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Lemma A1. *When at least one strategic type is at y_j:*

(a) $k_j \leq 2$.

(b) $k_j = 2$ *for* $j = 0, r$.

(c) *If* $k_j = 2$, $L_j = R_j$.

(d) *All strategic candidates who enter, tie and win.*

Proof: When all candidates at a given location are strategic, the proofs are identical to Cox (1987, Lemma 1) and Osborne (1993, Lemma 1) where all candidates are strategic (note that Cox does not have part (d) as he studies exogenous entry). In fact, so long as there is at least one strategic type at a given location, their proofs continue to hold, so I do not repeat them. ■

Proposition 1 (Without idealists). *For any unimodel density f, when no idealist candidates enter, no equilibrium with $n > 2$ exists.*

Proof: This proof uses some of the structure of that of Osborne (1993, Lemma 2), but goes further using the fact that f is assumed unimodal. If $n = 3$, then Lemma A1 (a) and (b) cannot be satisfied, so there is no equilibrium for any F. If $n = 4$, then by Lemma A1 (b), $k_0 = k_1 = 2$, $y_0 = F^{-1}\left(\frac{1}{4}\right)$, $y_1 = F^{-1}\left(\frac{3}{4}\right)$ and $L_0 = R_0$. The last condition implies $m_0 = F^{-1}\left(\frac{1}{2}\right)$. In turn this implies

$$\text{(A1)} \qquad F^{-1}\left(\tfrac{1}{n}\right) + F^{-1}\left(\tfrac{3}{n}\right) = 2F^{-1}\left(\tfrac{2}{n}\right).$$

This is not satisfied for almost any F. Furthermore, because f is unimodal, it can only be satisfied when the maximizer of f is in the interval $\left(F^{-1}\left(\frac{1}{4}\right), F^{-1}\left(\frac{3}{4}\right)\right)$: Suppose not, and without loss of generality that f were increasing throughout this interval. Then, F is convex over this interval and it must be that $F^{-1}\left(\frac{1}{2}\right) - F^{-1}\left(\frac{1}{4}\right) > F^{-1}\left(\frac{3}{4}\right) - F^{-1}\left(\frac{1}{2}\right)$, which contradicts (A1). Given the maximizer of f is in this interval, and this interval contains density of $\frac{1}{2}$, a strategic entrant can profitably enter: When an entrant locates at the maximizer of f, they will reduce the vote share of all the incumbent candidates and will gain a total vote share $> \frac{1}{4}$ (there is density of $\frac{1}{2}$ in $\left[F^{-1}\left(\frac{1}{4}\right), F^{-1}\left(\frac{3}{4}\right)\right]$ and the entrant's left (right) constituency is greater than the reduced right (left) constituency of their neighbor to the left (right)).

If $n \geq 5$, then by Lemma A1 (b) $k_0 = 2$. I now show that $k_1 = 1$. Suppose not. By Lemma A1 (a) $k_1 = 2$. If $n = 5$ then $k_2 = 1$ which contradicts Lemma A1 (b). If $n > 5$, we can follow the proof above for $n = 4$ to find that (A1) holds, and therefore that the maximizer for f must be in the interval $\left(F^{-1}\left(\frac{1}{n}\right), F^{-1}\left(\frac{3}{n}\right)\right)$, but then an entrant could enter at the maximizer of f and win outright. Therefore, $k_1 = 1$. Now I show that the maximizer of f must be to the left of y_1. Suppose not. Then F is convex in the interval (y_0, y_1) which means that $L_1 > R_0 = \frac{1}{n}$, but then the candidate at y_1 wins outright, contradicting Lemma A1 (d). Now I show that $k_j = 1$ for $j \geq 2$. Take $j = 2$ and suppose that instead $k_2 = 2$. Then by Lemma A1 (c) $L_2 = \frac{1}{n}$. However, because the maximizer of f is to the left of y_1, F is concave over (y_1, y_2), hence $R_1 > L_2 = \frac{1}{n}$, but then the candidate at y_1 wins outright, contradicting Lemma A1 (d). Similarly, one can show $k_j = 1$ for $j \geq 3$. However, $k_r = 1$ contradicts Lemma A1 (b). ■

