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Quasi-Static and Dynamic Behaviors of Helical Gear System with Manufacturing Errors

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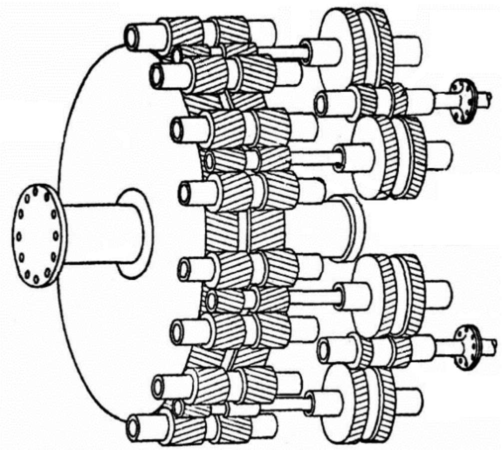
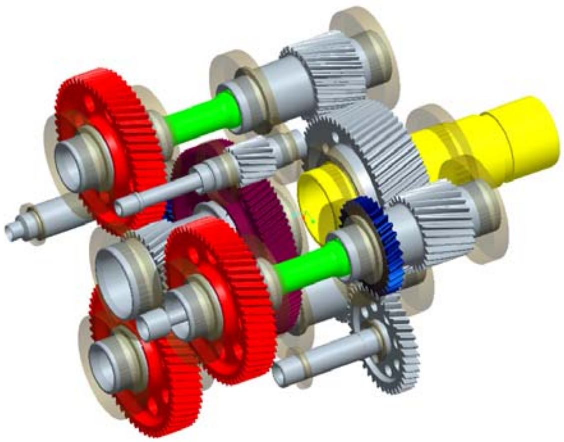
Outline

1. Background and Motivation
2. Improved LTCA Model
3. Dynamic Model
4. Results and Discussion
5. Conclusions

Background and Motivation

- Helical /double-helical gears are widely used in marine, automotive and other industry applications

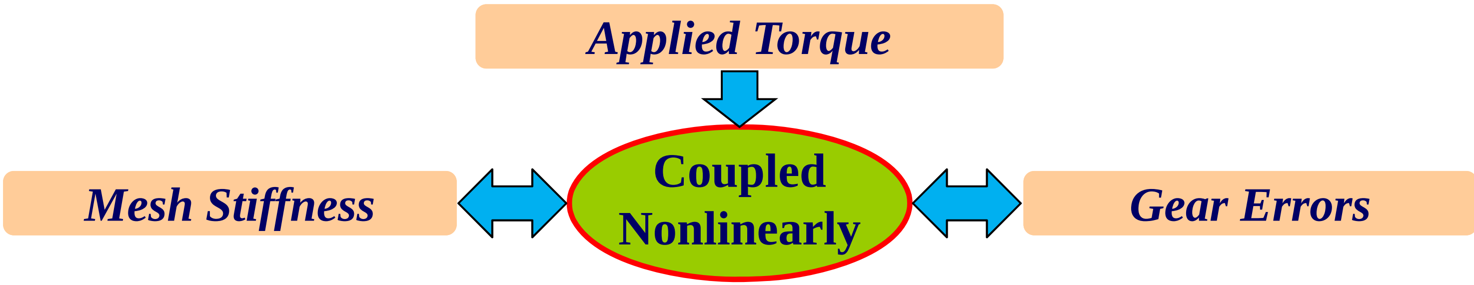




- Prediction and control of system vibration and noise

- Relationship between coupled dynamic excitations and dynamic behaviors of gear system

Background and Motivation

- **Time-varying mesh stiffness and gear errors are the two main excitations for gear transmission**
- **Gear errors include short-term and long-term components**



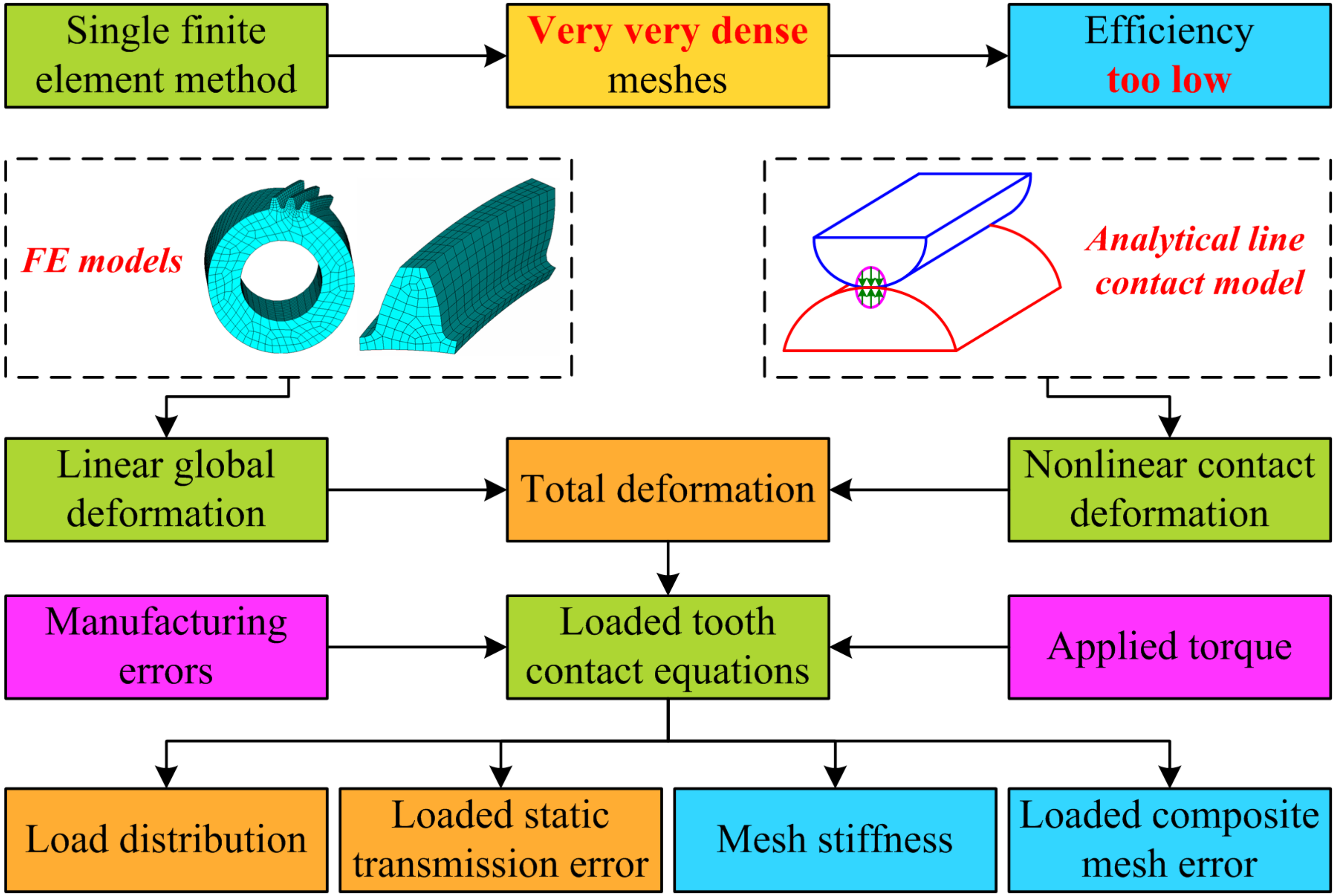
Objective

- **To reveal the coupling mechanism among time-varying mesh stiffness, gear errors and applied torque**
- **To investigate the dynamic behavior of helical gear system based on coupling model of dynamic excitations**

