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BEIHANG UNIVERSITY

Effect of Degree-of-Symmetry on Kinetostatic Characteristics of Flexure Mechanisms: A Comparative Case Study

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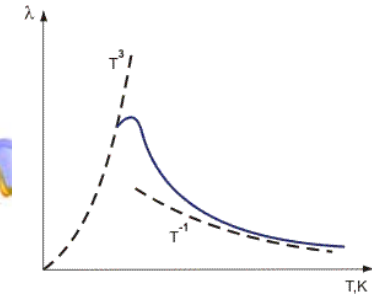
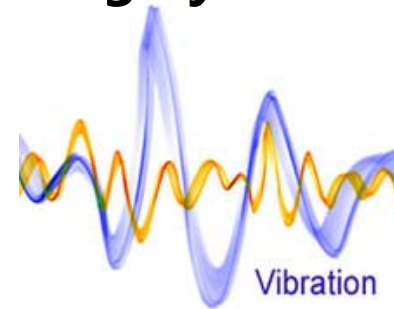


Part1 : Question 🙄

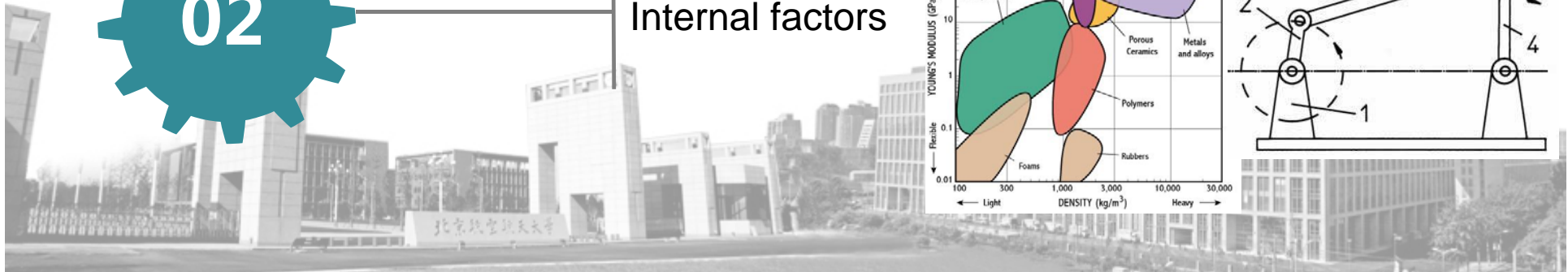
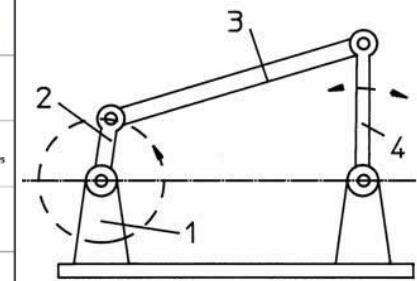
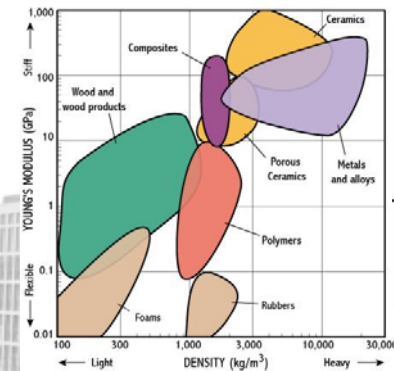
➤ Accuracy of flexure mechanisms is highly sensitive



External disturbances

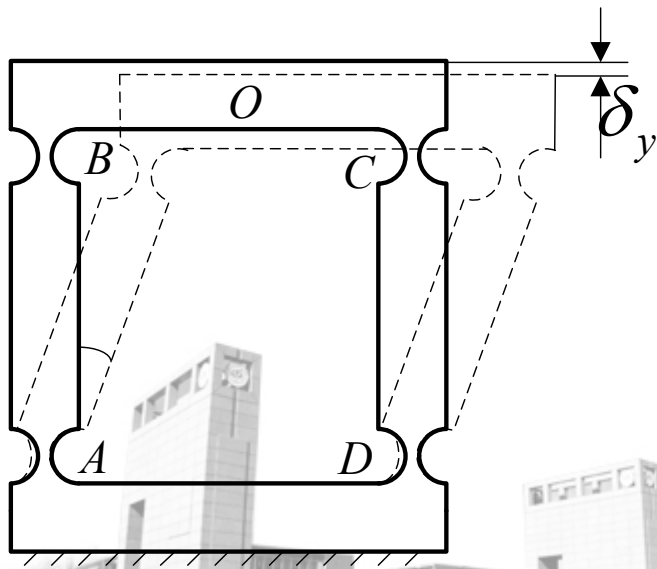


Internal factors



Part1 : Question 😞

➤ **P**arasitic motion is an index for accuracy



Parasitic motion is defined as any undesirable motion along the constraint directions of a mechanism.

➤ **M**ethods to reduce or eliminate the parasitic motion

parameter

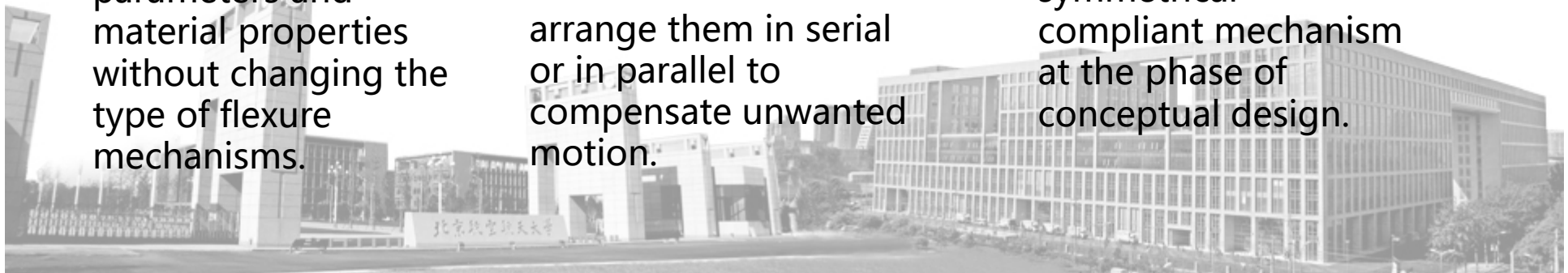
Tune the structural parameters and material properties without changing the type of flexure mechanisms.