Here I describe the special nature of the distributions of voter preferences which are ruled out of the analysis of this paper. Take B points $x_b \in X$ where $b = 1, \ldots, B$, denote $x = (x_1, \ldots, x_B)$ and index them, without loss of generality, such that $x_1 < \cdots < x_B$. Take also the parameters $\beta_b \in \mathbb{R}_{\neq 0}$ for $b = 0, \ldots, B$ and denote $\beta = \beta_0, \ldots, \beta_B$.

Definition A1. *Denote the set of continuous unimodal density functions,* $\mathcal{U}$. *Let* $\mathcal{F}_{x,\beta} \subset \mathcal{U}$ *be such that if* $f \in \mathcal{F}_{x,\beta}$, *its corresponding distribution function* F *satisfies:*

(A2) $$\beta_0 + \sum_{b=1}^{B} \beta_b F(x_b) = 0.$$

Define $\mathcal{F}^c_{x,\beta} \subset \mathcal{U}$ *as the complement of* $\mathcal{F}_{x,\beta}$ *within* $\mathcal{U}$.

Definition A2. *Denote* $g^\epsilon_f \subset \mathcal{U}$ *such that if* $g \in g^\epsilon_f$, $g \in \mathcal{U}$, *has corresponding cdf* G, *and* $|g(x) - f(x)| \leq \epsilon$ *for all* $x \in X$ *where* $\epsilon > 0$.

Lemma A2. $\mathcal{F}^c_{x,\beta}$ *is an open set: for any* $f \in \mathcal{F}^c_{x,\beta}$, *there exists* $\epsilon > 0$ *such that for all* $g \in g^\epsilon_f$, $g \in \mathcal{F}^c_{x,\beta}$.

Proof: For $f \in \mathcal{F}^c_{x,\beta}$ we have $\beta_0 + \sum_{b=1}^{B} \beta_b F(x_b) \neq 0$. Because F is continuous, there exist $\bar{\epsilon}_1, \ldots, \bar{\epsilon}_B \in \mathbb{R}_{\neq 0}$ such that for any $\epsilon_1 \in [0, |\bar{\epsilon}_1|], \ldots, \epsilon_B \in [0, |\bar{\epsilon}_B|]$ we have $\beta_0 + \sum_{b=1}^{B} (\beta_b F(x_b) + \epsilon_b) \neq 0$. Define $\epsilon = \min\{|\bar{\epsilon}_1|, \ldots, |\bar{\epsilon}_B|\}$ and take any $g \in g_f^{\epsilon}$. Notice that $\beta_0 + \sum_{b=1}^{B} \beta_b G(x_b) \neq 0$ and hence $g \in \mathcal{F}_{x,\beta}^c$. ■

Lemma A3. *$\mathcal{F}_{x,\beta}^c$ is dense in $\mathcal{U}$: for any $f \in \mathcal{U}$ and $\epsilon > 0$, there exists $g \in g_f^{\epsilon}$ such that $g \neq f$ and $g \in \mathcal{F}_{x,\beta}^c$.*

Proof: Take $f \in \mathcal{U}$. If $f \in \mathcal{F}_{x,\beta}^c$, Lemma A2 completes the proof. If $f \in \mathcal{F}_{x,\beta}$, take $\epsilon > 0$ and define g such that $g(x) = f(x)$ for $\{x \in X : x \leq x_{B-1}\}$, and $g(x)$ for $\{x \in X : x > x_{B-1}\}$ in any of the (uncountably) many ways such that $G(x_B) \neq F(x_B)$ and $g \in g_f^{\epsilon}$. Because $G(x_b) = F(x_b)$ for $b = 1, \ldots, B-1$, but $G(x_B) \neq F(x_B)$, we have $g \in \mathcal{F}_{x,\beta}^c$. ■