Outline

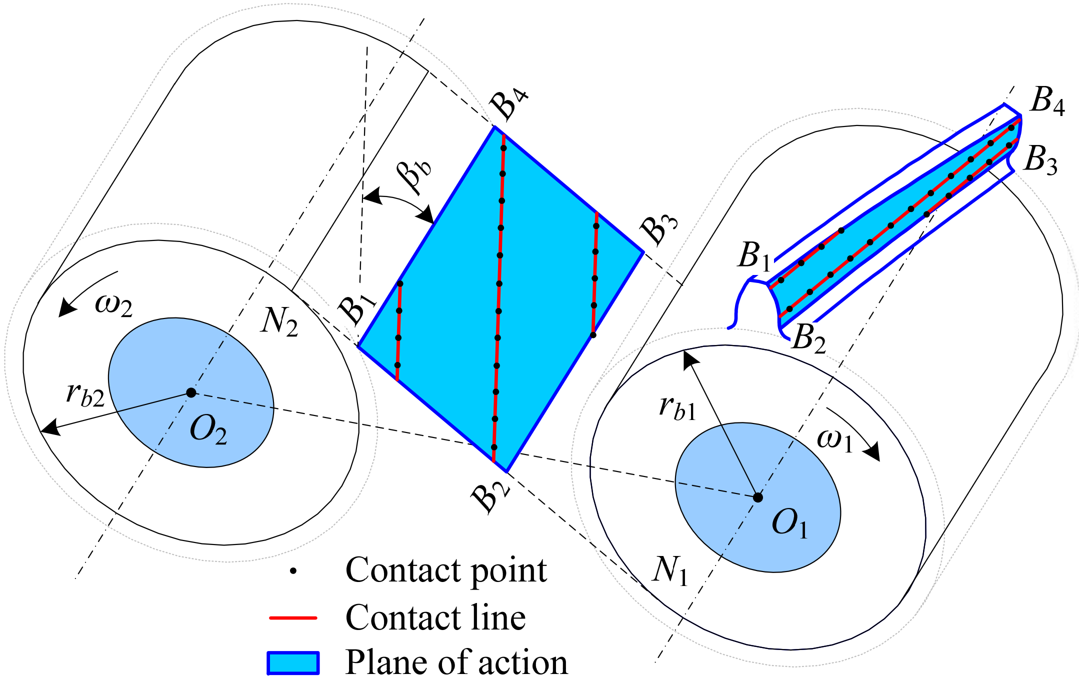
Improved LTCA Model

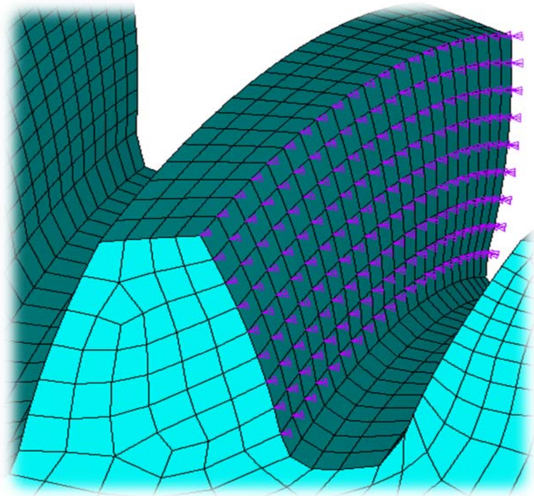
Technology Strategy



Improved LTCA Model

Plane of Action for a Helical Gear Pair





Sub-structure method

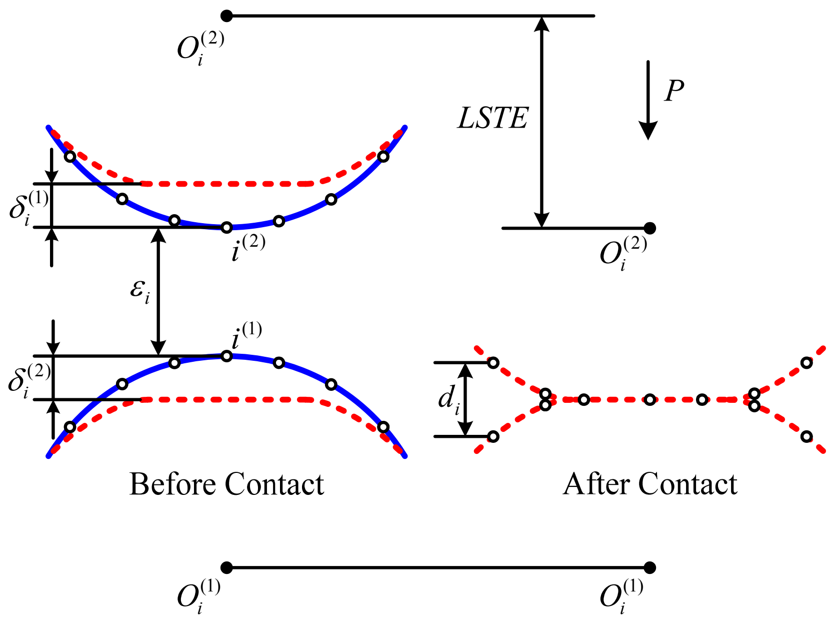
Interpolation method

Global deformation compliance

Three-dimensional line contact problem → Point contact problem

Improved LTCA Model

Tooth Contact in an Engagement Position



Global Deformation $\delta_{Global} = [\lambda]_{Global}\{F\}$

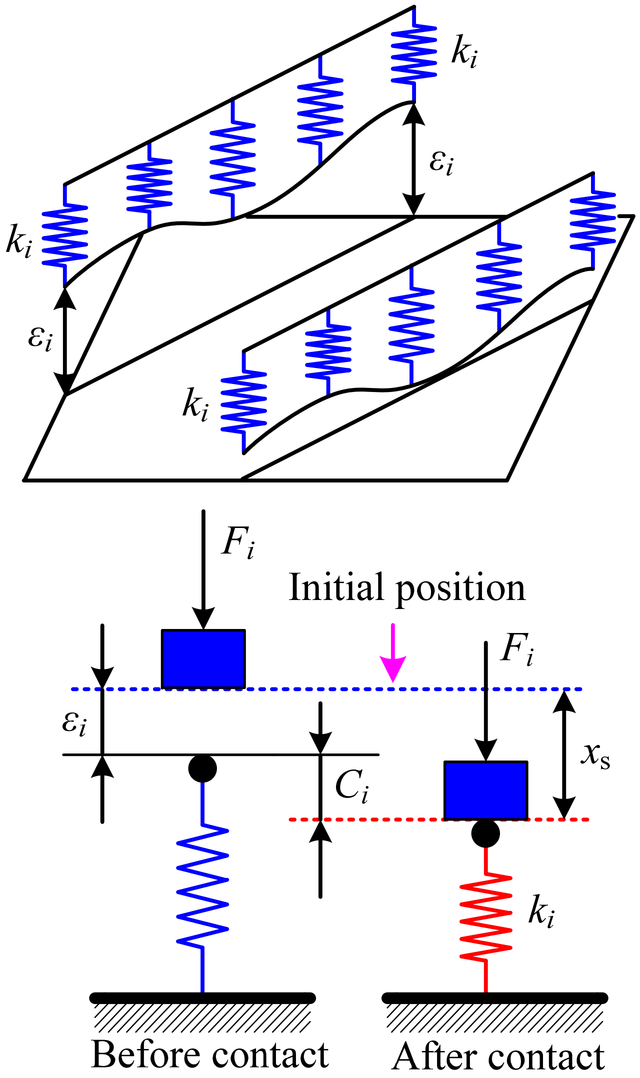
Local Contact Deformation u_{Local}

$$\begin{cases} [\lambda]_{Global}\{F\} + \{u\}_{Local} + \{\varepsilon\} - LSTE - \{d\} = 0 \\ \sum_{i=1}^{n} F_i = \{I\}\{F\} = P \\ if\ \ F_i > 0,\ d_i = 0 \\ if\ \ F_i = 0,\ d_i > 0 \end{cases}$$

- **Analytical Formula is employed to determine the Nonlinear Local Contact Deformation of gear teeth**
- **Iteration method is used to solve the LTCA equations to obtain the Load Distribution {*F*} and loaded Static Transmission Error *LSTE***

Improved LTCA Model

