Proof of Proposition 1 (No verification). To prove this proposition take the Lagrangian of problem (P1)

$$\mathcal{L}_1 = \sum_{i=1}^{n} \pi_i(1) s_i - \lambda \left(\sum_{i=1}^{n} (\pi_i(1) - \pi_i(0)) s_i - c_H \right) - \sum_{i=1}^{n} \xi_i s_i.$$

Then the FOCs read as

$$\frac{\partial}{\partial s_i} \mathcal{L}_1 = \pi_i(1) (1 - \lambda (1 - \delta_i)) - \xi_i = 0. \tag{P11}$$

Of course, in equilibrium the agent's incentive constraint is binding. Suppose this constraint would not be binding; that is, $\lambda = 0$. Then $\xi_i > 0$ and, hence $s_i = 0$ for all $i = 1, ..., n$. But this violates the incentive constraint (ICA), a contradiction. Hence $\lambda > 0$ in equilibrium. Consider now an outcome with $\delta_i > 1$. Then,

$$(1 + \lambda (\delta_i - 1)) > 0;$$

hence $\xi_i > 0$, which implies that $s_i^* = 0$. Suppose finally that $s_k^* > 0$ for an outcome with $\delta_k < 1$ but $k < n$. Then $\xi_k = 0$; hence $\lambda = 1/(1 - \delta_k)$. However, this implies that, for all outcomes with $\delta_i < \delta_k$, the LHS of (P11) is strictly negative; hence the equation cannot be satisfied, a contradiction. Consequently, only $s_n^* > 0$ and using the incentive constraint

$$s_n^* = \frac{c_H}{\pi_n(1)(1 - \delta_n)}.$$

The principal's expected costs then are

$$\frac{c_H}{(1 - \delta_n)} = \frac{\pi_n(1)}{\pi_n(1) - \pi_n(0)} c_H.$$

Q.E.D. ■

Proof of Proposition 2 (Auditing). Suppose that $(s_{1L}^*, s_{1H}^*, ..., s_{nL}^*, s_{nH}^*, v_1^*, ..., v_n^*)$ is a solution of the principal's problem (P2) of minimizing

$$\sum_{i=1}^{n} \pi_i(1) (p(v_i) s_{iH} + (1 - p(v_i)) s_{iL} + c v_i) \text{ such that} \tag{P2}$$

$$\sum_{i=1}^{n} \pi_i(1) (p(v_i) s_{iH} + (1 - p(v_i)) s_{iL}) - c_H \geq \tag{ICA}$$

$$\sum_{i=1}^{n} \pi_i(0) (p(v_i) s_{iL} + (1 - p(v_i)) s_{iH})$$

$$\sum_{i=1}^{n} \pi_i(1) (p(v_i) s_{iH} + (1 - p(v_i)) s_{iL}) - c_H \geq 0. \tag{IRA}$$

The Lagrangian then reads as

$$
\begin{aligned}
\mathcal{L}_2 &= \sum_{i=1}^{n}\pi_i(1)\left(p\left(v_i\right)s_{iH}+(1-p\left(v_i\right))s_{iL}+cv_i\right) \\
&\quad -\lambda\left(\sum_{i=1}^{n}\pi_i(1)\left(p\left(v_i\right)s_{iH}+(1-p\left(v_i\right))s_{iL}\right)-c_H\right) \\
&\quad +\lambda\left(\sum_{i=1}^{n}\pi_i(0)\left(p\left(v_i\right)s_{iL}+(1-p\left(v_i\right))s_{iH}\right)\right) \\
&\quad -\sum_{i=1}^{n}\xi_{iH}s_{iH}-\sum_{i=1}^{n}\xi_{iL}s_{iL}-\sum_{i=1}^{n}\xi_{iv}v_i.
\end{aligned}
$$

