SUPPLEMENT B

Mathematical model for automatic passenger counting

Let D_i be the relative differences (also referred to as relative errors) as defined in Section 3 of the manuscript. Their distribution (i.e. expected value and variance) may be dependent on the related stop door event and thus not necessarily identically distributed for $i = 1, .., n$. For example, one would expect bigger relative differences at the stop door events with large passenger flow than at stop door events with little passenger flow. Additionally, other (unobserved) factors may play a role, like the time of the day, the weather, the operated route, frequency of luggage or children as of many other covariables. Addressing this, the D_i need to be regarded as errors, conditional on all characteristics of the related stop door events. In the following, we use the common notation of conditional distributions and conditional probabilities, marking them by a vertical bar $|$. More precisely, D_i is defined as $D_i := \frac{K_i|\omega_i - M_i}{\overline{M}} \big| \Omega$ (which is simply notated as $D_i := \frac{K_i - M_i}{\overline{M}}$ in Section 3 of the manuscript). The D_i may thus be distributed differently, but with this (conditional) approach D_i and D_j are (conditionally) independent for $i \neq j$. Extending the notation from the manuscript we use the following table of definitions:

Term	Description and explanation
$\mu_\Omega := \frac{1}{n}\sum_{i=1}^{n} E(D_i\|\omega_i)$	Average expectation as an average of the expected values of the relative differences
$\sigma^2_\Omega := \frac{1}{n}\sum_{i=1}^{n} \mathrm{Var}(D_i\|\omega_i)$	Average variance of the sample, resulting from different stop door event characteristics with different single variances
$\nu^2_\Omega := \frac{1}{n}\sum_{i=1}^{n} E(D_i - \mu_\Omega)^2$	Average squared deviation of D_i towards the average expectation

Because of the independence of D_i and D_j for $i \neq j$ and under some regularity assumptions towards the third moments of the D_i, the Lindeberg-Feller version of the central limit theorem can be used to show that the limiting distribution is Gaussian:

$$\frac{\frac{1}{n}\sum_{i=1}^{n} D_i \; - \; \frac{1}{n}\sum_{i=1}^{n} E(D_i)}{\frac{1}{\sqrt{n}} \cdot \sqrt{\frac{1}{n}\sum_{i=1}^{n} \mathrm{Var}(D_i)}} = \frac{\overline{D} \; - \; \mu_\Omega}{\frac{1}{\sqrt{n}} \cdot \sigma_\Omega} \quad \dot{\sim} \quad \mathcal{N}(0,1)$$

$$\text{and thus } \overline{D} \;\dot{\sim}\; \mathcal{N}\left(\mu_\Omega, \frac{\sigma_\Omega}{\sqrt{n}}\right) \tag{1}$$

The assumption of limited third moments will hold in practice since extremely large errors should be investigated separately beforehand in a quality control step of automatic passenger counts and bounded errors imply reasonably small higher moments.

The standard deviation σ_Ω is in general not estimable without further model assumptions, but for our purposes, it is sufficient to use the upward estimate

$$\sigma_\Omega^2 \leq \frac{1}{n}\sum_{i=1}^{n} E(D_i - \mu_\Omega)^2 = \nu_\Omega^2 \tag{2}$$

instead for which we can simply use the empirical variance estimator based on all D_i as

$$\widehat{\nu}^2 := \frac{1}{n-1}\sum_{i=1}^{n}(D_i - \overline{D})^2 \quad . \tag{3}$$

With the Gaussian limiting distribution and the estimation of the variability, it is possible to construct a $1-2\alpha$ confidence interval. The equivalence test can then be carried out by checking whether the confidence interval is entirely covered by the equivalence range $[-\Delta, +\Delta]$, i.e.:

$$\left|\overline{D} \pm z_{1-\alpha} \cdot \frac{\widehat{\nu}}{\sqrt{n}}\right| \leq \Delta$$

$$\text{or simply:} \quad |\overline{D}| \leq \Delta - z_{1-\alpha} \cdot \frac{\widehat{\nu}}{\sqrt{n}} \tag{4}$$

If parameters are chosen as $\Delta = 0.01$, $\alpha = 0.025$ the equivalence test is then evaluated by the criterion $|\overline{D}| \leq 0.01 - 1.96 \cdot \widehat{\nu}/\sqrt{n}$.