Online-Only Appendix for
Properties of Profit Premium in an Equilibrium Framework

This appendix has three sections. Section 1 presents results on how the profit premium behaves as a function of brand equity in the simple logit version of the model while Section 2 presents summary statistics for the variables used in the estimation. Section 3 presents robustness checks of profit premium calculations based on two alternative versions of the model.

1 Properties of profit premium in the simple logit

According to the definition from Section 3 of the paper, profit premium can be regarded as an explicit function of brand equity. Here we present results on how profit premium behaves as a function of brand equity in a version of the model that does not allow for consumer heterogeneity. Specifically, we provide conditions under which profit premium is either increasing or decreasing in brand equity. We observe that if profit is increasing in brand equity, then profit premium is also increasing in brand equity. Therefore, we study whether profit is increasing or decreasing in brand equity by computing its derivatives with respect to its own brand equity.

An exogenous increase in the brand equity of a product implies changes in virtually all endogenous variables of the model, that is, prices and market shares of all products in the market.[1] Intuitively, when the brand equity of a product increases, the market share of the same product will increase, if prices stay unchanged, but the price of the product is also expected to increase. Now, even if the prices of the other products change only insignificantly, the effect on the market share of the product will be ambiguous because the price increase lowers the market share. The literature has not clarified whether price will increase or not, if the corresponding brand equity increases, and the effect on profit is even more complicated. Therefore, in this section we derive comparative statics for profit by computing the derivative with respect to brand equity.

[1] In reality brand equity is endogenous, as modeled for example by Borkovsky et al. (2017) in a dynamic context. However, in the literature it is not uncommon that for analytical purposes, marketing activities devoted for creating brand equity or brand equity itself are treated as exogenous. For example, Sriram et al. (2007) treat advertising as exogenous, whereas Stahl et al. (2012) study the effect of exogenous changes in brand equity.

The simple logit model maintains all the variables but eliminates the random coefficients.[2] This yields a model that is analytically tractable in certain dimensions. Exploiting this analytical tractability, below we present comparative statics results on the signs of the derivative of profit. Here we consider the case when brands are firm-specific. As mentioned, we consider the special case when all random coefficients are deterministic (i.e., $\alpha_i = \alpha$, $\beta_i = \beta$, and $\delta_i = \delta$). Then, by denoting $d_j = x_j\beta + \delta M_j + \xi_j$, we obtain that the market share of product j is the logit expression

$$s_j = \frac{\exp\left(\beta_f - \alpha p_j + d_j\right)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp\left(\beta_g - \alpha p_r + d_r\right)}. \tag{1}$$

In the simple logit model, the first order condition for profit maximization (eq. (4) in the paper) can be written in closed form as

$$p_j - c_j = \frac{1}{\alpha} \frac{1}{1 - \sum_{r \in \mathcal{G}_j} s_r} \quad \text{for} \;\; j \in \mathcal{G}_f, \;\; f = 1, \ldots, F.$$

We know that a unique Nash equilibrium in prices exists under the assumption $\alpha > 0$ (Konovalov and Sándor 2010), and therefore, we maintain this assumption. We obtain the following results on the derivative of profit (proofs are available upon request from the authors).

Proposition 1 *The following statements hold for any firm f.*

1. *If $\phi_f = \min\{\phi_g : g = 1, \ldots, F\}$ and $\rho_f = 1 - \alpha\phi_f > 0$, the derivative $d\pi_f/d\beta_f$ is positive.*

2. *If $\phi_f = \max\{\phi_g : g = 1, \ldots, F\}$ and $\rho_f = 1 - \alpha\phi_f < 0$, the derivative $d\pi_f/d\beta_f$ is negative.*

As a corollary, part 1 of Proposition 1 for $\phi_f = 0$ for all $f = 1, \ldots, F$ implies that the derivatives $d\pi_f/d\beta_f$ are positive. In other words, profit premium is positive when marginal costs do not contain brand-specific intercepts. This case is important because it is the case considered by Goldfarb et al. (2009).

[2] Monte Carlo simulation results for the random coefficient logit (available from the authors upon request) suggest that the results derived for the simple logit hold for the random coefficient logit as well.