Mesh Stiffness and Loaded Composite Mesh Error



- **Stiffness of Single Contact Point Pair**

$$k_i = F_i / (x_s - \varepsilon_i)$$

- **Mesh Stiffness of the Gear Pair**

$$k_m = \sum_{i=1}^{n} k_i$$

- **Static Load Balance Equation**

$$\sum_{i=1}^{n} F_i = \sum_{i=1}^{n} [k_i (x_s - \varepsilon_i)] = \sum_{i=1}^{n} k_i x_s - \sum_{i=1}^{n} k_i \varepsilon_i$$

$$= k_m x_s - \sum_{i=1}^{n} k_i \varepsilon_i = P$$

$$\Rightarrow \quad x_s - \sum_{i=1}^{n} (k_i \varepsilon_i) / k_m = P / k_m$$

- **Loaded Composite Mesh Error**

$$e_{\mathrm{m}} = \sum_{i=1}^{n} (k_i \varepsilon_i) / k_m$$

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3. **Dynamic Model**
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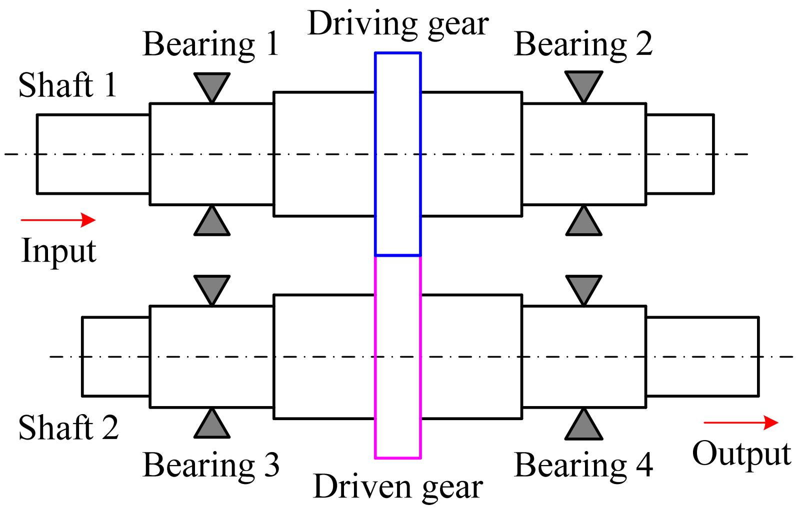
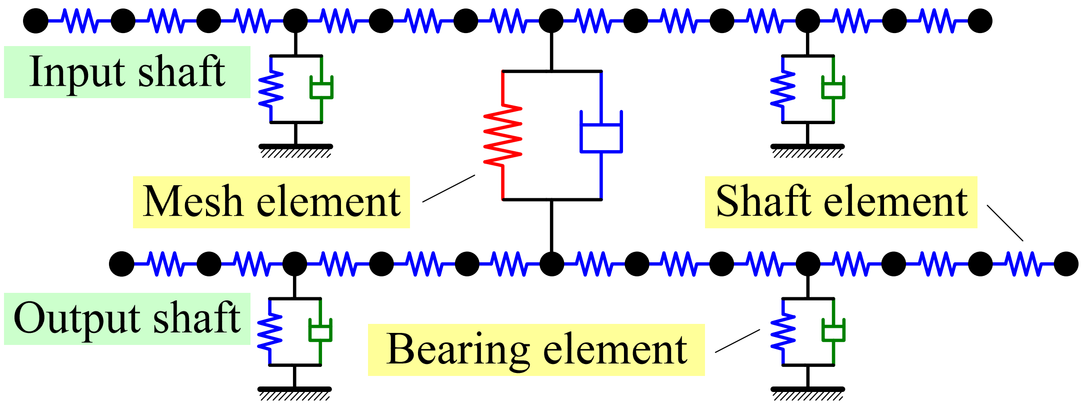
Dynamic Model

Helical Gear-Rotor-Bearing System



Number of Teeth	20/20
Normal Module (mm)	10
Pressure Angle (°)	20
Helix Angle (°)	25/-25
Face Width (mm)	60
Accuracy Grade	ISO 6

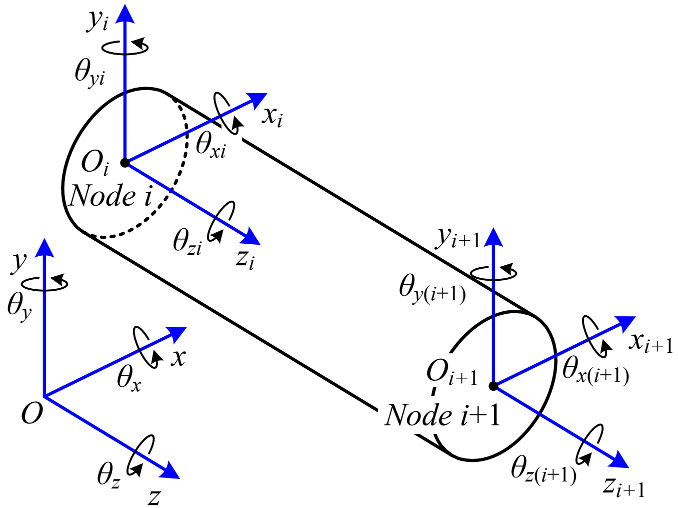
Generalized Finite Element Dynamic Model



