Supplementary Information for

Electrical Conductivity Determination of Semiconductors by utilizing Photography, Finite Element Simulation and Resistance Measurement

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Part I: Error estimation for specimen with non-infinitesimal electrodes

A 3d-Model of a sample with specified dimensions for the FE software COMSOL Multiphysics is shown in Fig. S1a. All the electrodes have a width of 2 mm and thickness of 100 µm. Another 3D model of the same geometry but where the four wide electrodes are replaced with infinitely thin electrodes is also constructed in COMSOL Multiphysics. Same materials properties and boundary conditions are applied in both models to simulate the potential difference of the two inner electrodes. Fig. S1b shows that the deviation of potential difference of the inner electrodes in these two models can be about 20 % at various conductivity of the bulk material of the specimens (σ_s).

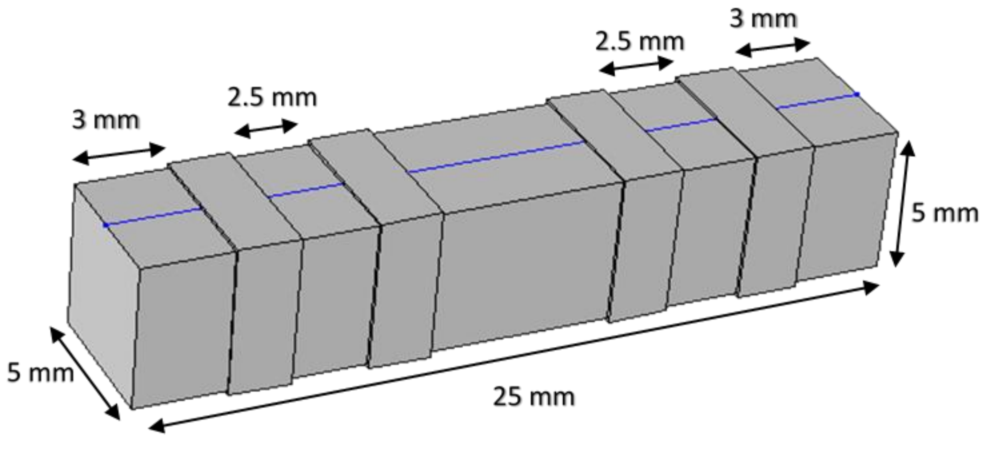

Fig. S1a Dimensions of the 3D model. The electrodes have width of 2 mm and a thickness of 100 µm

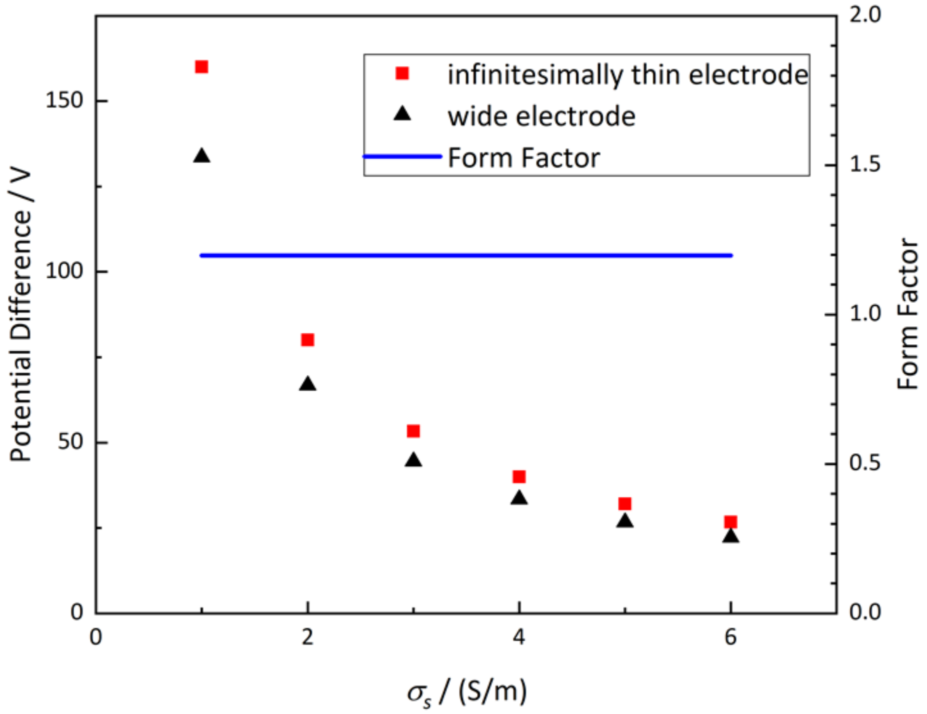

Fig. S 1b Simulated potential difference of the two inner electrodes of the sample in Fig. S1a and a sample with the same dimension and material properties as in Fig. S1a but with infinitesimally thin electrodes. Compared with the case where the electrodes are infinitesimally thin, the potential difference between the wide electrodes is approximately 20% smaller