2 Descriptive statistics

Table 1 presents the means weighted by sales for the main variables used in the estimation of demand. The number of different car models sold increased from 208 in 2003 to 236 in 2008. The lowest sales are observed in 2005 while the highest in 2007. Prices had an increasing tendency in real terms with a remarkable drop in 2008, which is probably the result of the upcoming economic crisis. Regarding the share of European cars sold, the period 2003-2008 witnessed a downward trend, which was accompanied by an upward trend of brands that originate from Eastern Asia. Acceleration of cars measured by the ratio of horsepower to weight (denoted HP/weight) showed a steady increase. The average size of cars (measured as length times width times height) increased slightly during the sampling period. The share of cars sold with cruise control as standard equipment went up significantly in 2004, but afterwards showed a slight decrease. Average fuel efficiency improved during the sampling period as shown by the kilometers per liter (denoted KPL) variable. This improvement, however, was not matched by kilometers per euro (denoted KP€), which is a fuel efficiency variable more relevant to consumers. This means that the improvement in fuel efficiency was not sufficient to offset the increase in gasoline prices over the sampling period. The proportion of family cars sold is rather high in each year and shows a slight decrease. The proportion of luxury cars has a similar trend, although the overall share of luxury cars is rather low. The share of sport cars is extremely low and does not change much during the sampling period. The proportion of MPV's shows a decreasing trend while the proportion of SUV's shows an increasing trend. Finally, we observe that advertising expenditures peaked in 2004, after which they dropped considerably.

Table 2 presents the means weighted by sales for additional variables used in the estimation of marginal cost. Cylinder volume (CC) shows an upward trend between 2003-2007, while it drops to its lowest average in 2008. Acceleration (seconds elapsed for reaching 100km/h) and maximum speed both improve during the sampling period, which means that acceleration decreases and maximum speed increases. The proportions of cars having air-conditioning, power steering, sport chairs, and xenon lights show increasing trends and tend to reach their peaks around 2007 after which they

Table 1. Summary statistics

Year	No. of Models	Sales	Price	HP/ Weight	Size	Cruise Control	KPL	KP€	Family Car	Luxury	Sport	MPV	SUV	Adver- tising
2003	208	480,309	19,553	0.786	7.152	0.230	14.482	12.498	0.426	0.031	0.006	0.184	0.033	1.686
2004	222	475,032	19,945	0.788	7.184	0.309	14.699	11.739	0.408	0.029	0.008	0.198	0.038	2.152
2005	227	457,094	20,540	0.794	7.270	0.301	14.863	10.989	0.403	0.030	0.007	0.189	0.054	1.802
2006	229	474,452	20,388	0.804	7.271	0.309	15.122	10.709	0.360	0.030	0.006	0.172	0.063	0.843
2007	233	491,723	20,552	0.809	7.328	0.282	15.121	10.363	0.359	0.027	0.007	0.171	0.071	0.867
2008	236	483,807	18,699	0.812	7.268	0.297	15.834	10.305	0.377	0.021	0.006	0.127	0.061	0.855
All	1,355	477,070	19,941	0.799	7.246	0.288	15.023	11.097	0.389	0.028	0.007	0.173	0.053	1.361

Notes: Prices are in 2006 euros. Advertising is in million euros. All variables are sales weighted means, except for the number of models and sales.

decline. The proportion of cars with a board computer is rather variable. The proportion of cars with an anti-roll bar increases steadily over the sampling period.

Table 2. Summary statistics additional marginal cost variables

Year	CC	Accel- eration	Maximum speed	Aircon- ditioning	Board computer	Power steering	Sports chairs	Anti-roll bar	Xenon lights
2003	1,478	13.555	173.655	0.248	0.409	0.882	0.037	0.141	0.001
2004	1,476	13.523	173.805	0.283	0.495	0.917	0.038	0.163	0.003
2005	1,500	13.402	174.936	0.350	0.527	0.908	0.042	0.229	0.006
2006	1,498	13.334	174.578	0.380	0.465	0.915	0.058	0.224	0.006
2007	1,512	13.287	174.756	0.396	0.442	0.920	0.060	0.239	0.006
2008	1,464	13.163	175.112	0.353	0.464	0.904	0.047	0.298	0.004
All	1,488	13.376	174.473	0.335	0.466	0.908	0.047	0.216	0.004

Notes: All variables are sales weighted means.