Lemma A4. (A2) *is a non-generic property on $\mathcal{U}$.*

Proof: By Lemmas A2 and A3, $\mathcal{F}_{x,\beta}^c$ is an open set and is dense in $\mathcal{U}$. Because (A2) holds on $(\mathcal{F}_{x,\beta}^c)^c \subset \mathcal{U} = \mathcal{F}_{x,\beta}$, (A2) is a non-generic property on $\mathcal{U}$. ■

Definition A3. *If a result holds for $f \in \mathcal{F}_{x,\beta}^c$ for some finitely many (x, β)-pairs, it holds "for almost any" f.*

Lemma A5. *For almost any unimodal density f, not all candidates tie.*

Proof: Suppose not. Firstly, consider the case where there are two candidates at an extreme location and without loss of generality, suppose this is on the left i.e., $k_0 = 2$. I now show that for any unimodal f, $k_j = 1$ for all $j \geq 1$. At least one of the candidates at y_0 is strategic hence by Lemma A1 (c), $y_0 = F^{-1}\left(\frac{1}{n+2}\right)$ and $m_0 = F^{-1}\left(\frac{2}{n+2}\right)$. If $n = 1$, $k_1 = 1$. If $n \geq 2$, suppose $k_1 = 2$. At least one of the candidates at y_1 is strategic, hence $y_1 = F^{-1}\left(\frac{3}{n+2}\right)$ which implies $F^{-1}\left(\frac{1}{n+2}\right) + F^{-1}\left(\frac{3}{n+2}\right) = 2F^{-1}\left(\frac{2}{n+2}\right)$. Following the proof of Proposition 1 (for $n \geq 4$ there, which covers $n \geq 2$ here) shows that for any unimodal f, $k_j = 1$ for all $j \geq 1$. However, unlike the proof of Proposition 1, we do not conclude that $k_r = 1$ is a contradiction. Instead, it must be that $y_r = z_2$. For all candidates to tie, $m_j = F^{-1}\left(\frac{j+2}{n+2}\right)$ for $j = 0, \ldots, n-1$. Solving recursively yields $y_0 = (-1)^n z_2 + 2\sum_{j=0}^{n-1}(-1)^j F^{-1}\left(\frac{j+2}{n+2}\right)$. However, we also required $y_0 = F^{-1}\left(\frac{1}{n+2}\right)$. These two expressions are not satisfied simultaneously for almost any unimodal density.

Now consider the case where there is one candidate at each extreme location $k_0 = k_r = 1$, which by Lemma A1 implies $y_0 = z_1$ and $y_r = z_2$. For all to tie, $F(m_j) = F(m_{j-1}) + s_j$ for $j = 0, \ldots, r-1$ where $s_j = \frac{k_j}{n+2}$. Solving recursively yields $z_1 = (-1)^r z_2 + 2\sum_{j=0}^{r-1}(-1)^j F^{-1}(S_j)$, where $S_j = \sum_{i=1}^{j} s_i$ which is not true for almost any unimodal density. ■

Proposition 2 (Extreme idealism). *For almost any unimodal f: $y_0 = z_1$, $y_r = z_2$ and $k_0 = k_r = 1$ in equilibrium.*

Proof: Suppose not. Either $k_0 = 2$ or $k_r = 2$ by Lemma A1 (b). Without loss of generality say $k_0 = 2$, which implies $L_0 = R_0$ by Lemma A1 (c). Denote the equilibrium vote share of the winning candidates by s.