Part II: How to come from the camera image to the form factor

A detailed explanation of the procedure (recipe) to derive the form factor in six steps is given here. The entire workflow is depicted schematically in Fig. S2. It comprises the six steps from (1) to (6) from the main article and shows how the conductivity is derived based on measurements and simulation of calculating the form factor that contains the information about the geometry of the specimen and the shape of the electrodes.

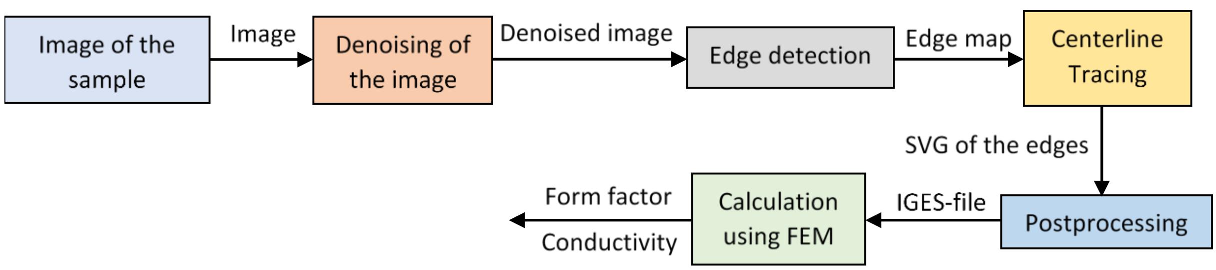

Fig. S2 Routine to calculate the conductivity and the form factor of the sample. After the image of the sample is taken, the image is denoised using a technique called morphological anisotropic diffusion that preserves the edge. The edge of the denoised image is extracted using a canny edge detector. The resulted edge map is a raster image. The pixels of the edged will be traced using centerline tracing to produce a standard vector graphic (SVG). The redundant and false edges will be eliminated during the postprocessing and output a IGES-file. This IGES-file will be imported into the FE software and the form factor can be calculated using the method that is described in the section "Theoretical background of the new method"

Step 1: Image of the sample

Inside a photo light box, place a scale next to the specimen. Place the camera (here the front camera of Samsung Galaxy S8) above the specimens and keep the camera lens parallel to the surface of the specimens. Images may be taken using the automatic mode. Typical images for two specimens are given in Fig. S3.

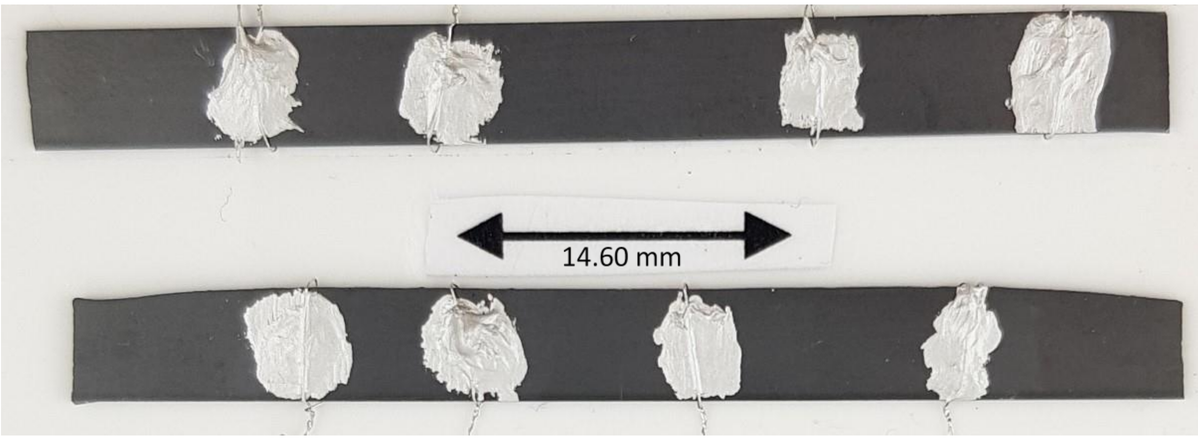

Fig. S3 Images of the two specimens A1 (top) and B1 (bottom) from Fig. 4 from the main article. Images taken by the internal camera in Samsung Galaxy S8 in automatic mode