3 Robustness checks

This section presents robustness checks based on two versions of the model. First, using the original demand model (specification A-B in Table 1 of the paper), we estimate the marginal cost equation by including a proxy on log(production).[3] This is relevant because this way we relax the constant returns to scale assumption. Second, we estimate the demand side by omitting advertising. Although we follow the literature (e.g., Goldfarb et al. 2009) by including advertising in the specifications

[3]Using data from the Global Market Data Book, International Organization of Motor Vehicle Manufacturers, and manufacturers' annual reports, we construct a proxy for model-specific yearly production based on the total global production and sales for each of the 16 manufacturers in our data, and use European sales data at the model level (using data from carsalesbase.com).

discussed in the paper, this specification is potentially interesting because advertising affects brand equity and can therefore be considered part of it. In both cases we calculate the implied profit premiums.

Table 3. Estimation results

	(C)		(D)	
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.
Base coefficients				
constant	-19.486	(7.407)***	-24.074	(8.392)***
HP/weight	2.444	(0.548)***	2.185	(0.424)***
cruise control	0.478	(0.110)***	0.427	(0.112)***
KM per euro	0.764	(0.298)***	1.975	(0.311)***
size	10.910	(1.494)***	11.919	(1.435)***
advertising	0.557	(0.040)***	—	
family car	-0.773	(0.165)***	-0.711	(0.129)***
luxury	-0.193	(0.216)	-0.431	(0.227)*
sport	-0.775	(0.238)***	-0.927	(0.237)***
MPV	-0.240	(0.153)	-0.417	(0.142)***
SUV	0.544	(0.217)**	0.573	(0.223)**
Random coefficients				
price/income	-6.755	(2.323)***	-5.884	(1.359)***
constant	3.352	(5.418)	5.473	(5.481)
Marginal cost parameters				
constant	-0.475	(0.812)	-0.478	(0.829)
log(HP/weight)	0.162	(0.064)**	0.176	(0.065)***
cruise control	0.047	(0.013)***	0.047	(0.013)***
log(KM per liter)	-0.582	(0.048)***	-0.609	(0.048)***
log(size)	1.085	(0.084)***	1.095	(0.085)***
log(CC)	0.330	(0.044)***	0.334	(0.045)***
log(acceleration)	-0.191	(0.069)***	-0.196	(0.071)***
log(maximum speed)	0.166	(0.112)	0.171	(0.115)
airconditioning	0.055	(0.013)***	0.060	(0.013)***
board computer	0.033	(0.012)***	0.033	(0.012)***
power steering	0.096	(0.021)***	0.095	(0.021)***
sports chairs	0.067	(0.020)***	0.069	(0.020)***
anti-roll bar	0.066	(0.014)***	0.067	(0.015)***
xenon lights	0.116	(0.023)***	0.121	(0.024)***
log(production)	-0.012	(0.004)***	—	
Brand fixed effects supply side	yes		yes	
R^2 supply side	0.953		0.952	

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,355. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors are in parenthesis. The demand side estimates for specification (C) are the same as those for specifications (A) and (B) (see paper). The demand specifications (upper panel) contain brand-specific intercepts. Kia is the base brand.