compensation

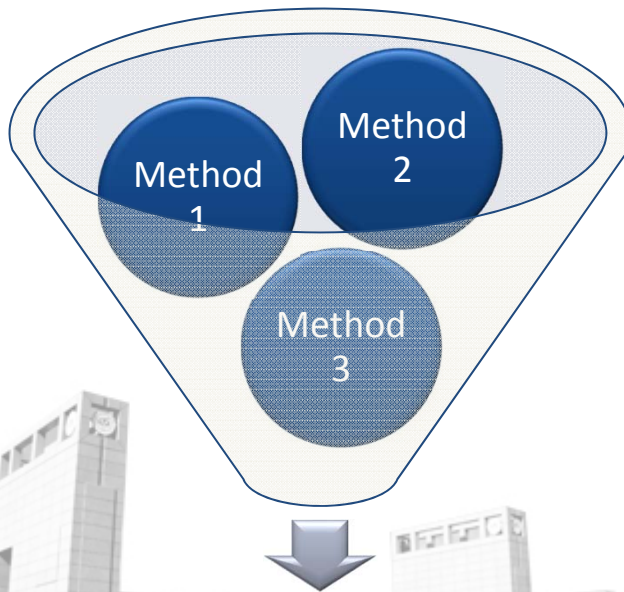
Mirror two identical flexure modules and arrange them in serial or in parallel to compensate unwanted motion.

design

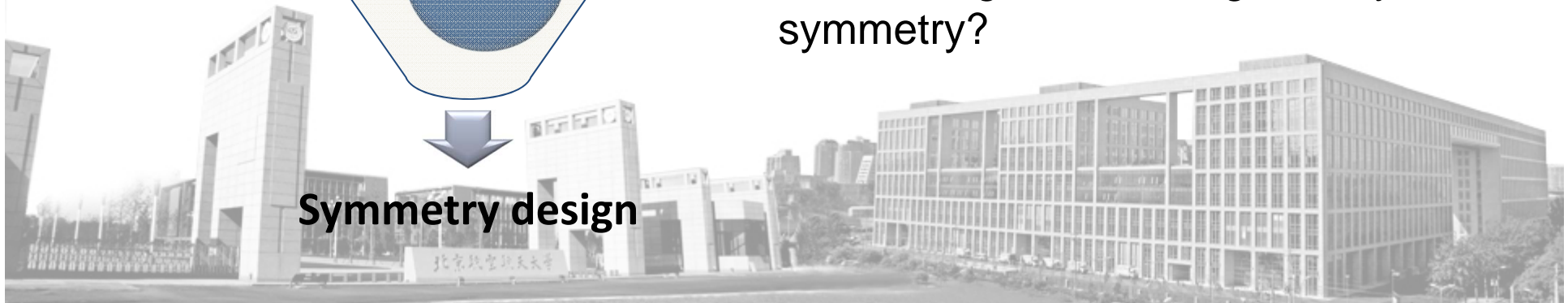
Design a practical full-symmetrical compliant mechanism at the phase of conceptual design.



➤ Common knowledge of symmetry



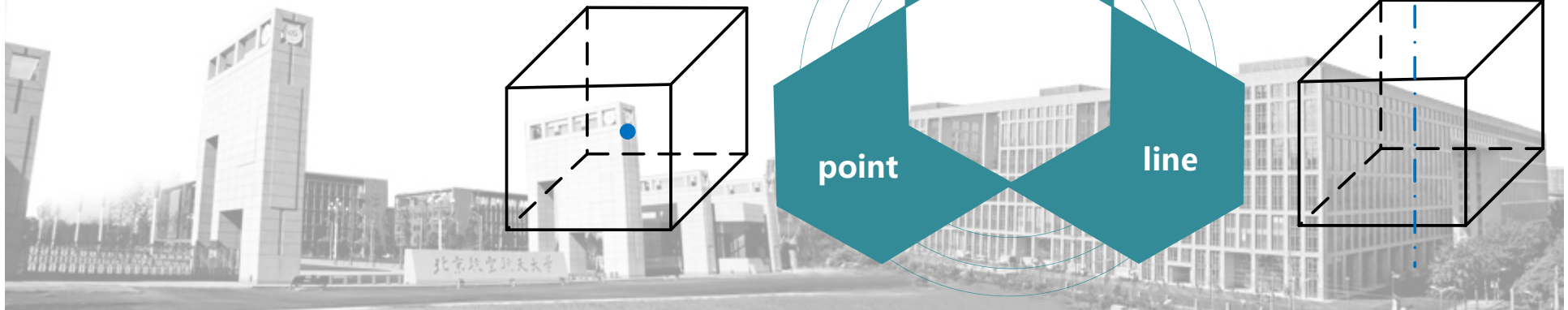
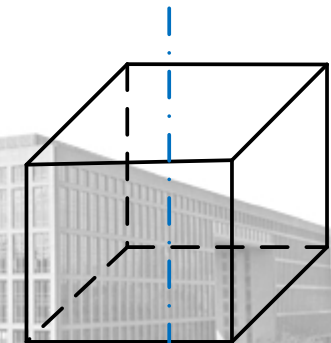
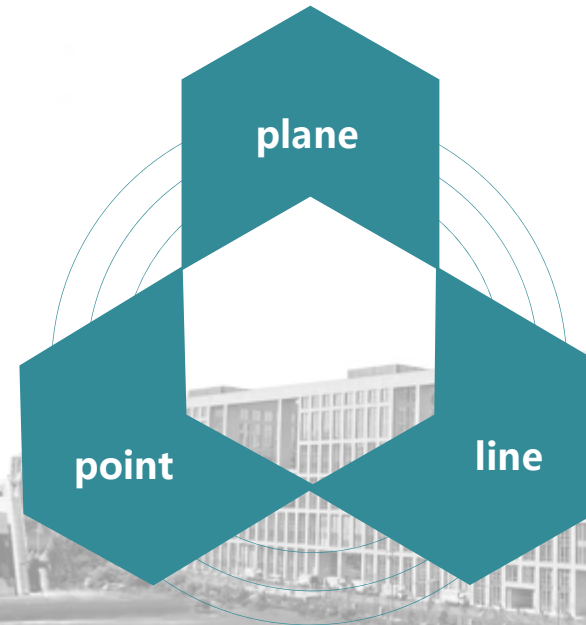
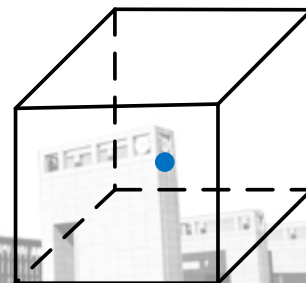
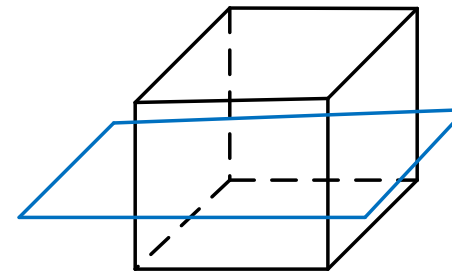
is it necessary to design the flexure mechanisms with symmetrical features as many as possible for better kinetostatic performance, when considering the resulting cost by the symmetry?