If $n = 1$ this imposes $F(z_1) = F(\frac{1}{2}(z_1 + z_2)) - F(z_1)$, which is not true for almost any F. If $n = 2$, $s \geq \frac{1}{4}$. If $s = \frac{1}{4}$, all candidates tie, which is ruled out by Lemma A5. If $s > \frac{1}{4}$, then by Lemma A1 (d), z_2 is the sole loser. It must be that the strategic candidate is located at $y_1 < z_2$: if they were located at z_2, then they would tie with z_2; if they were located right of z_2, they could profitably deviate slightly to the left. If $f(m_0) > f(m_1)$, then the candidate at y_1 can profitably deviate by moving slightly to the left (they increase their share, and decrease the shares of candidates at y_0). If $f(m_0) \leq f(m_1)$, $R_0 < L_1$ because f is unimodal. But $L_0 = R_0 = s$, hence the candidate at y_1 must get strictly more than s votes and wins outright, a contradiction.

For $n \geq 3$ strategic candidates, $y_0 = F^{-1}(s)$ and $m_0 = F^{-1}(2s)$. If there is a strategic candidate at y_1 and $k_1 = 2$, then $y_1 = F^{-1}(3s)$ which implies $\frac{1}{2}(F^{-1}(s) + F^{-1}(3s)) = F^{-1}(2s)$. Following the proof of Proposition 1 (there for $n \geq 5$) where $s \equiv \frac{1}{n}$ shows $k_j = 1$ for each $j \geq 1$ when there are only strategic types at each y_j. To deal locations with idealists, denote y_l as the left-most position after y_0 where there is an idealist. What I have shown so far is that for almost any F, $k_l = 1$. Now I consider two cases, both of which end in a contradiction. (Recall that by Lemma A1 (d) and Lemma A5, z_2 must lose for almost any F.)

(i) If there are no strategic candidates to the right of y_l, then for the unimodal density f: if $f(m_{l-2}) \leq f(m_{l-1})$, then $L_1 > s$ because $R_0 = s$, which contradicts Lemma A1 (d); if

$f(m_{l-2}) > f(m_{l-1})$, then the candidate at y_l has a profitable deviation slightly to the left (by increasing their own vote share and decreasing that of the winning candidates).

(ii) If there is a strategic candidate to the right of y_l, let the right-most such candidate be at y_j. If $f(m_{l-1}) \leq f(m_l)$, $L_1 > s$. If $f(m_{l-1}) > f(m_l)$, I consider two sub-cases: If $j = r$, then $k_r = 2$ and $R_r = L_r = s < R_{r-1}$. If $j < r$, then $j = r - 1$ and there is a lone idealist at y_r, in which case y_j can deviate profitably by moving slightly to the left (by increasing their own vote share and decreasing that of the winning candidates). ■

Lemma A6. *For almost any unimodal f, $k_j = 1$ for all j when $n = 2$.*

Proof: Suppose not. Then by Proposition 2 and Lemma A1 (c), $k_1 = 2$ and $L_1 = R_1$. If z_1 gets a strictly lower (higher) vote share than z_2, an entrant can locate slightly to the right (left) of the strategic candidates at y_1 and win outright. Thus all candidates tie, contradicting Lemma A5. ■

Lemma A7. *For almost any unimodal f, exactly one idealist must tie with the strategic candidates when $n \geq 2$.*

Proof: First, I show that it cannot be that both idealists lose. Suppose they do and consider first $n = 2$. By Lemma A6, $k_1 = k_2 = 1$. If $f(m_0) < f(m_1)$, the candidate at y_1 can move slightly to the right, increasing their vote-share and decreasing that of the other strategic candidate; if $f(m_0) \geq f(m_1)$ then because f is unimodal, the maximizer of f must lie to the left of m_1, which implies $f(m_1) > f(m_2)$ and hence that the candidate at y_2 can profitably deviate by moving slightly to the left, increasing their vote-share and decreasing that of the other strategic candidate.