- ✓ Shaft Deformation
- ✓ Power Flow Direction
- ✓ Bearing Location
- ✓ Gear Mesh Excitation

Dynamic Model

Shaft Element – Timoshenko beam



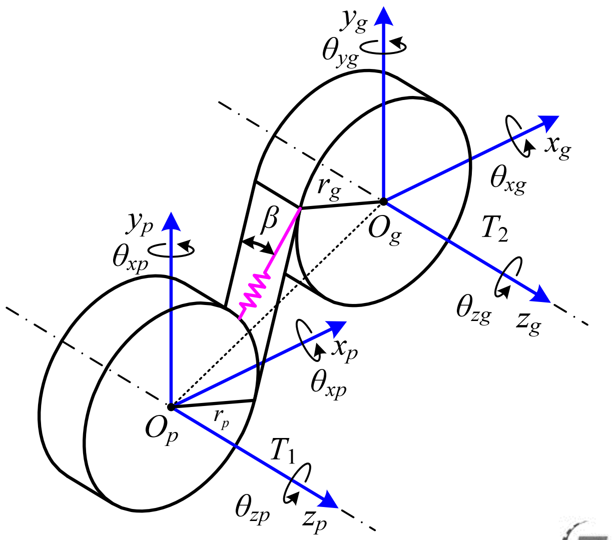
- **The generalized coordinate vector**

$$q_s = \{x_i, y_i, z_i, \theta_{xi}, \theta_{yi}, \theta_{zi}, x_{i+1}, y_{i+1}, z_{i+1}, \theta_{x(i+1)}, \theta_{y(i+1)}, \theta_{z(i+1)}\}^T$$

- **The matrix form of motion equation**

$$M_s \ddot{q}_s(t) + C_s \dot{q}_s(t) + K_s q_s(t) = 0$$

Mesh Element



- **The generalized coordinate vector**

$$q_m = \{x_p, y_p, z_p, \theta_{xp}, \theta_{yp}, \theta_{zp}, x_g, y_g, z_g, \theta_{xg}, \theta_{yg}, \theta_{zg}\}^T$$

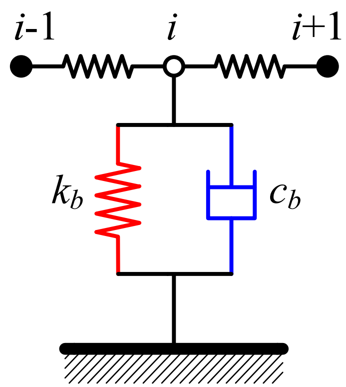
- **The matrix form of motion equation**

$$M_m \ddot{q}_m(t) + C_m \dot{q}_m(t) + K_m(t)[q_m(t) - e_m(t)] = 0$$

Dynamic Model

Bearing Element

- A spring-damping element that connects shaft node with foundation



- The matrix form of motion equation

$$M_b\ddot{q}_b(t)+C_b\dot{q}_b(t)+K_bq_b(t)=0$$

- Both roller bearing and sliding bearing can be considered

Finite Element Dynamic Model of the System

- The matrix form of static balance equation of gear system

$$K_G(t)[X_{Gs}(t)-E_G(t)]=P_{G0}$$

- The matrix form of global motion equation of the system

$$M_G\ddot{X}_G(t)+C_G\dot{X}_G(t)+K_G(t)[X_G(t)-E_G(t)]=P_{G0}$$

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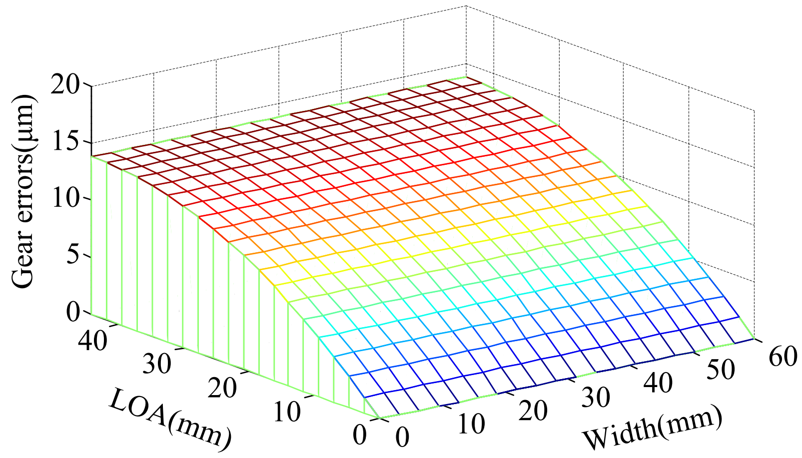
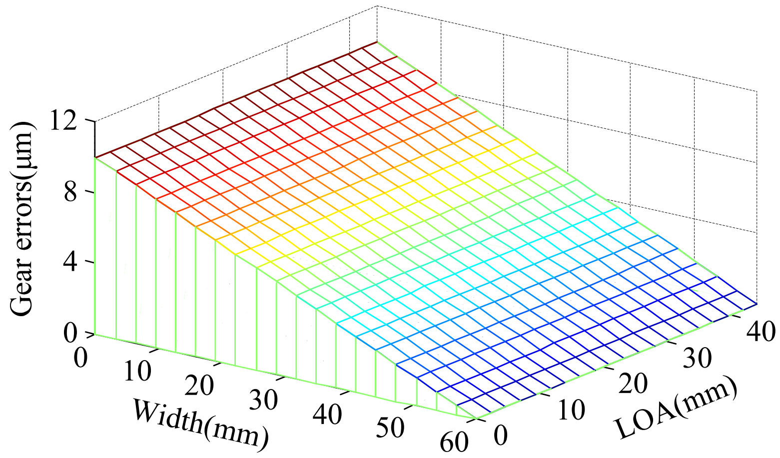
Results and Discussion – Short-Term Errors

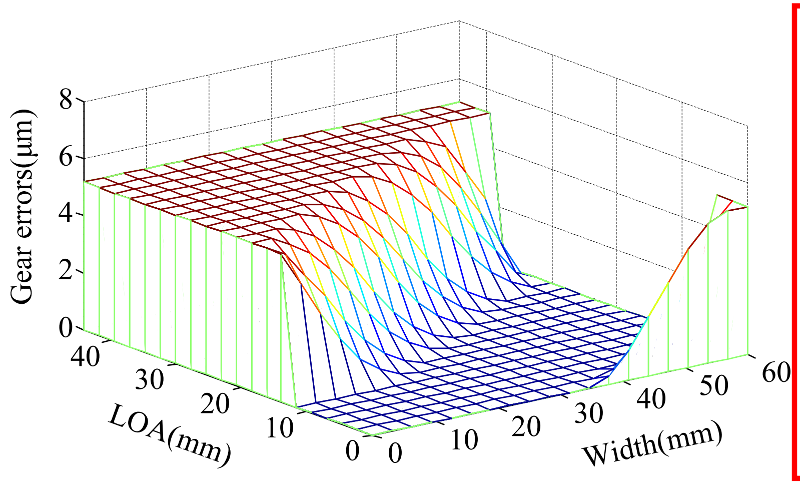
Assumption and Composite of Short-Term Gear Errors