Symmetry design

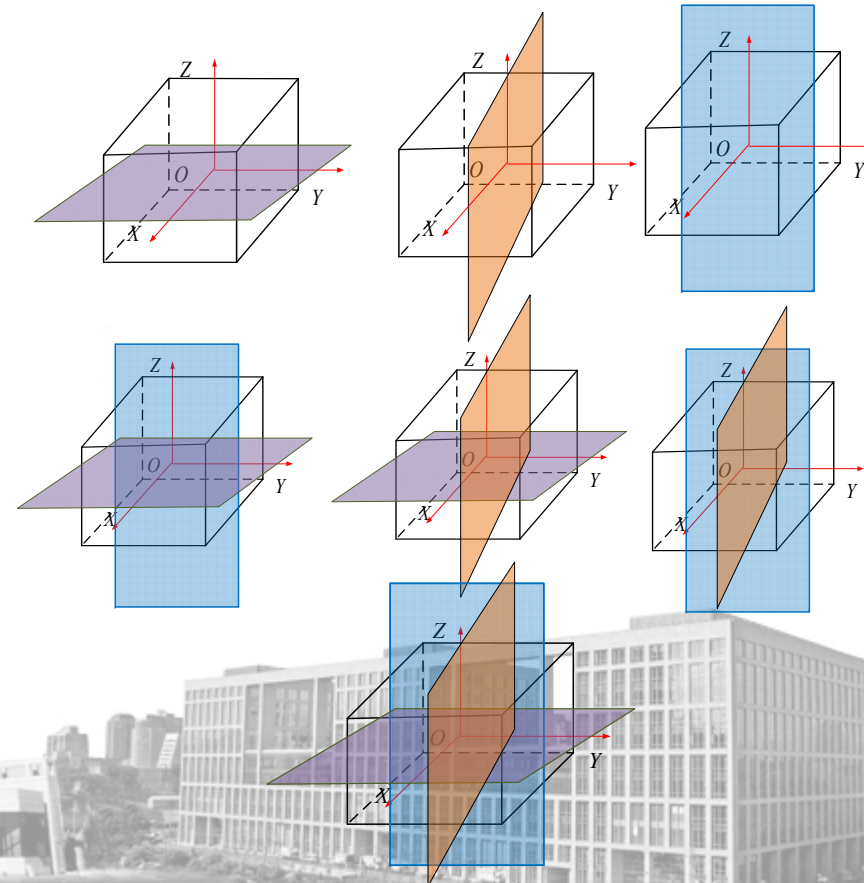
➤ Degree of symmetry

Here, the Degree-of-Symmetry (DoS) is specifically constrained with plane symmetry.



Part2 : Analysis

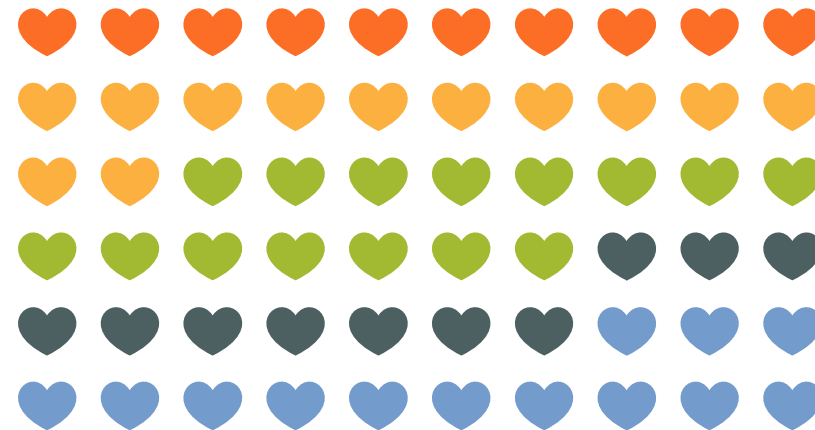
➤ X-DoS



Part2 : Analysis

➤ **A** comparative study

So,
what is the effect of DoS on
kinetostatic characteristic of
flexure mechanisms?



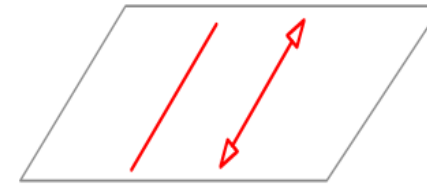
3-DoS > 2-DoS > 1-DoS ? ? ?



Part2 : Analysis

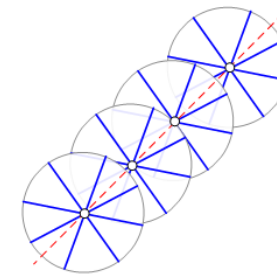
➤ Build the model

The desired freedom space

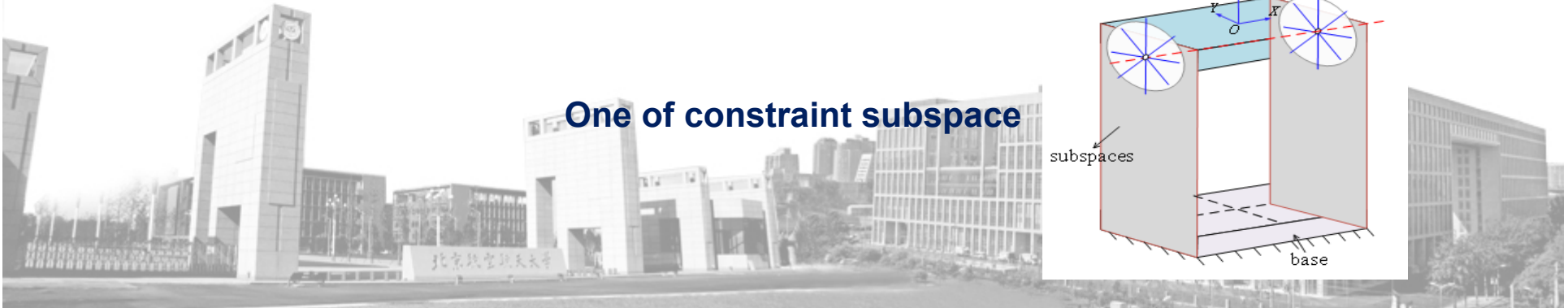
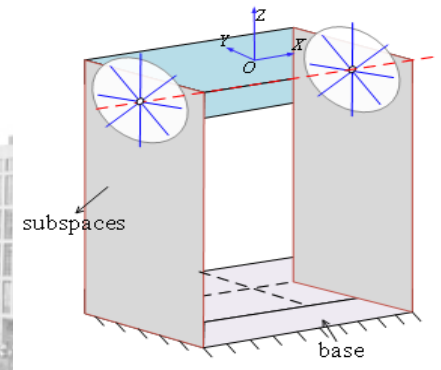