Now consider $n \geq 3$. Denote the equilibrium vote share of strategic candidates as s. If y_2 is weakly to the left of the maximizer of f, then $k_1 = 1$ because if $k_1 = 2$, $s = R_1 < L_2$, which contradicts Lemma A1 (d). Because $k_1 = 1$ and z_1 loses, the candidate at y_1 can profitably deviate slightly to the right. Now consider the case where y_2 is to the right of the maximizer. There can be no more strategic candidates to the right of y_2. If there were, then $k_j = 1$, $j > 2$ because if $k_j = 2$ for one such j, then $R_{j-1} > L_j = s$. Note now that the candidate at y_{r-1} has a profitable deviation to the left because z_2 loses. Next I show that it must be that $k_1 = k_2 = 2$

and hence that $n = 4$. If $k_2 = 1$ and $f(m_1) > f(m_2)$, the candidate at y_2 can profitably deviate to the left; if $k_2 = 1$ and $f(m_1) \leq f(m_2)$, $k_1 = 1$ (else $s = R_1 < L_2$) and the candidate at y_1 can profitably deviate right. Hence $k_2 = 2$. If $k_1 = 1$ and $f(m_0) < f(m_1)$, the candidate at y_1 can profitably deviate right; if $k_1 = 1$ and $f(m_0) \geq f(m_1)$ then $f(y_1) > f(m_1)$ implying $R_1 > L_2 = s$ as $k_2 = 2$. As $k_1 = k_2 = 2$, by Lemma A1 (c) and (d), $L_1 = R_1 = L_2 = R_2$. But with only two free variables (y_1 and y_2) these three conditions will not be satisfied for almost any F.

Hence, for almost any unimodal distributions at least one idealist must tie, but by Lemma A5, exactly one idealist must tie. ■

Lemma A8. *For almost any unimodal f, $k_j = 1$ for all j when $n \geq 3$.*

Proof: By Proposition 2, $y_0 = z_1$ and $y_r = z_2$ while by Lemma A1 (d) all strategic entrants tie for the win. This implies $F(z_1) < \frac{1}{n+2}$ and $F(z_2) > \frac{n+1}{n+2}$ in any equilibrium. By Lemma A7, exactly one idealist ties with the strategic types and without loss of generality, let this be z_1. Now consider the following *spacing procedure* which spaces candidate locations throughout the distribution F for some arbitrary number of candidates n, where $k_0 = k_r = 2$, $k_j = 1, 2$ for $j = 1, \ldots, r-1$ and strategic types tie with the idealist z_1.

Spacing Procedure:

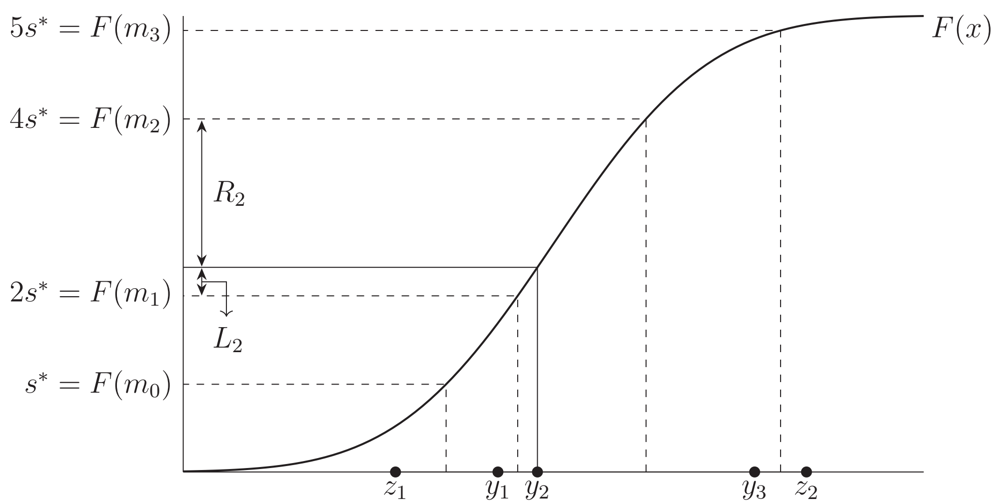
1. Choose y_1 such that $s \equiv F(m_0) \in \left(F(z_1), \frac{1}{n+2}\right)$.
2. Place the remaining $r-2$ candidate locations at y_j for $j = 2, ..., r-1$ in turn, such that $F(m_{j-1}) = F(m_{j-2}) + k_{j-1}s$.
3. Observe whether $\frac{1}{2}(y_{r-1} + z_2) = m_{r-1}$. If yes, stop and denote s as s^*; if $m_{r-1} < (>)$ $\frac{1}{2}(y_{r-1} + z_2)$ return to step 1 and choose a higher (lower) value of s.