Step 2: Denoising of the image

Denoise the image of the sample using morphological anisotropic diffusion to maintain edge locations while smoothing small scale feature leading to Fig. S4. The filter size is defined by the matrix size used to perform the discrete calculations for the following diffusion equation:

$$\frac{\partial I}{\partial t} = \nabla \cdot (c(i,j,t)\nabla I)$$

$$\text{With } c(i,j,t) = exp\left\{-\left[\frac{\nabla S}{k}\right]^2\right\} \quad (1)$$

$$\text{Here } \nabla S = (I \cdot B)°B$$

I is the image, *i* and *j* are the pixel coordinates, *t* is time, ***B*** is structure element matrix, is the morphological operation in image processing “close” and ° is the morphological operation in image processing “open”. The discrete form of equation (1) is:

$$I_{i,j,t+\Delta t} = I_{i,j,t} + \Delta t(c_N\nabla_N + c_S\nabla_S + c_E\nabla_E + c_W\nabla_W), \quad (2)$$

Where $I_{i,j,t}$ is the image pixel value at location (*i*,*j*) for solution time *t*, $\nabla_N, \nabla_S, \nabla_E, \nabla_W$ are the image gradients in north, south, east and west directions respectively, c_N, c_S, c_E, c_W are the corresponding diffusion coefficients and Δt is the size of time step. The following parameters are used. The structure element B is a square structuring element of width 5, the time step Δt = 1/3, iteration number is 10 and gradient threshold *k* = 70.

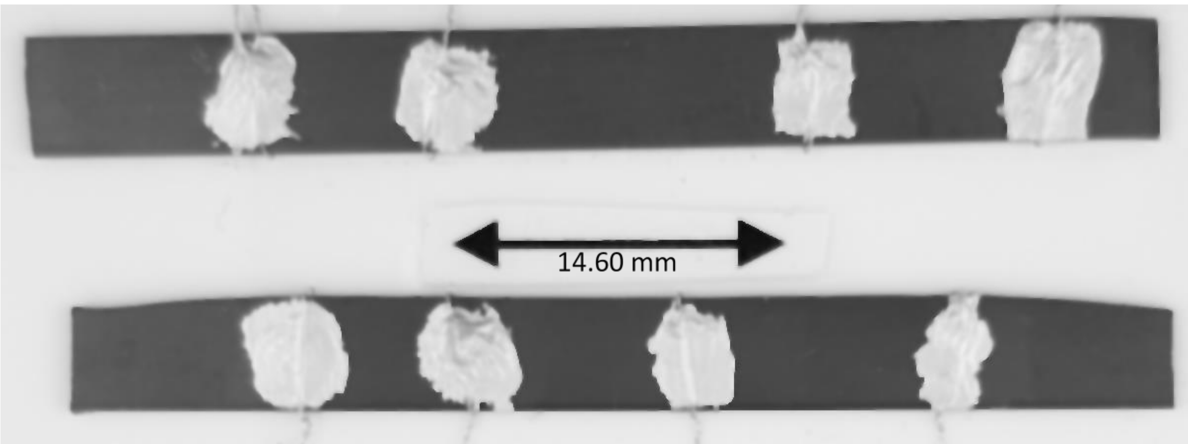

Fig. S4 Image after applying morphological anisotropic diffusion with filtering matrix width 5, iteration times 10 and gradient threshold 70

Step 3: Edge detection

Use a Canny edge detector to calculate the edge location in the denoised image (see Fig. S5 Top). Lay the edge map over the original image for quality inspection (see Fig. S5, bottom).

