the control present a stable and acceptable response. We notice that the responses of speed (Figure 6.a), couple (Figure 6.b) and flux (Figure 6.c) shows that the influence of this variation is null, and that estimation errors of speed (Figure 6.d), flux (Figure 6.e)and stator resistance remains null (Figure 6.f).

Parameters		Values	Units
Rated power	Р	3	kw
Voltage	U	380	V
Rated current	Ι	7.3	А
Rated speed	n	1440	rpm
Stator Resistance	Rs	2.2	$\Omega$
Rotor Resistance	Rr	2.68	$\Omega$
Mutual Inductance	Lm	0.217	Н
Stator Inductance	Ls	0.229	H
<b>Rotor Inductance</b>	Lr	0.229	H
Motor load inertia	J	0.047	kg.m <sup>2</sup>
Viscous friction coefficient	f	0.004	N.s/rad
Number of pole pairs	р	2	

The parameters of Induction Machine used in simulation are given below: Table 1. The parameters of the Induction Machine:



Figure 4. Simulation responses for steps of reference speed for 10N.m in load torque.



Figure 5. Simulation responses for steps of load torque with 100 rad/s in reference speed.



Figure 6. Simulation responses for stator resistance variation with 100 rad/s in reference speed and 10N.m in load torque.

# 6. Conclusion

In this article, we made a study of the sensorless speed control of the induction machine using exact input-output linearization control technique, associated to a special MRAS observer that provides simultaneous estimations of rotor speed and stator resistance, governed

by an adaptation law according to the stator current and rotor flux estimated errors. The estimation of flux component was obtained by using a sliding mode observer. The simulation results showed that this control and observation techniques combination presents of maid good performances and allows a complete decoupling between the flux and the torque. These performances are obtained with stator current, stator voltage and torque that respect the physical limits of the induction machine.

## Notation

Are respectively the estimation and estimation error of the x,  $\hat{x}$ ,  $\tilde{x}$ 

Time-derivative of the x,

 $\dot{x} \text{ or } \frac{dx}{dt}$  $\Phi_r$ Rotor flux,

 $\Phi_{r\alpha}, \Phi_{r\beta}$  Flux components in the stationary  $(\alpha, \beta)$  axis,

Stator currents in the stationary  $(\alpha, \beta)$  axis,  $i_{s\alpha}$ ,  $i_{s\beta}$ 

Stator voltages in the stationary  $(\alpha, \beta)$  axis,  $v_{s\alpha}, v_{s\beta}$ 

- Rotor speed, Ω
- $T_L$  ,  $T_e$ Are Load torque, and Electromagnetic torque.

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