Iterating on this procedure, the value of s will converge to s^*. As F is continuous, s^* exists, and as F is strictly increasing, s^* is unique. An example result of the procedure is illustrated below in Figure A1. The points $y_1, \ldots, y_{r-1}$ associated with s^* pin-down the necessary locations of the strategic candidates in equilibrium.[13]

[13]Notice that although s^* is necessarily the equilibrium share of the vote for the winning

It is now straightforward to see that for almost any distribution F, $k_i = 1$ for all i. Suppose instead that $k_j = 2$ for some $j = 2, ..., r$. By Lemma A1 (c) we must have that $L_j = R_j$. However, as is illustrated in Figure A1 for the example of $j = 2$, this extra condition will not be satisfied for all except very particular distributions. ■

Figure A1: An example result of the spacing procedure



The example shown has $n = 4$ and $r = 4$ where $k_i = 1$ for all i except $k_2 = 2$. F is the standard Normal distribution and $z_1 = F^{-1}(0.10)$, $z_2 = F^{-1}(0.98)$. Solving the procedure yields $s^* = 0.19$ (2 d.p.) with candidate positions as shown.

Proposition 3 (No platform sharing). *For almost any unimodal f, $k_j = 1$ for all j when $n \geq 2$ in equilibrium.*

Proof: Immediate from Lemmas A6 and A8. ■

Lemma A9. *For any symmetric, unimodal f, when there is $n = 1$ strategic entrant, the idealists' vote shares are equal.*

Proof: Suppose not. Without loss of generality, suppose that the idealist z_1 has a higher vote share than z_2 which implies that $f(m_0) > f(m_1)$. The strategic candidate at y_1 can move

candidates, this procedure is not sufficient to define an equilibrium as for example, it may not be that $y_j > y_{j-1}$ for all $j = 1, ..., r - 1$.

slightly to the left, simultaneously increasing their own vote share and reducing the vote share of z_1, giving strictly higher utility. ■

Proposition 4 (Symmetric distributions). *For almost any symmetric, unimodal f, a unique equilibrium exists wherein $n = 1$ strategic candidate enters at location y_1, where y_1 solves (1):*

$$F(m_0) = 1 - F(m_1). \tag{1}$$

Here, $m_0 = \frac{1}{2}(z_1 + y_1)$ and $m_1 = \frac{1}{2}(y_1 + z_2)$, whenever the positions of the idealists (z_1, z_2) satisfy (2) and (3):

(2) *not too moderate:* $m_0 < F^{-1}\left(\frac{1}{3}\right) \iff m_1 > F^{-1}\left(\frac{2}{3}\right)$

(3) *not too extreme: if z_1 is closer to the maximizer of f than z_2 is,* $F(y_1) \geq 1 - 2F(m_0)$
if z_2 is closer to the maximizer of f than z_1 is, $F(y_1) \leq 2F(m_0)$

Proof: Firstly I show that $n = 1$ in equilibrium. Suppose instead $n > 1$. By Proposition 2 and Lemmas A6 and A8, for almost any unimodal f, the strategic candidates occupy the non-extreme locations and $k_j = 1$ for all j. As f is symmetric, there must be at least one strategic candidate on either side of the maximizer of f, else Lemma A1 (d) is violated. I now show this implies that both idealists tie with the strategic candidates. Suppose not and without loss of generality that z_1 loses. As f is symmetric, this implies $f(m_0) < f(m_1)$ (if not, z_1 gets at least as many votes as the candidate at y_2). The candidate at y_1 then can profitably deviate slightly to the right. But by Lemma A5, not all candidates can tie.