1R1T

The desired constraint space

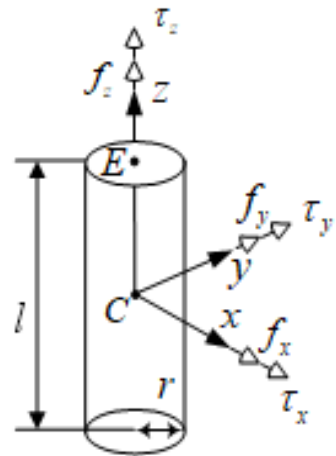


One of constraint subspace

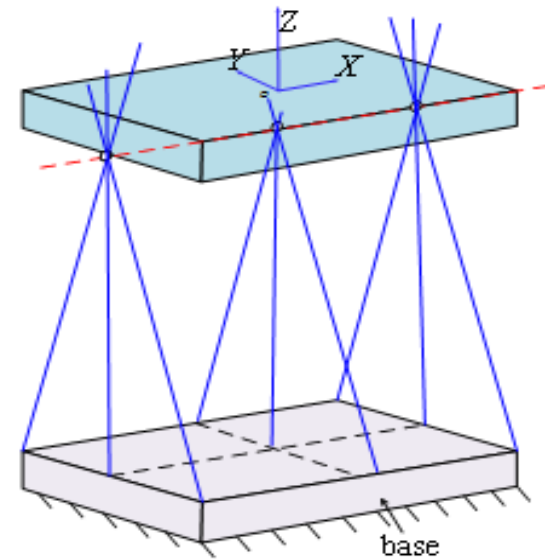


Part2 : Analysis

➤ Build the model



Each mechanism is a cylindrical joint that is composed of several identical beams distributed in two planes orthogonal to the direction of motion axes.

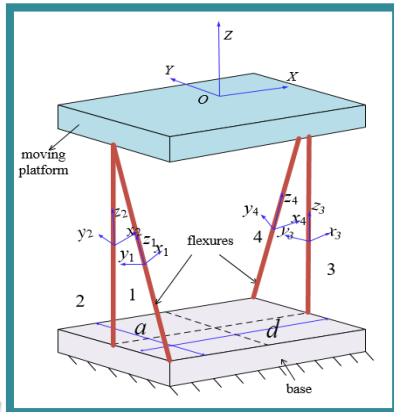


$$\frac{c_{c11}}{c_{c33}} = \frac{c_{c22}}{c_{c33}} = \frac{l^3}{12EI_y} \bigg/ \frac{l}{EA} = \frac{1}{3} \left(\frac{l}{r}\right)^2 = 133,$$

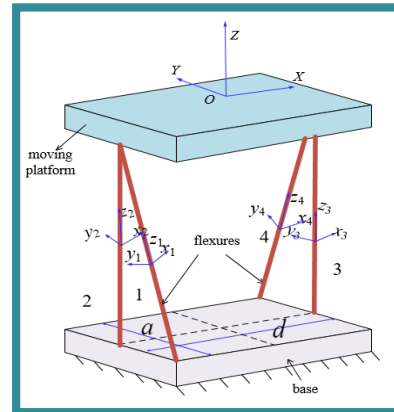
$$\frac{c_{c44} l^2}{c_{c33}} = \frac{c_{c55} l^2}{c_{c33}} = 1600, \quad \frac{c_{c66} l^2}{c_{c33}} = \frac{2l^2}{3(1+\mu)r^2} \geq 1000$$

Part2 : Analysis

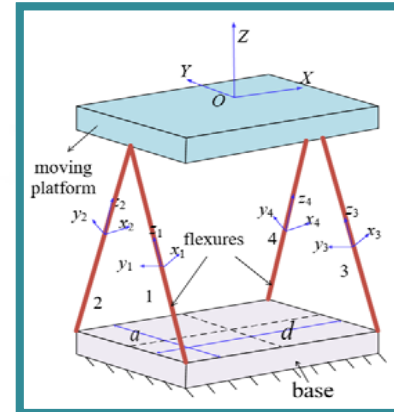
➤ Build the model



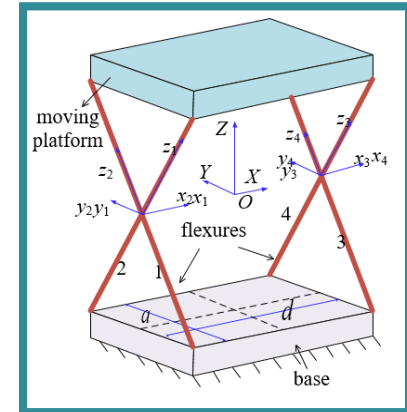
0-DoS



1-DoS



2-DoS



3-DoS



➤ Overall compliance matrix

According to the screw theory and the theory of linear elasticity, the deformation denoted by the twist

$$\xi = (\boldsymbol{\theta}; \boldsymbol{\delta}) = (\theta_x, \theta_y, \theta_z; \delta_x, \delta_y, \delta_z)$$

and the load wrench

$$\mathbf{F} = (\boldsymbol{\tau}; \mathbf{f}) = (\tau_x, \tau_y, \tau_z; f_x, f_y, f_z)$$

are connected by the generic 6×6 compliance matrix.

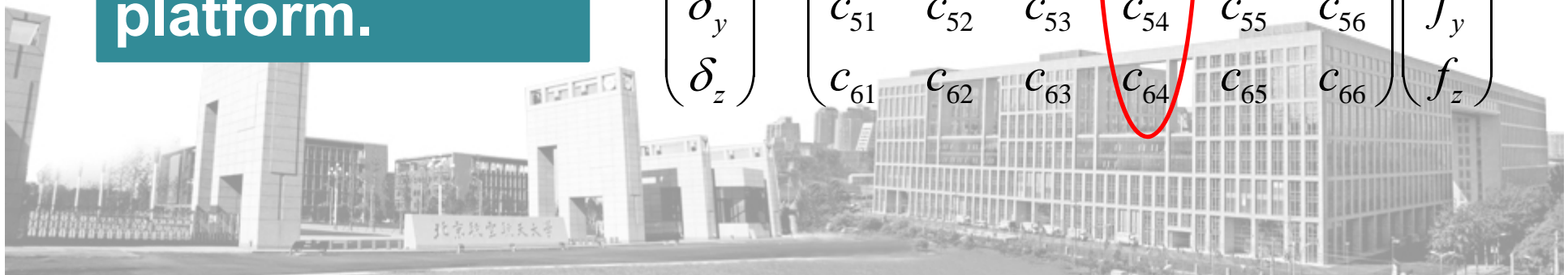
$$\begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \delta_x \\ \delta_y \\ \delta_z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_x \\ f_y \\ f_z \end{pmatrix}$$



➤ Overall compliance matrix

Only a force f_x is imposed on the mobile platform.