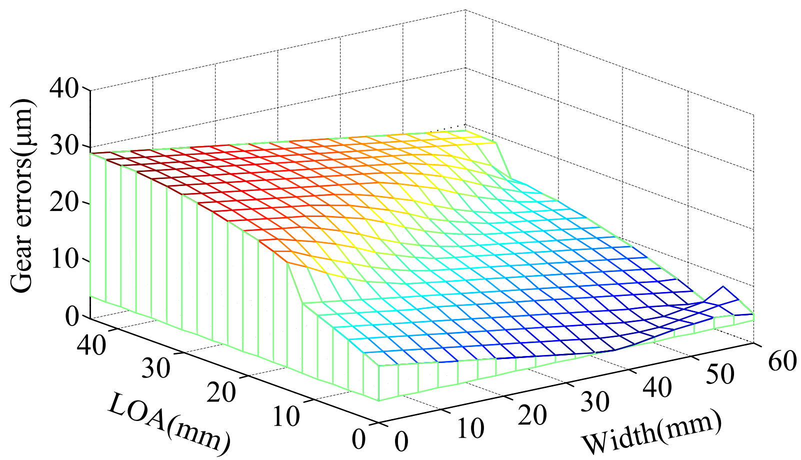
Profile errors



Helix errors



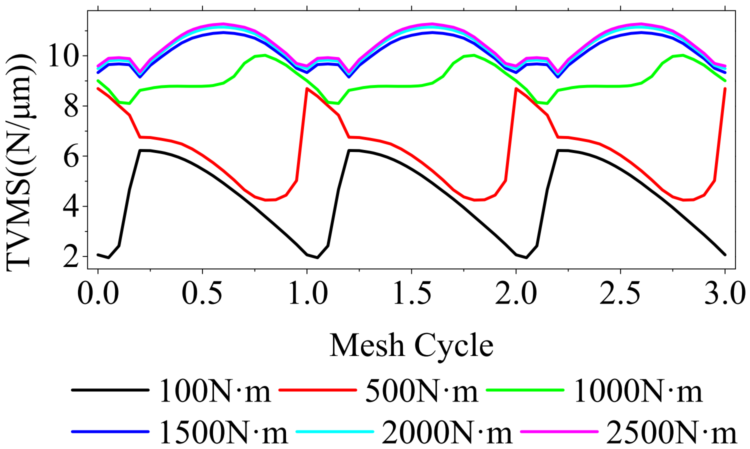
Pitch errors



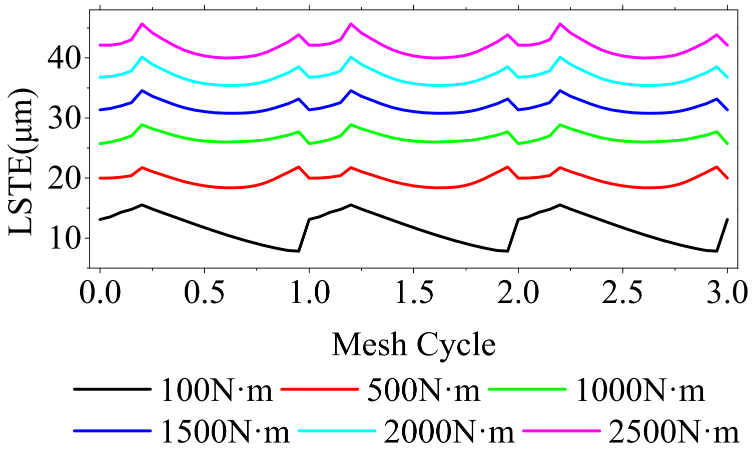
Composite errors

Results and Discussion – Short-Term Errors

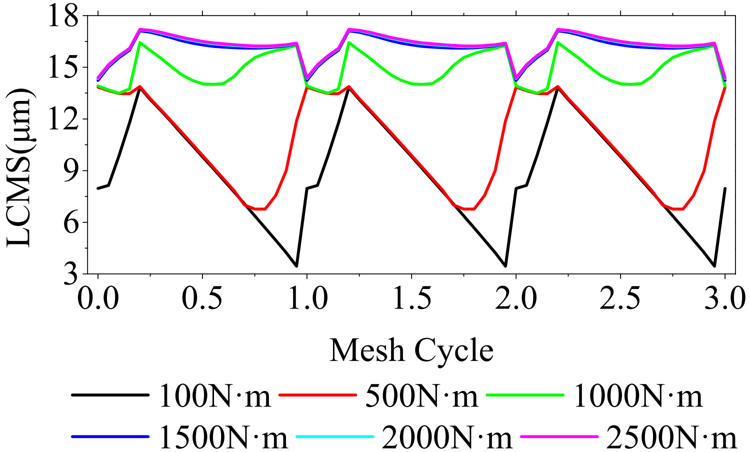
Quasi-Static Analysis – TVMS, LSTE and LCMS



Time-Varying Mesh Stiffness (TVMS)



Loaded Static Transmission Error (LSTE)