The upper part of Table 3 contains the demand estimates obtained in the first stage, where the demand side of specification (C) is the same as that of specifications (A) and (B) in the paper. The signs of the base coefficients in specification (D) do not change compared to (C); we observe notable changes in magnitude for the coefficient of Km per euro and the special car class dummy coefficients

Table 4. Brand-specific intercepts and profit premiums

	Brand-specific intercepts						Profit premiums	
	Brand equity (D)		Marginal cost (C)		Marginal cost (D)			
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Coeff.	Std. Err.	(C)	(D)
bmw	3.258	(0.386)***	0.576	(0.034)***	0.557	(0.034)***	42.603	44.637
mini	2.514	(0.396)***	0.521	(0.067)***	0.505	(0.068)***	8.577	8.838
chrysler	0.458	(0.374)	0.191	(0.037)***	0.190	(0.038)***	-2.456	-1.569
jeep	0.890	(0.441)**	0.272	(0.041)***	0.266	(0.042)***	-0.277	-0.510
mercedes-benz	3.463	(0.435)***	0.610	(0.032)***	0.592	(0.032)***	30.517	33.564
smart	0.981	(0.515)*	0.921	(0.049)***	0.939	(0.050)***	0.785	-0.038
alfa romeo	1.205	(0.336)***	0.311	(0.036)***	0.305	(0.036)***	2.671	2.825
fiat	0.554	(0.278)**	0.184	(0.032)***	0.174	(0.032)***	17.797	15.673
lancia	-0.133	(0.329)	0.326	(0.042)***	0.324	(0.043)***	-1.556	-3.299
ford	1.820	(0.277)***	0.182	(0.031)***	0.153	(0.031)***	103.865	115.059
jaguar	2.794	(0.500)***	0.546	(0.040)***	0.536	(0.041)***	0.825	0.721
land rover	2.835	(0.556)***	0.508	(0.040)***	0.491	(0.040)***	4.583	4.749
mazda	0.728	(0.309)***	0.309	(0.031)***	0.297	(0.031)***	2.133	-0.716
volvo	2.739	(0.369)***	0.389	(0.032)***	0.374	(0.032)***	54.668	56.307
subaru	-0.052	(0.308)	0.307	(0.035)***	0.312	(0.036)***	-3.107	-6.916
cadillac	-0.027	(0.391)	0.196	(0.053)***	0.212	(0.054)***	-1.158	-1.410
chevrolet	0.051	(0.276)	0.047	(0.030)*	0.042	(0.031)	3.466	-0.902
opel	1.969	(0.317)***	0.370	(0.030)***	0.351	(0.030)***	79.959	95.430
saab	1.837	(0.341)***	0.323	(0.046)***	0.310	(0.047)***	3.738	4.361
honda	0.593	(0.301)**	0.388	(0.035)***	0.382	(0.035)***	-3.297	-11.887
hyundai	0.301	(0.278)	0.118	(0.029)***	0.115	(0.029)***	10.337	3.863
kia	0.000		0.000		0.000		0.000	0.000
mitsubishi	0.719	(0.295)**	0.231	(0.031)***	0.232	(0.032)***	7.723	4.664
porsche	4.037	(0.584)***	0.699	(0.048)***	0.681	(0.048)***	1.959	2.201
citroen	1.226	(0.288)***	0.263	(0.030)***	0.243	(0.031)***	32.951	39.102
peugeot	1.467	(0.322)***	0.238	(0.031)***	0.218	(0.031)***	62.902	85.151
nissan	1.204	(0.319)***	0.245	(0.031)***	0.231	(0.032)***	13.288	16.950
renault	1.948	(0.297)***	0.276	(0.031)***	0.255	(0.031)***	74.948	92.520
suzuki	0.788	(0.267)***	0.177	(0.033)***	0.179	(0.034)***	18.597	16.466
daihatsu	0.278	(0.324)	0.352	(0.037)***	0.370	(0.038)***	7.188	-3.534
lexus	2.128	(0.414)***	0.353	(0.043)***	0.342	(0.044)***	1.322	2.592
toyota	1.864	(0.307)***	0.390	(0.031)***	0.378	(0.031)***	69.765	79.448
audi	2.955	(0.382)***	0.530	(0.035)***	0.509	(0.035)***	51.090	53.398
seat	0.972	(0.325)***	0.213	(0.033)***	0.197	(0.034)***	19.697	19.573
skoda	1.044	(0.360)***	0.267	(0.040)***	0.246	(0.041)***	16.194	13.917
volkswagen	1.997	(0.340)***	0.347	(0.029)***	0.325	(0.030)***	115.064	116.254
Correlations (p-values) with corresponding brand equity:								
Pearson			0.72 (0.00)		0.62 (0.00)		0.32 (0.05)	0.38 (0.02)
Kendall			0.59 (0.00)		0.51 (0.00)		0.32 (0.01)	0.45 (0.00)