The edges of the diffused images in Fig. S4 are detected using a canny detector in Matlab R2019a with threshold $T_1 = 0.001$ and $T_2 = 0.1$

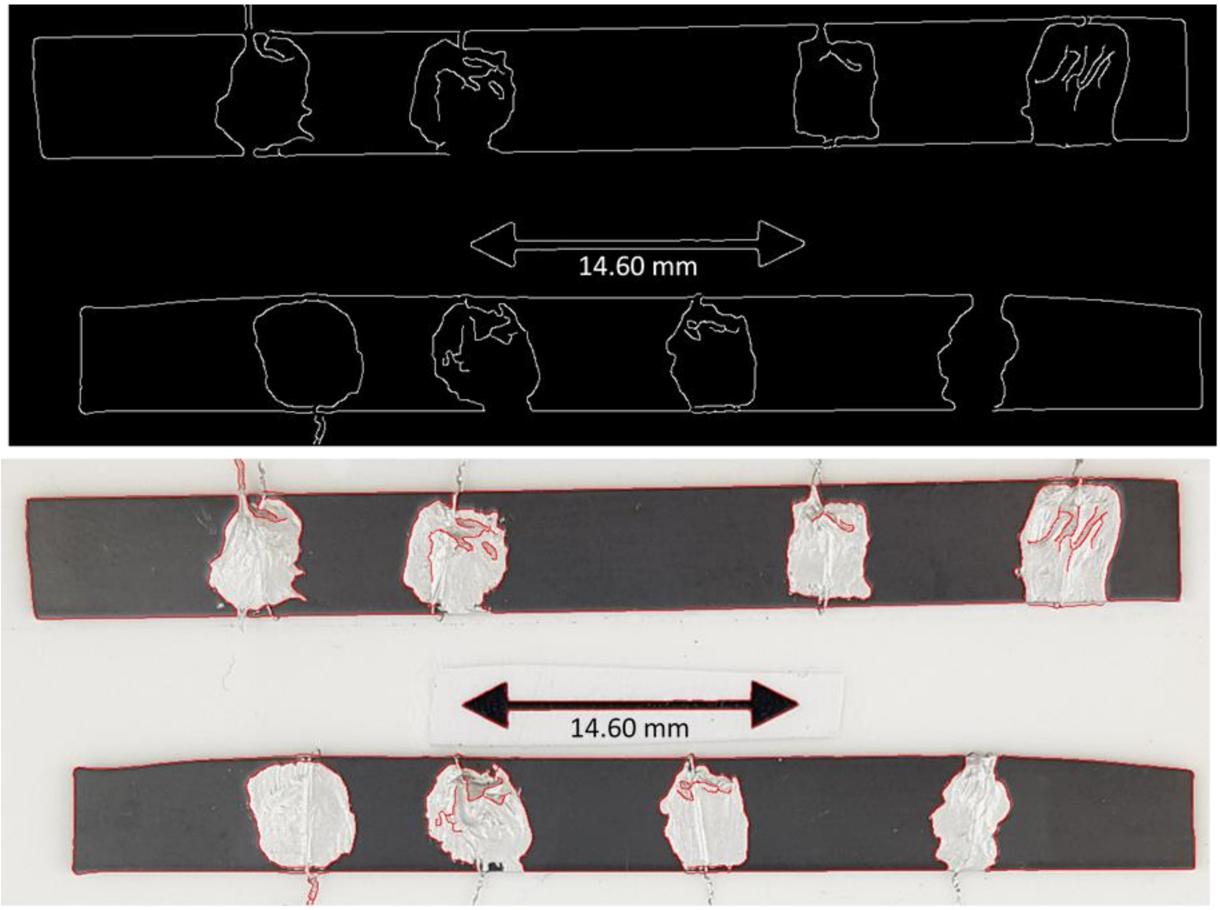

Fig. S5 Detected edges of the image (above) and the edge map (red) laid over the original image

Step 4: Centerline tracing

In Inkscape, put the edge map on top of the original image. Set the two images with the same size. Use the plug-in Centerline Tracing under <Extentions> choose <Images>, then click <Centerline Trace v0.8c>, in the opening window of "Centerline Trace v0.8c", the following parameters were used to trace the pixels of the edge(see Fig. S6).