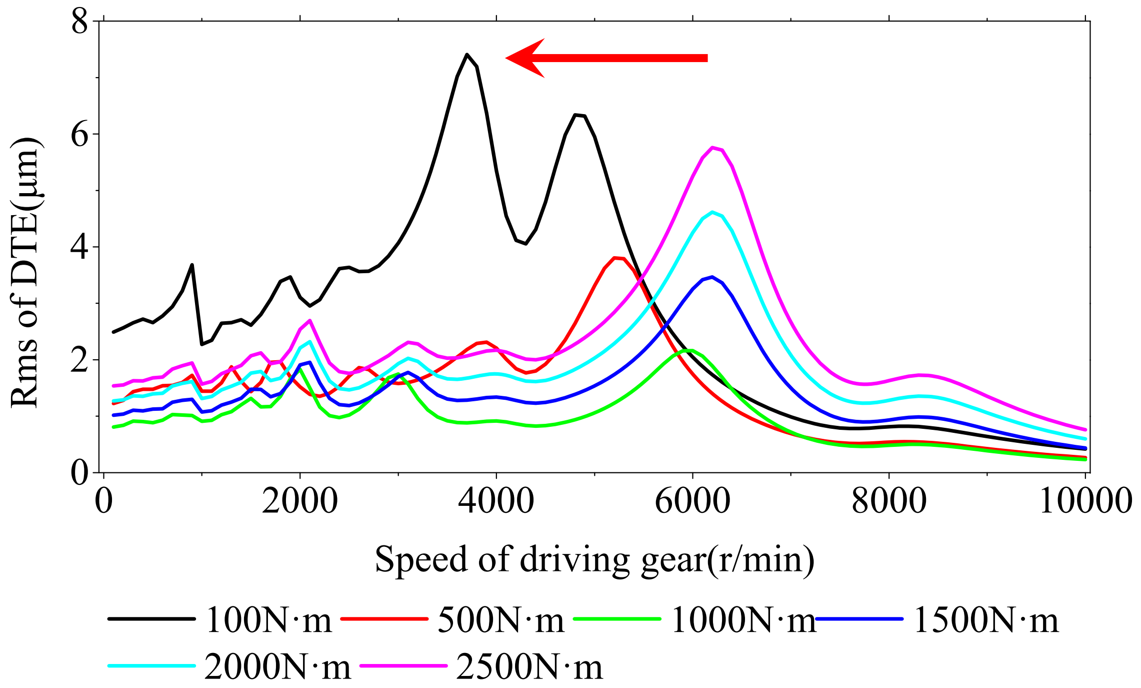


Loaded Composite Mesh Error (LCMS)

- **T_{out} ↓ TVMS ↓ as a result of partial contact loss**
- **T_{out} ↑ LSTE ↑ due to increased deformation of gear teeth**
- **T_{out} ↑ LCMS ↑ due to enlarged contact pattern**

Results and Discussion – Short-Term Errors

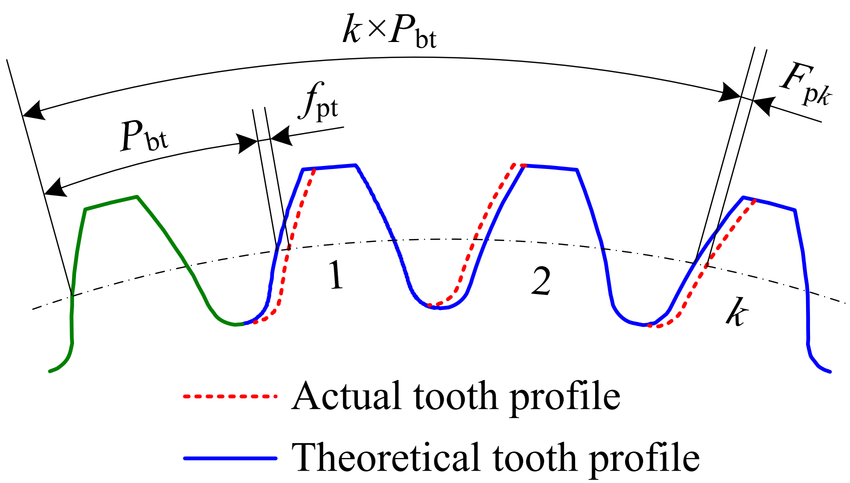
Dynamic Analysis



- **The resonance speed of the system decreases in lower applied torque as a result of the decreased mesh stiffness**
- **Lower applied torque will bring larger system vibration**

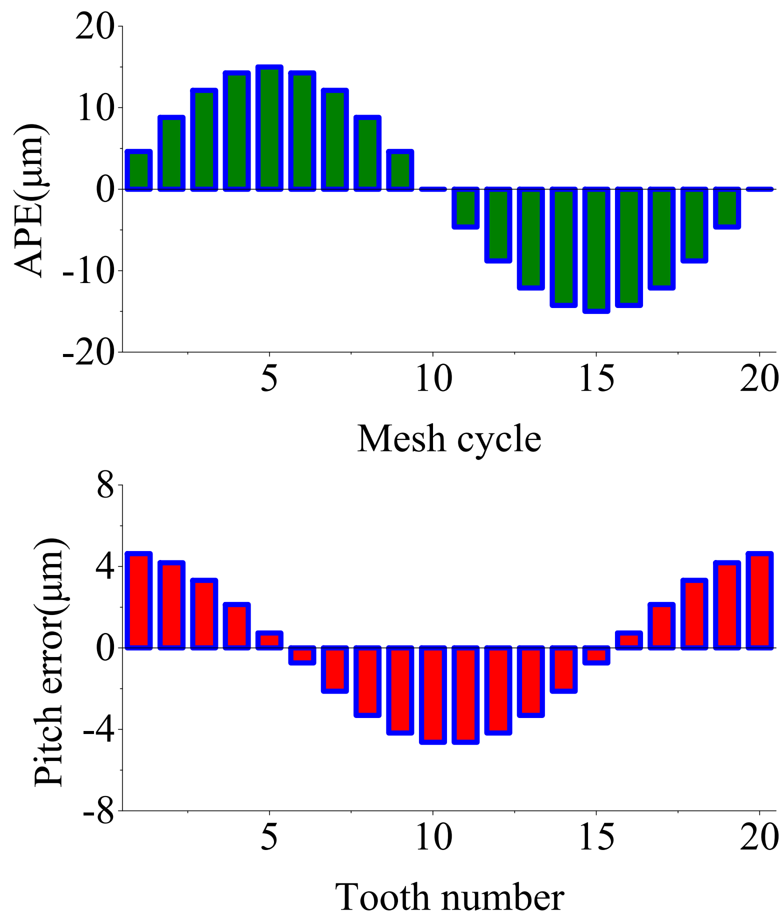
Results and Discussion – Long-Term Errors

Assumption of Accumulative Pitch Error



- ✓ P_{bt} - **Theoretical Circular Pitch**
- ✓ f_{pt} - **Circular Pitch Error**
- ✓ $k \times f_{pt}$ – **Accumulative Pitch Error**

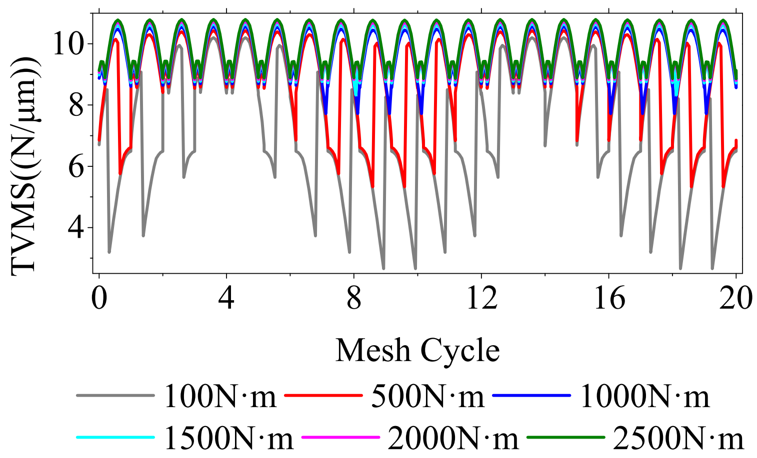
Accumulative pitch error (APE)