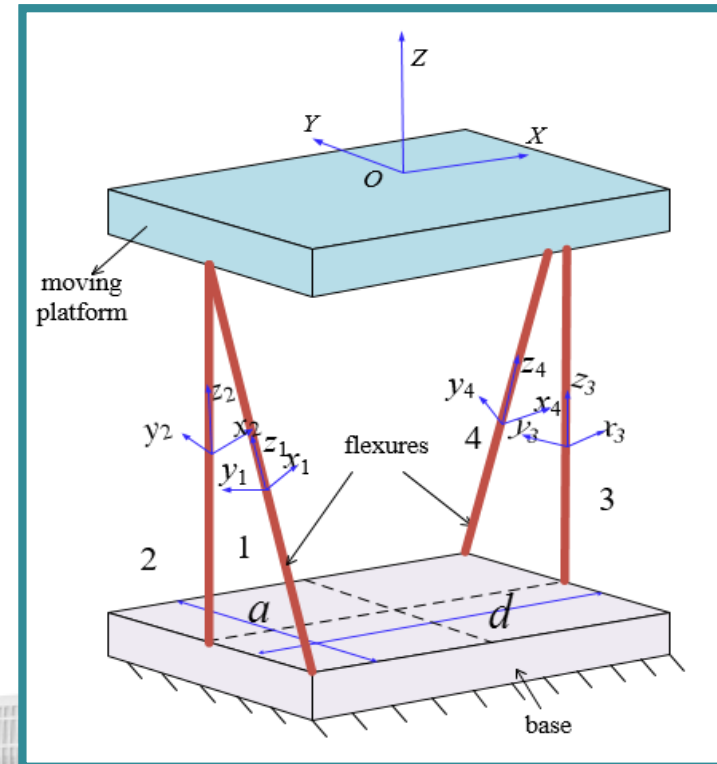
$$\begin{pmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \delta_x \\ \delta_y \\ \delta_z \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_x \\ f_y \\ f_z \end{pmatrix}$$



Part2 : Analysis

➤ Parasitic motion error analysis

$$C_{0-DOS} = \begin{pmatrix} c_{11} & c_{12} & 0 & c_{14} & c_{15} & 0 \\ c_{21} & c_{22} & 0 & c_{24} & c_{25} & 0 \\ 0 & 0 & c_{33} & 0 & 0 & c_{36} \\ c_{41} & c_{42} & 0 & c_{44} & c_{45} & 0 \\ c_{51} & 0 & c_{53} & c_{54} & c_{55} & 0 \\ 0 & 0 & c_{63} & 0 & 0 & c_{66} \end{pmatrix}$$



$$\xi_0 = \theta_x + \theta_y + \delta_x + \delta_y = c_{14} f_x + c_{24} f_x + c_{44} f_x + c_{54} f_x$$

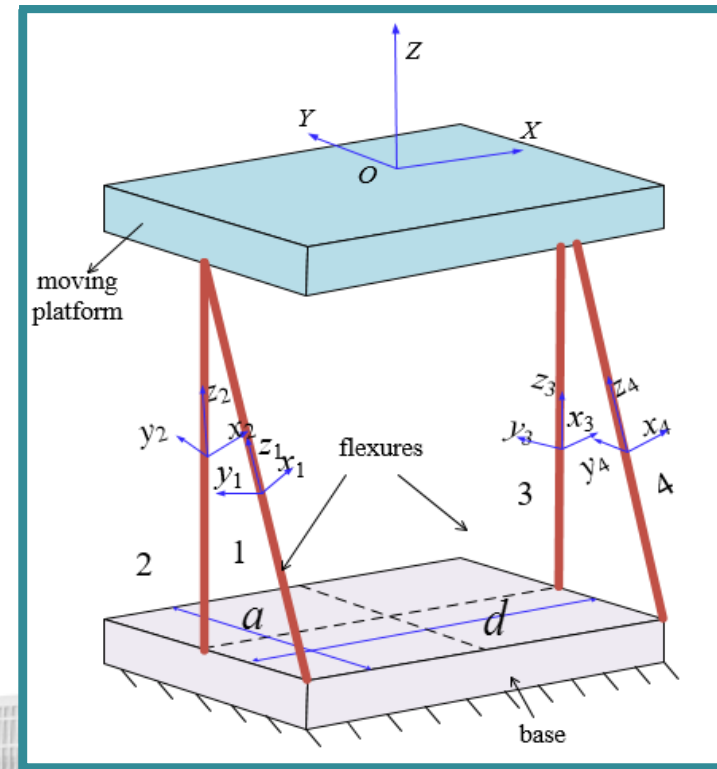
Part2 : Analysis

➤ Parasitic motion error analysis

$$C_{1-DOS} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & c_{16} \\ 0 & c_{22} & c_{23} & c_{24} & 0 & 0 \\ 0 & c_{32} & c_{33} & c_{34} & 0 & 0 \\ 0 & c_{42} & c_{43} & c_{44} & 0 & 0 \\ c_{51} & 0 & 0 & 0 & c_{55} & c_{56} \\ c_{61} & 0 & 0 & 0 & c_{65} & c_{66} \end{pmatrix}$$



$$\xi_1 = \theta_y + \theta_z + \delta_x = c_{24} f_x + c_{34} f_x + c_{44} f_x$$



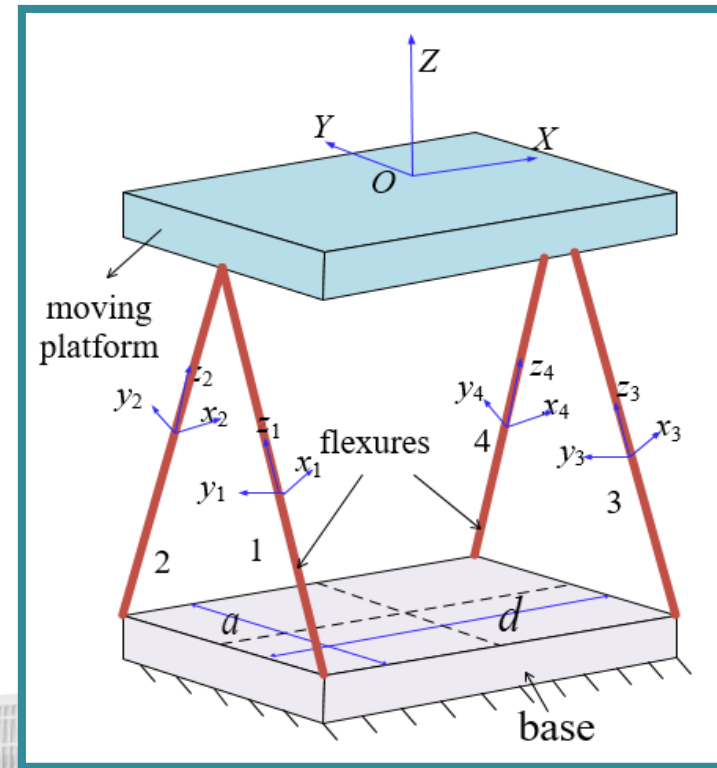
Part2 : Analysis

➤ Parasitic motion error analysis

$$C_{2-DOS} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & c_{15} & 0 \\ 0 & c_{22} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & c_{42} & 0 & c_{44} & 0 & 0 \\ c_{51} & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$