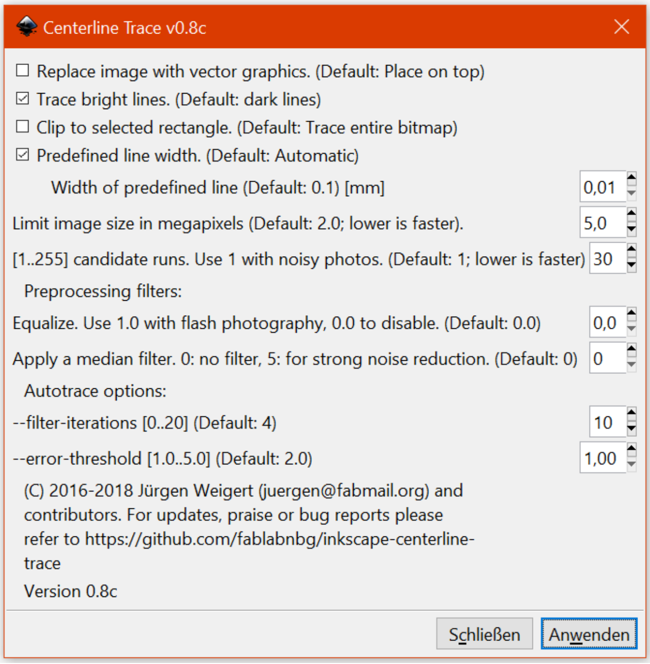

Fig. S6 Set-up in Centerline Trace v0.8c

Step 5: Postprocessing the edge into a CAD-file

Still in the same working window from Step 4, draw a line segment with its vertices locating at the endpoints of the scale arrow of the edge map to indicate the scale (see the blue line in the center in Fig. S7). This line in is later used for scaling in the CAD programm Fusion 360. After deleting the artefacts and completing of the edges, a vector image of the edges is obtained, see Fig. S7. This vector graphic is saved as a SVG-file. Import the SVG-file into Fusion 360 as a new sketch. Extrude the contour of the sample and the electrodes with a length of 10 mm to form a single body. Under the menu <Modify> use the tool <Rescale> to scale the body using the ratio of the length of the scale and the length of the line segment at the beginning of Step 6 now in Fusion 360. Under the menu <Modify> use the tool <Press Pull> to set the thickness of the electrodes to 100 µm and the thickness of the sample as measured. Export the modified body as an IGES-file.

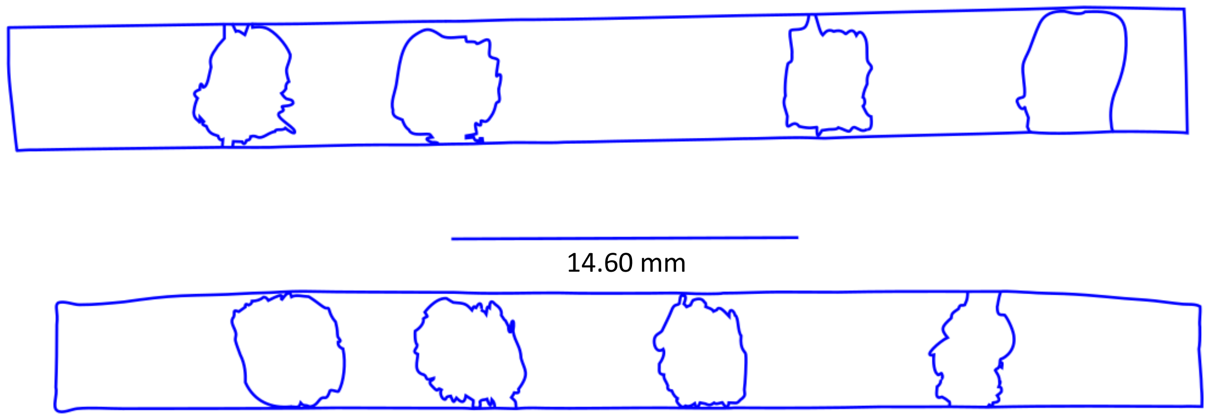

Fig. S7 Vector image of the edges

Step 6: Calculation of the form factor and the conductivity

Import the IGES-file as derived from above into the FE software, here COMSOL Multiphysics 5.2. Define the contact face of electrodes and sample as a new working plane. Under the menu <Geometry> in the tool box <Boolean Operations> use <Partition> to separate the CAD-Model into two parts using the new working plane. Set the conductivity of the electrodes to be 10^8 S/m and the conductivity of sample in the simulation to be σ_s (arbitrary). Set the input current in one of the outer electrodes to be as large as the current used in the measurement. Set the other outer electrode to have ground potential. Place an edge probe on each of the inner electrodes. Mesh the CAD-Model using physics-controlled mesh. Set the mesh to be extreme fine. Now start the simulation. The results will be the potentials on the edge probes. Calculate the potential difference of the two inner electrodes U_s. The measured resistance of the sample is R_m. During the measurement the direct current is I. So, according to equations (8) and (9) in the main article, the conductivity of the sample is $\frac{\sigma_\mathrm{s} U_\mathrm{s}}{I R_\mathrm{m}}$ and the form factor is $K_m = \frac{\sigma_\mathrm{s} U_\mathrm{s}}{I}$.