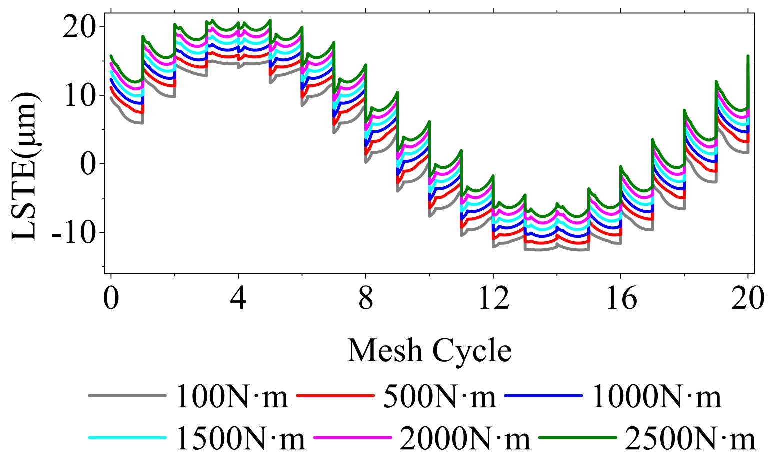
Assumption of Accumulative pitch error

Results and Discussion – Long-Term Errors

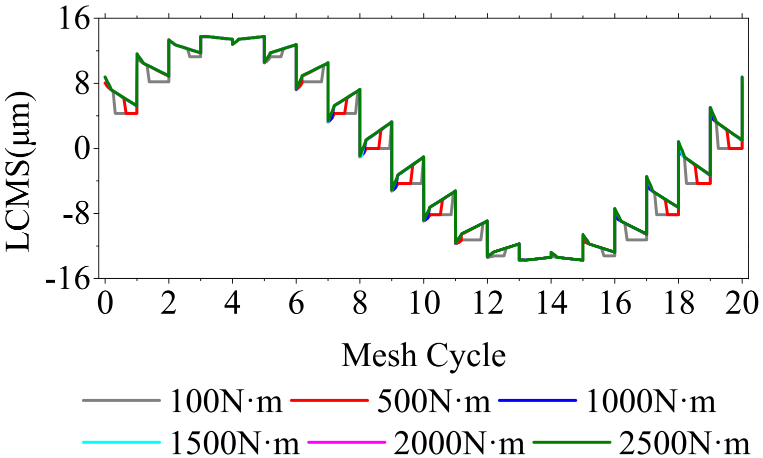
Quasi-Static Analysis – TVMS, LSTE and LCMS



Time-Varying Mesh Stiffness (TVMS)



Loaded Static Transmission Error (LSTE)



Loaded Composite Mesh Error (LCMS)

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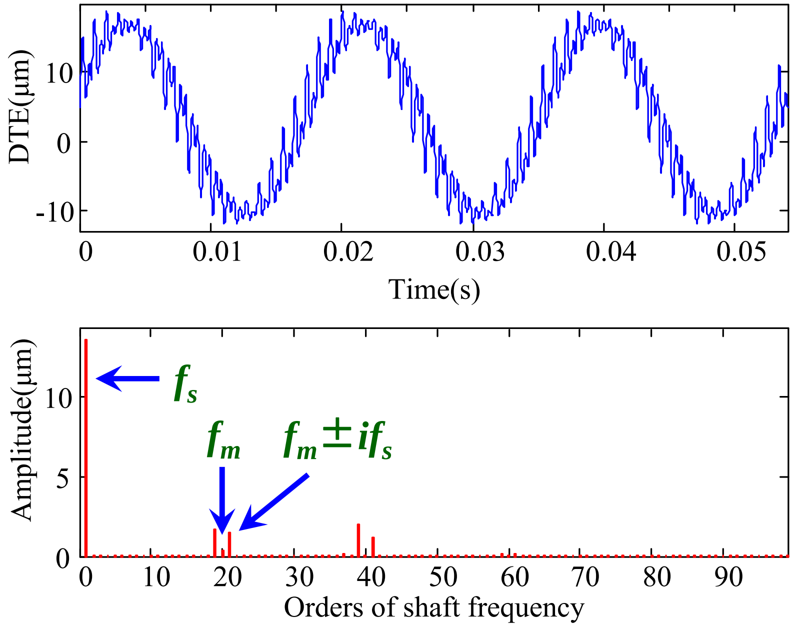
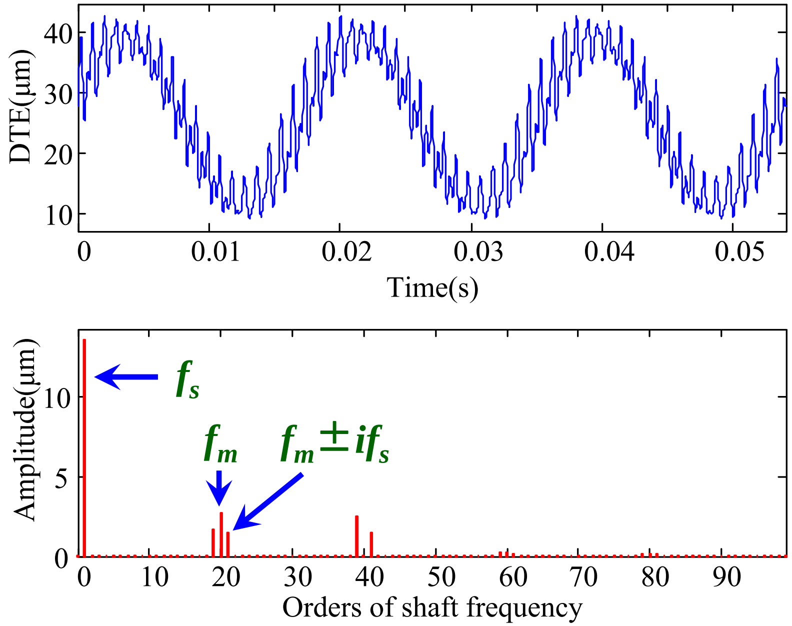
Results and Discussion – Long-Term Errors

Dynamic Analysis – Dynamic Transmission Error (DTE)