$$\xi_2 = \theta_y + \delta_x = c_{24} f_x + c_{44} f_x$$



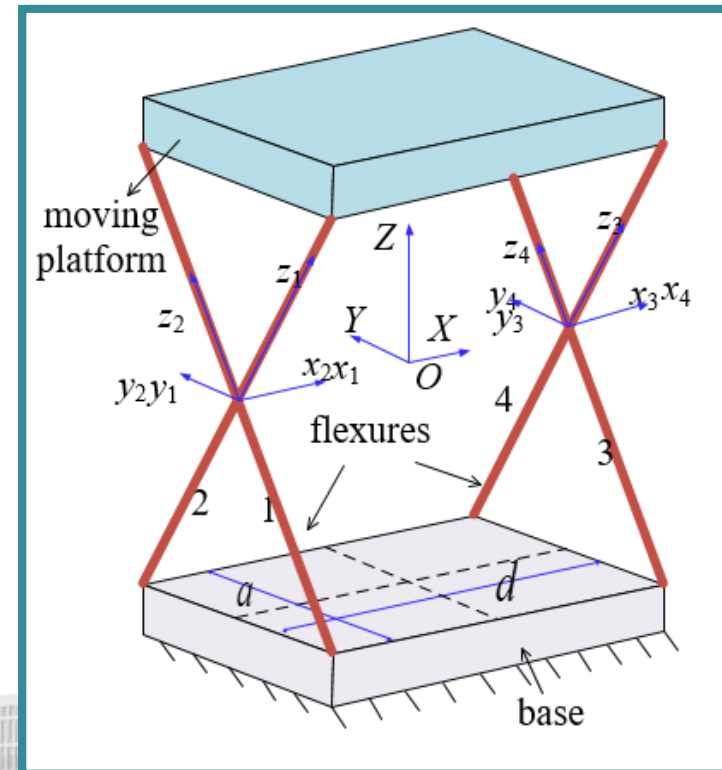
Part2 : Analysis

➤ Parasitic motion error analysis

$$C_{3\text{-DOS}} = \begin{pmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$



$$\xi_3 = \delta_x = c_{44} f_x$$



➤ The best design

3-DoS type

Because of all symmetric planes



Part3 : Verification

➤ Finite element analysis

ANSYS Rigid platform: SOLID 186
15.0

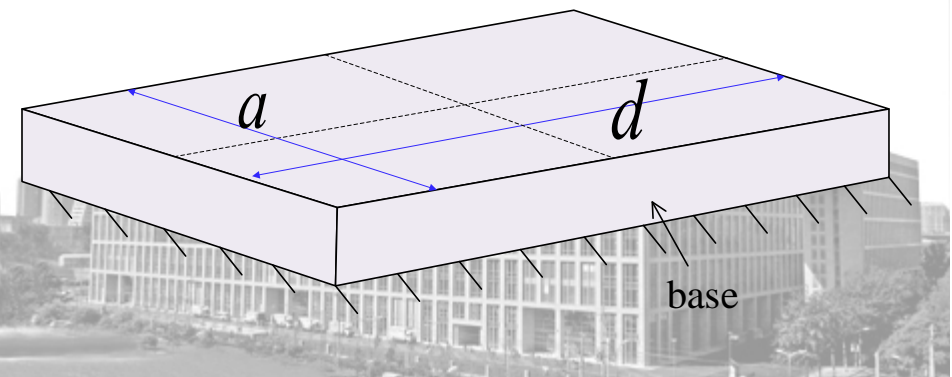
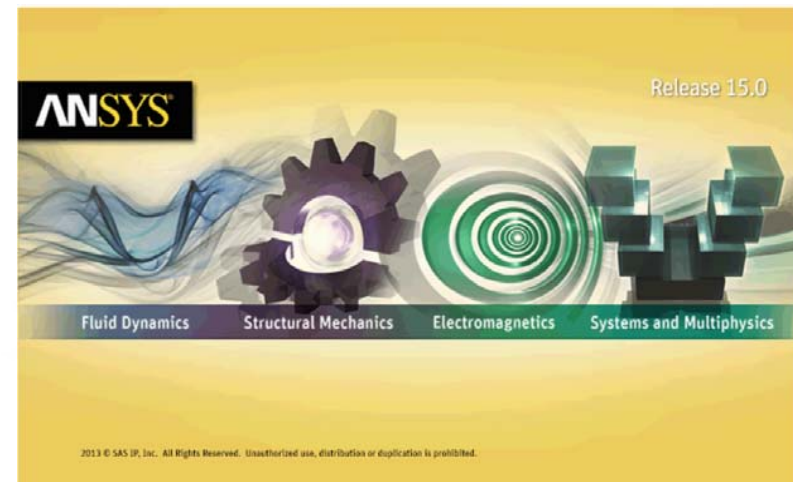
Flexure beam: BEAM 189

$E=70 \text{ GPa}$, $\mu=0.34$

Cross section radius: 5 mm

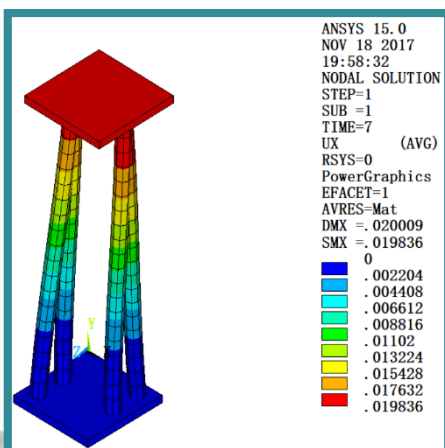
Height between platforms: 200 mm

$a=b= 60 \text{ mm}$

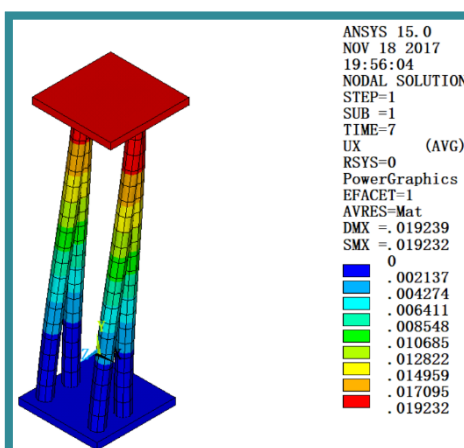


Part3 : Verification

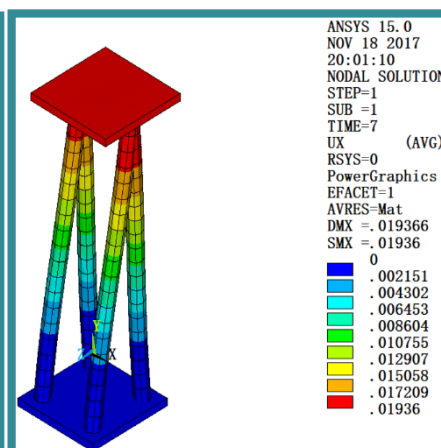
➤ FEA simulations of each mechanism



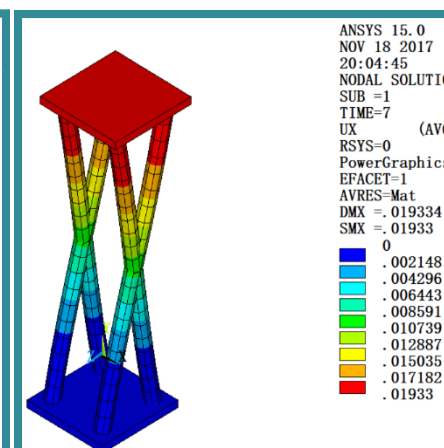
0-DoS
DMX=0.020009
SMX=0.019836



1-DoS
DMX=0.019239
SMX=0.019232



2-DoS
DMX=0.019366
SMX=0.01936



3-DoS
DMX=0.019334
SMX=0.01933

➤ Optimize the 3-DoS flexure mechanism



The entry c_{11} and c_{44} should be the dominant ones for ensuring a rotation along the x-axis and a translational about x-axis.