I now characterize the equilibrium. By Lemma A9, the idealists' vote shares must be equal, meaning that the strategic candidate's position y_1 must solve (1). To be an equilibrium, the strategic candidate must win, which implies $F(m_1) - F(m_0) > \frac{1}{3}$. Using (1), this becomes (2).

In equilibrium, the strategic candidate must not want to deviate to the left of z_1 or the right of z_2. Note that (2) implies that $z_1 < F^{-1}(\frac{1}{3})$ and $z_2 > F^{-1}(\frac{2}{3})$. As the strategic candidate gets at least $\frac{1}{3}$ of the vote share in order to win, there is no such profitable deviation. The strategic candidate would also lose if they deviated to an idealist's location as the other idealist would win outright. Finally, the strategic candidate does not have incentive to deviate to another location in (z_1, z_2): Without loss of generality, consider such a deviation to the left. By Lemma A9 this increases z_2's vote share (and z_2 now beats rather than ties with z_1). However, as f symmetric, this deviation also decreases the strategic candidate's vote share and hence also their plurality.

In equilibrium, inactive strategic candidates must not wish to enter. Notice that an inactive candidate could only profitably locate in (z_1, z_2). Assume first that z_1 is closer to the maximizer of f than z_2, so that y_1 is to the left of the maximizer. Notice that the payoff of the entrant is increasing as their location approaches y_1 from the right. Hence, entry is not profitable if the right constituency of y_1 is less than the vote share of the idealists $F(m_1) - F(y_1) \leq F(m_0)$ which gives (3). Similarly, the case of z_2 being closer to the maximizer gives the second expression in (3). ■

Corollary 1. *For almost any unimodal f where Mo(f) = Md(f), $n = 1$.*

Proof: Suppose instead $n > 1$. By Lemma A7 exactly one idealist loses and without loss of generality assume this is z_2. This implies that $f(m_{r-2}) \leq f(m_{r-1})$ else the candidate at y_{r-1} deviates left. This implies that m_{r-2} is strictly to the left of the maximizer of f. For the candidate at y_{r-2} and z_1 to tie (along with any number of others on the left of the maximizer), there must be strictly more than half the density to the left of the maximizer, contradicting Mo(f) = Md(f). ■

Lemma A10. *For almost any unimodal f, when $n \geq 2$, strategic candidates and one idealist tie for the win with vote share s^*, where:*

If Mo(f) < Md(f), then s^* solves (A4), locations are given by (A5) and the left extremist loses (A6);

(A4) $$z_1 = (-1)^{n+1} z_2 - 2\sum_{i=1}^{n+1} (-1)^{n+i} F^{-1}(1 - is)$$

(A5) $$y_j = (-1)^{n+1-j} z_2 - 2\sum_{i=1}^{n+1-j} (-1)^{n-j+i} F^{-1}(1 - is^*), \quad \text{s.t. } z_1 < y_j < y_{j+1}, \quad j = 1, ..., n$$

(A6) $$z_1 < 2F^{-1}(s^*) - y_1.$$

If Mo(f) > Md(f), s^* solves (A7), locations are given by (A8) and the right extremist loses (A9);

(A7) $$z_1 = (-1)^{n+1} z_2 + 2\sum_{i=1}^{n+1} (-1)^{n+i} F^{-1}(is)$$

(A8) $$y_j = (-1)^j z_1 + 2\sum_{i=1}^{j}(-1)^{j+i}F^{-1}(is^*), \quad \text{s.t. } z_1 < y_j < y_{j+1}, \quad j = 1, ..., n$$