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,355. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors of estimates other than correlations are in parenthesis. Numbers are based on the estimates in Table 3. Profits are measured in € mln. Profit premium is calculated as profits minus counterfactual profits (setting brand to the minimum brand found).

of Luxury, Sport, and MPV. In specification (D) the coefficient of price/income is lower in absolute value than in (C), which is in line with the expectation that advertising is positively correlated with price.

The brand-specific intercept estimates presented in Table 4 are obtained –as before– by using Kia as the base brand for both the demand and supply sides. We note that most estimates of brand equity for specification (D) are significant at a 5% level, but there are some statistically insignificant negative values, namely those of Lancia, Subaru, and Cadillac. The main patterns noted for the brand equities presented in the paper remain valid.

The lower part of Table 3 contains the marginal cost estimates obtained in the second stage. The estimates across specifications (C) and (D) do not differ much from each other and neither do they from specification (B) in the paper. In specification (C) the newly added log(production) variable is statistically significant and has a negative sign, which is according to our expectations.

The marginal cost brand-specific intercept estimates presented in Table 4 for specifications (C) and (D) are rather similar to each other and also to specification (B) from the paper. Nevertheless, we note that most intercept estimates from specification (C) are slightly higher than those from specifications (B) and (D); this is most likely due to the extra log(production) variable included. The similarity is also supported by the R^2's, which are virtually the same as that in specification (B) in the paper.

The correlation coefficients between the brand equities and the corresponding marginal cost intercepts are reported at the bottom of Table 4. They confirm the strong positive relationship between these two variables. Both the Pearson and Kendall correlations are rather large and statistically significant: 0.72 and 0.59, respectively, for specification (C) and 0.62 and 0.51, respectively, for specification (D) with all p-values equal to 0.000. The fact that in specification (D) the correlation is somewhat lower is related to the fact that in this model the measured brand equities capture some variation of the omitted advertising variable, and therefore, they have variance higher than in specification (C). Still the correlations in specification (D) are not very different from those in specification (B).

The profit premium estimates based on data from 2008 are presented in Table 4. The profit premiums in different specifications are very similar based on pairwise correlations (all Pearson correlations are above 0.99 and Kendall correlations range between $0.87 - 0.997$; not shown in the tables). Profit premiums reported for specification (C) have the same signs and are numerically very similar to those in specification (B) in the paper. Nevertheless, the profit premiums in specification (D) differ slightly from those in specifications (B) and (C). This is a consequence of the fact that advertising is omitted from the utility specification (D), and therefore, the brand equities for some brands (e.g., Smart, Mazda, Chevrolet, Daihatsu, Lancia, Subaru, Honda) are measured to be lower than in specification (B) in the paper. Lower brand equities imply higher ratios of the marginal cost intercept and brand equity, hence Proposition 1 from the paper implies that the profit premiums can get negative, as it happens for Smart, Mazda, Chevrolet, and Daihatsu, for which the profit premiums are positive in specifications (B) and (C). Along the same argument, the profit premiums of Lancia, Subaru, and Honda decrease substantially compared to the other two specifications.

In spite of these subtle differences, still the main conclusion with respect to profit premiums formulated in the paper remain valid for specification (D) as well. In this regard, highly popular brands like Volkswagen, Toyota, Ford, Opel, Renault, and Peugeot still have the highest profit premiums in specification (D). The profit premiums of the European luxury brands (Porsche, Mercedes, BMW, Audi, Jaguar, Land Rover) are comparable to those reported for specification (B) in the paper. The brands having the lowest profit premium are very much the same in specifications (B) (in the paper) and (D). In addition to the six brands with negative profit premium in specification (B), in specification (D) the brands Smart, Mazda, Chevrolet, and Daihatsu also have negative profit premiums. This is not completely unexpected, because these brands are reported as having rather small profit premiums in specification (B) in the paper.

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