$$c_{11} = \frac{a}{E\pi r^4 \sin \theta}$$

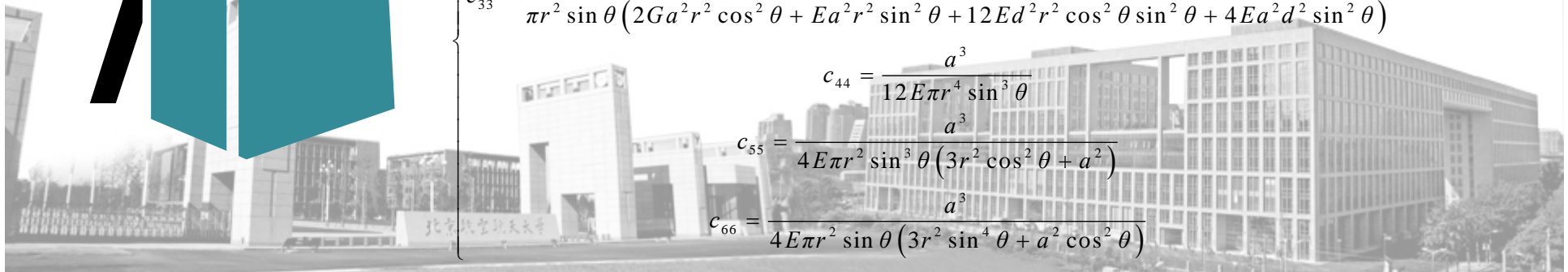
$$c_{22} = \frac{a^3}{\pi r^2 \sin \theta (2Ga^2 r^2 \sin^2 \theta + 12Ed^2 r^2 \sin^4 \theta + Ea^2 r^2 \cos^2 \theta + 4Ea^2 d^2 \cos^2 \theta)}$$

$$c_{33} = \frac{a^3}{\pi r^2 \sin \theta (2Ga^2 r^2 \cos^2 \theta + Ea^2 r^2 \sin^2 \theta + 12Ed^2 r^2 \cos^2 \theta \sin^2 \theta + 4Ea^2 d^2 \sin^2 \theta)}$$

$$c_{44} = \frac{a^3}{12E\pi r^4 \sin^3 \theta}$$

$$c_{55} = \frac{a^3}{4E\pi r^2 \sin^3 \theta (3r^2 \cos^2 \theta + a^2)}$$

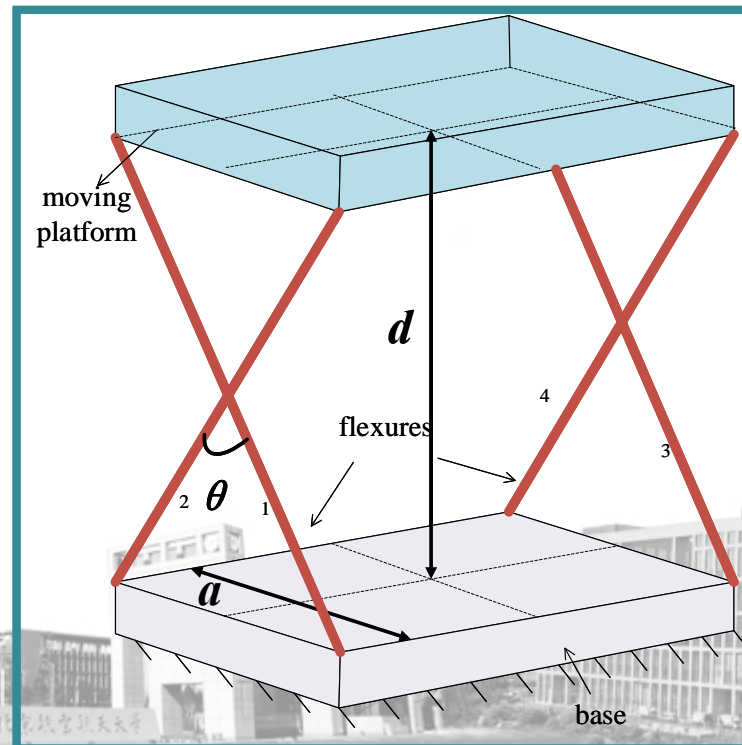
$$c_{66} = \frac{a^3}{4E\pi r^2 \sin \theta (3r^2 \sin^4 \theta + a^2 \cos^2 \theta)}$$