(A9) $$z_2 > 2F^{-1}(1 - s^*) - y_n.$$

Proof: I first show that if Mo(f) < Md(f) and $n > 1$, z_1 loses: If not, by Lemma A7 z_2 loses and one can then then follow the proof of Corollary 1 to show that there must be strictly more than half the density to the left of the maximizer, contradicting Mo(f) < Md(f). Given z_1 loses, z_2 must tie with the strategic candidates by Lemma A7 and $k_j = 1$ for all j by Lemmas A6 and A8. This implies that $r = n + 1$ and that $F(m_j) = F(m_{j-1}) + s$ for $j = 1, \ldots, n+1$ where s is the equilibrium vote share and $F(m_{n+1}) \equiv 1$. Solving recursively yields (A4) which the equilibrium s, s^*, solves, giving equilibrium locations as (A5) where (A6) is the requirement for z_1 to lose: $F(m_0) < s^*$. Similarly, one finds (A7)-(A9) in the case of Mo(f) > Md(f). ■

Proposition 5 (Asymmetric distributions). *For almost any asymmetric, unimodal f satisfying (4) - (6) where Mo$(f) \neq$ Md(f), an equilibrium exists with $n > 1$ strategic candidates where locations and vote-shares are given by Lemma A10.*

	If Mo(f) < Md(f)	*If Mo(f) > Md(f)*
(4)	$f(m_0) \in [f(m_1), 2f(m_1)]$	$f(m_n) \in [f(m_{n-1}), 2f(m_{n-1})]$
(5)	$f(m_{j-1}) \leq 2f(m_j) \quad j = 2, \ldots, n$	$f(m_j) \leq 2f(m_{j-1}) \quad j = 1, \ldots, n-1$
(6)	$f(m_0) \leq \max\{f(y_1), f(z_1)\}$	$f(m_n) \leq \max\{f(y_n), f(z_2)\}$

Proof: I show that conditions (4) - (6) are sufficient for an equilibrium by considering all possible deviations in the case of Mo(f) < Md(f); those for Mo(f) > Md(f) follow similarly.

Consider deviations of the candidate at y_1 within (z_1, y_2) (the candidate at y_1 is the only strategic candidate who could have a constituency boundary to the left of the maximizer of f) (i) to the left: the candidate at y_2 then becomes the candidate with the highest vote-share of all other candidates, hence if $f(m_0) \leq 2f(m_1)$ there is no profitable deviation within (z_1, y_1); (ii) to the right: for a small move, z_1 remains a loser and the candidate at y_2 becomes a loser. It must be that $f(m_0) \geq f(m_1)$ else the candidate at y_1 could profit from such a move. This implies that any deviation within (y_1, y_2) reduces this candidate's vote share, hence there is no such profitable deviation. This gives (4).

Next consider deviations for the candidate at y_j, $j > 1$ within (y_{j-1}, y_{j+1}) (i) to the left: their vote share would increase, but so will that of the candidate at y_{j+1} who then becomes the candidate with the highest share of all the others, but the plurality of the deviating candidate decreases if $f(m_{j-1}) \leq 2f(m_j)$ which gives (5); (ii) to the right: their own vote share would decrease while increasing that of the candidate at y_{j-1}.

Next consider an inactive candidate entering (i) at an occupied location: this is not profitable as it results in an outright loss; (ii) left of z_1 or right of z_2: this results in an outright loss; (iii) between two strategic candidates y_j and y_{j+1}, $j > 1$: such an interval does not contain the maximizer of f, hence the optimal such deviation is as close as possible to the candidate whose position is has higher density, y_j. But this cannot be profitable because the maximum vote share is bounded from above by $\max\{L_j, R_j\} < s^*$; (iv) between z_1 and y_1, which contains the maximizer of f: under (6), the optimal such deviation is to locate arbitrarily close to z_1 or y_1 (whichever has the higher density), but as in case (iii) this is unprofitable because $\max\{R_0, L_1\} < s^*$.

Finally, for deviations of the candidate at y_j to locations outside the interval (y_{j-1}, y_{j+1}), $j = 1, \ldots, n$, it suffices to follow the steps above relating to an inactive candidate. ■