DTE at 300N·m applied torque

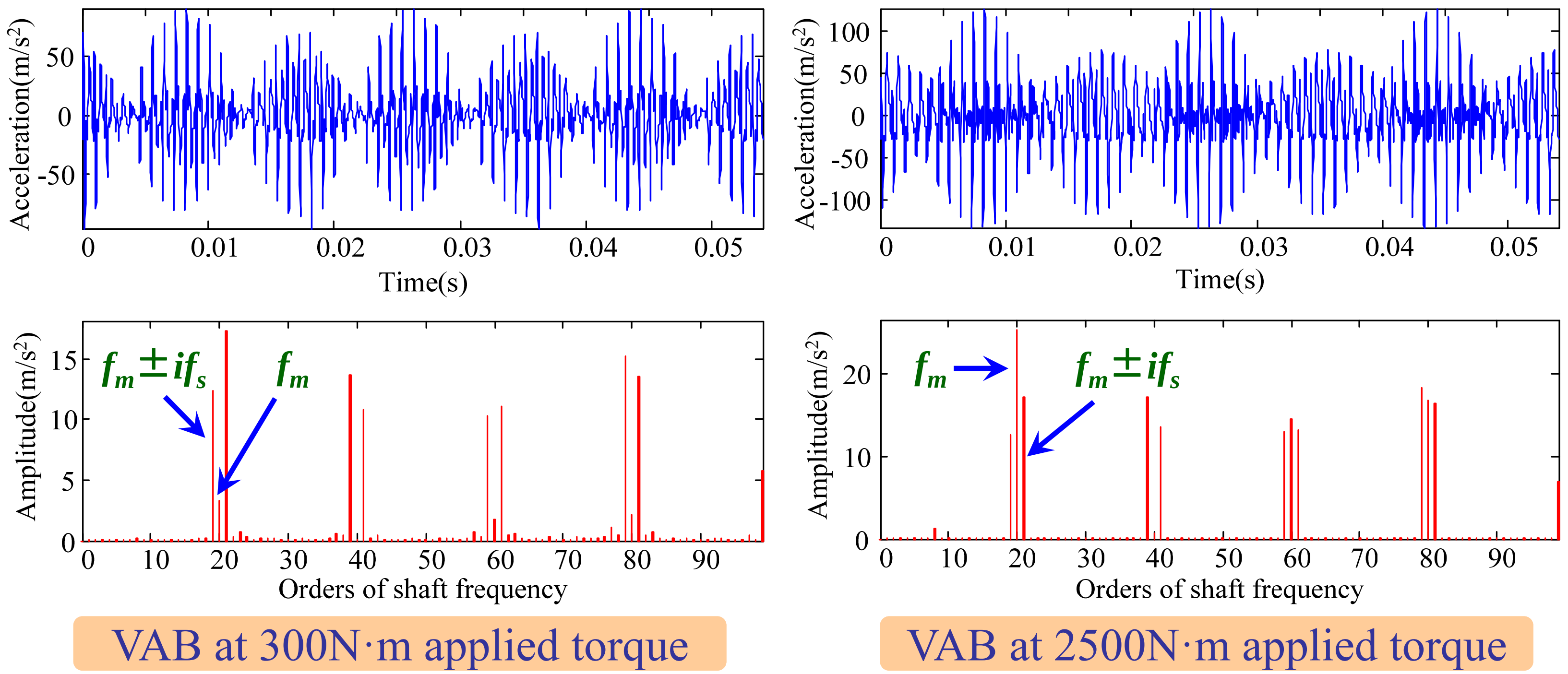


DTE at 2500N·m applied torque

- **Shaft frequency is predominant**
- **Amplitude and frequency modulation can be observed**
- **The mesh frequency and its harmonic components increase as a result of increased applied torque**

Results and Discussion – Long-Term Errors

Dynamic Analysis – Vibration Acceleration of Bearing (VAB)



- Amplitude and frequency modulation can be observed
- Sideband frequency components are predominant when applied torque is lower
- The mesh frequency and its harmonic components will be enhanced with the increase of applied torque

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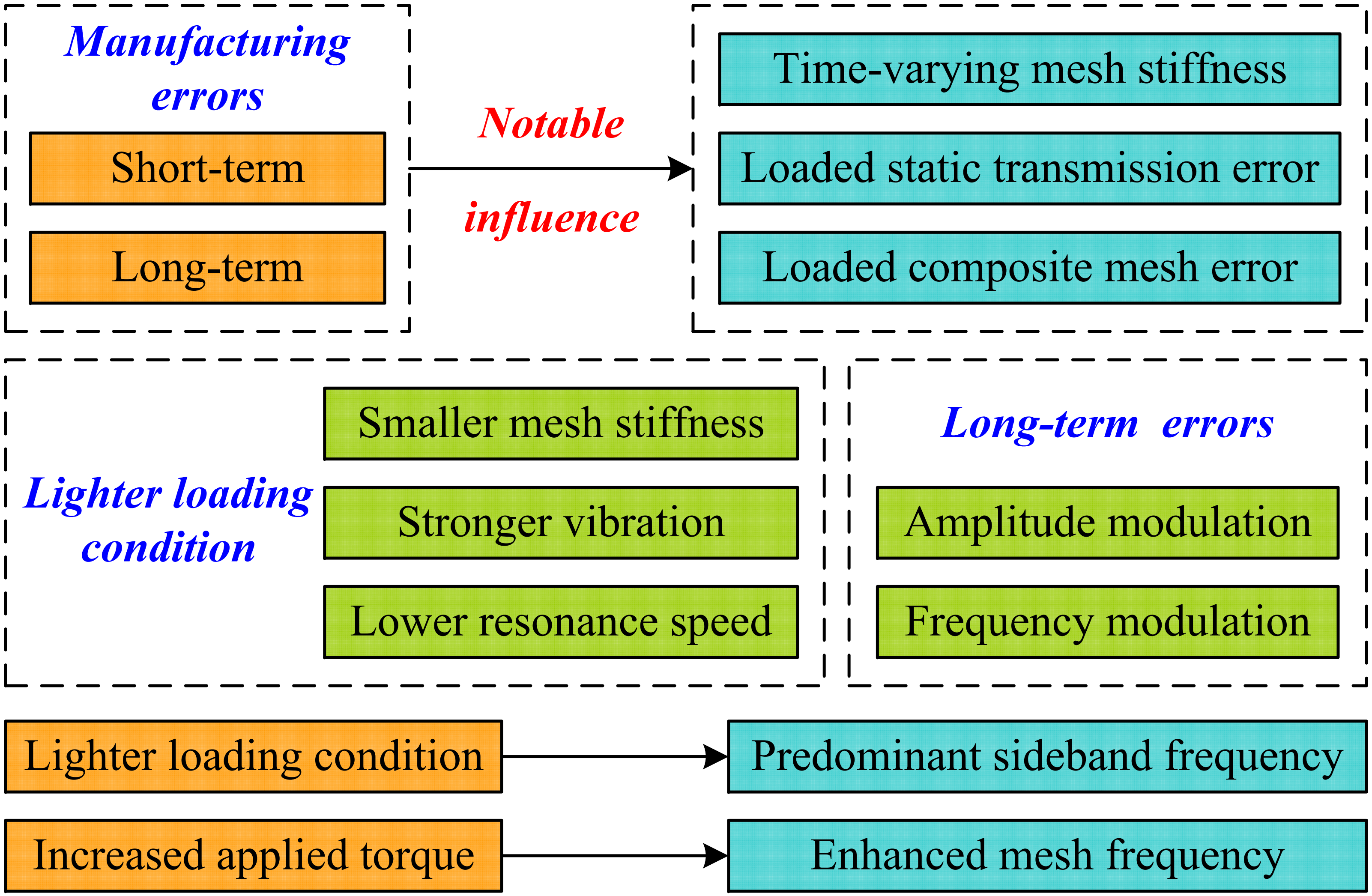
Conclusions



Thanks for your attention!

Questions?