Part4 : Optimization

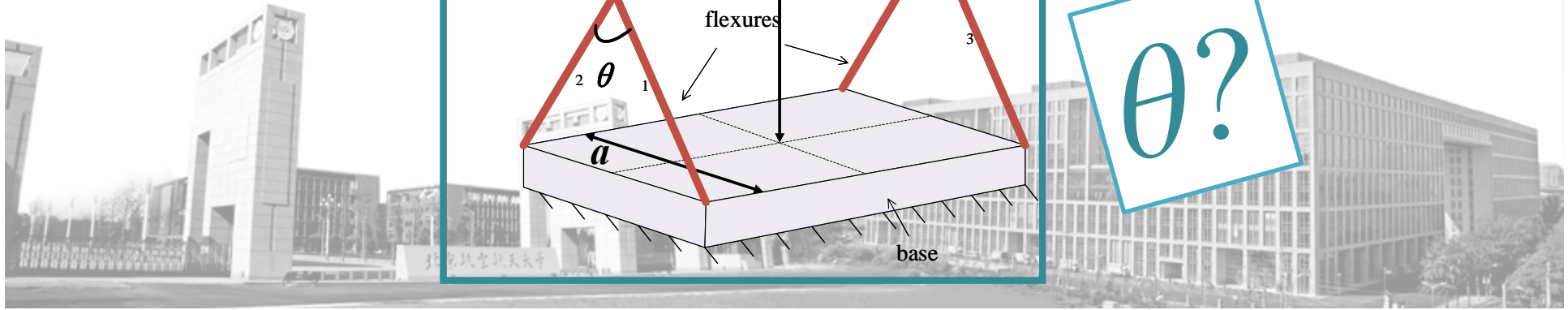
➤ Optimize the 3-DoS flexure mechanism

$a?$



$d?$

$\theta?$

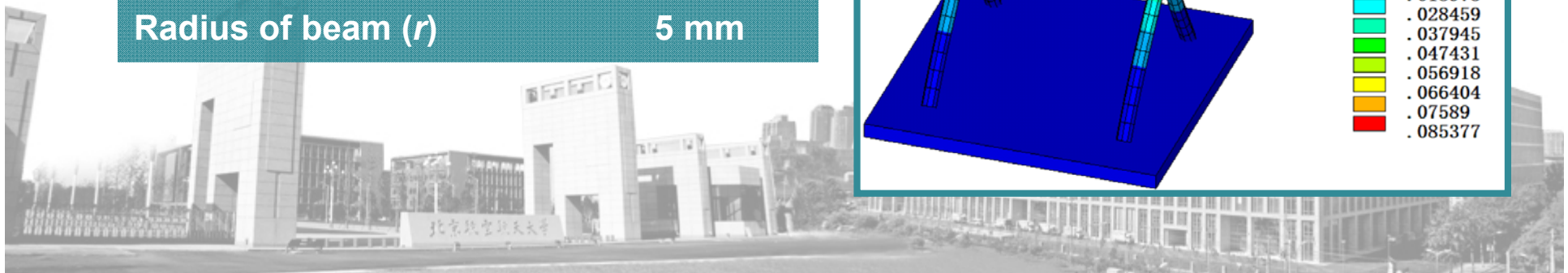
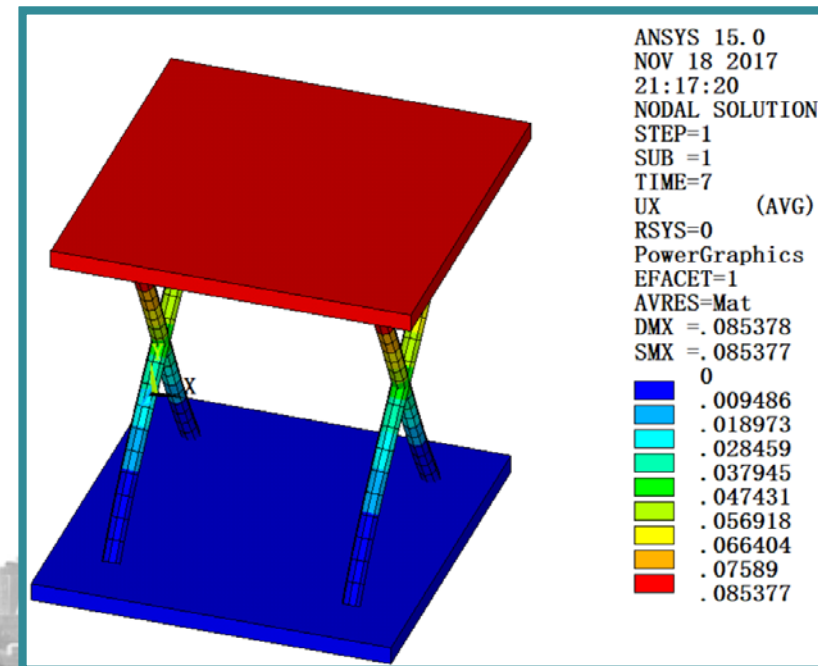


Part4 : Optimization

➤ Optimize the 3-DoS flexure mechanism

Optimal parameters for 3-DoS flexure model

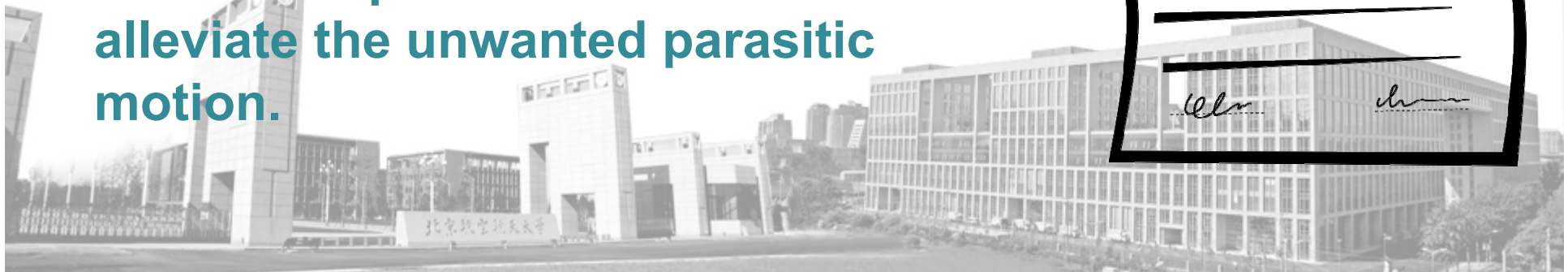
Beam orientation (θ)	$\theta = \pi/4$
Two end points distance (a)	200 mm
Interval distance (d)	200 mm
Radius of beam (r)	5 mm



Part5 : Discussion

➤ **L**arger DoS, better result.

In the design process, it is indeed better to generate as many DoS as possible from the very beginning. With the different DoS, people can further adopt certain methods to alleviate the unwanted parasitic motion.



➤ **A**pplication of flexure mechanisms with cylindrical motion

The joint for realizing a snake-link robot's three dimensions' gaits.

For the in-pipe inspection robots with fast and continuous rate together with required accuracy.



Part5 : Conclusion

01

Design

Using FACT method to design flexure mechanisms with different number of symmetric planes.

02

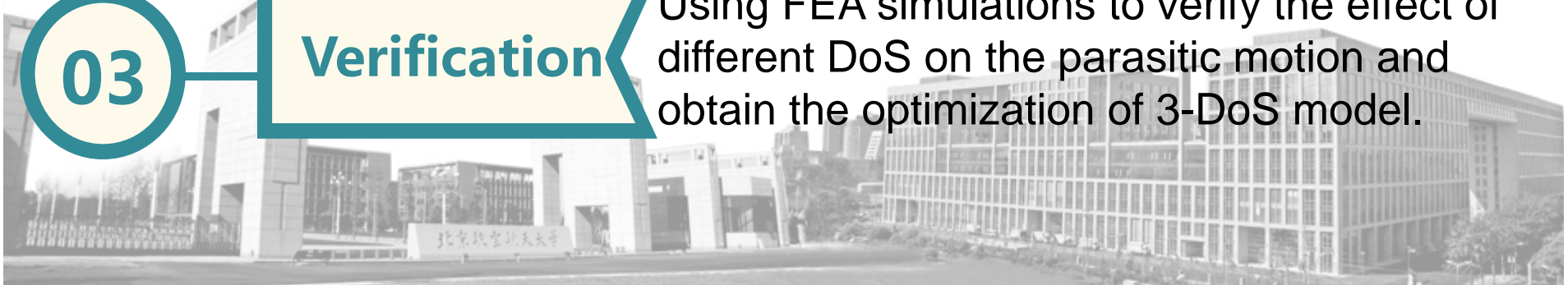
Analysis

Using overall compliance matrix to analyze the effect of symmetrical geometry on the kinetostatic characteristics.

03

Verification

Using FEA simulations to verify the effect of different DoS on the parasitic motion and obtain the optimization of 3-DoS model.



Acknowledge



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BEIJING UNIVERSITY OF AERONAUTICS AND ASTRONAUTICS

THANK YOU