Then the FOCs read as:

$$
\frac{\partial}{\partial s_{iH}}\mathcal{L}_2 = \pi_i(1)p\left(v_i\right)\left(1-\lambda\left(1-\delta_i\frac{(1-p\left(v_i\right))}{p\left(v_i\right)}\right)\right)-\xi_{iH}=0 \tag{P21}
$$

$$
\frac{\partial}{\partial s_{iL}}\mathcal{L}_2 = \pi_i(1)\left(1-p\left(v_i\right)\right)\left(1-\lambda\left(1-\delta_i\frac{p\left(v_i\right)}{(1-p\left(v_i\right))}\right)\right) -\xi_{iL}=0 \tag{P22}
$$

$$
\frac{\partial}{\partial v_i}\mathcal{L}_2 = \pi_i(1)p'\left(v_i\right)\left(s_{iH}-s_{iL}\right)\left(1-\lambda\left(1+\delta_i\right)\right)+\pi_i(1)c -\xi_{iv}=0. \tag{P23}
$$

I prove the proposition in three steps. First, I show that the incentive constraint must be binding. Second, I prove that it is never optimal to pay the agent if the signal indicates a normal level of effort. And third, I argue that auditing never takes place in two outcomes but only in the one where it is most likely that the agent chose $e=1$.

Step 1: Note that $\lambda>0$, since otherwise, $s_{iH}=s_{iL}=0$ for all $i=1,...,n$ which violates the agent's incentive constraint.

Step 2: Note that

$$
\frac{p\left(v_i\right)}{(1-p\left(v_i\right))}\geq 1\geq\frac{(1-p\left(v_i\right))}{p\left(v_i\right)}
$$

for $v_i\geq 0$, with equality signs for $v_i=0$. Hence comparing condition (P21) and (P22) implies that $s_{iH}\geq s_{iL}$. Suppose that it is optimal for the principal to verify the agent's behavior in outcome x_i; that is, $v_i>0$. Then $\xi_{iv}>0$, and condition (P23) implies $s_{iH}>s_{iL}$ as well as $1<\lambda\left(1+\delta_i\right)$. But then $\xi_{iH}=0$ and (P21) implies

$$
0=1-\lambda\left(1-\delta_i\frac{(1-p\left(v_i\right))}{p\left(v_i\right)}\right)<1-\lambda\left(1-\delta_i\frac{p\left(v_i\right)}{(1-p\left(v_i\right))}\right);
$$

hence $\xi_{iL} > 0$ and $s_{iL} = 0$. (P23) then becomes

$$p'(v_i)\, s_{iH} = \frac{c}{\lambda(1+\delta_i) - 1}.$$

Step 3: Suppose that there are two outcomes $x_k < x_j$ such that $v_i > 0$ and $v_k > 0$. Note that $\delta_j < \delta_k$. To see that this cannot be part of an equilibrium, note first that (P23) for both outcomes implies

$$p'(v_j)\, s_{jH} > p'(v_k)\, s_{kH};$$

hence either $s_{jH} > s_{kH}$ or $v_j < v_k$ or both. Suppose that $v_j > v_k$ and $s_{jH} > s_{kH}$ such that this inequality is satisfied. Since $\partial/\partial v\left[(1-p(v))/p(v)\right] = -p'(v)/(p(v))^2 < 0$, it follows that

$$\begin{aligned}\left(1-\lambda\left(1-\delta_k\frac{(1-p(v_k))}{p(v_k)}\right)\right) &> \left(1-\lambda\left(1-\delta_k\frac{(1-p(v_j))}{p(v_j)}\right)\right)\\ &> \left(1-\lambda\left(1-\delta_j\frac{(1-p(v_j))}{p(v_j)}\right)\right).\end{aligned}$$

Since $s_{jH} > 0$, the RHS of this inequality is zero; hence the LHS positive and $\xi_{kH} > 0$. This implies $s_{kH} = 0$, a contradiction. Hence $v_j < v_k$. Consider now the agent's expected utility from $e = 1$ in the outcomes x_j and x_k,

$$\pi_j(1)p(v_j)\, s_{jH} + \pi_k(1)p(v_k)\, s_{kH}$$

and suppose that, instead of auditing in both outcomes, the principal only verifies his behavior in one outcome x_j with an effort v_j and rewards the agent in case of a high signal with a payment

$$\tilde{s}_{jH} = s_{jH} + \frac{\pi_k(1)p(v_k)}{\pi_j(1)p(v_j)} s_{kH}.$$

Then the agent's expected utility remains unchanged for $e = 1$. Moreover, his expected utility remains unchanged for $e = 0$. To see this, note that $s_j > 0$ and $s_k > 0$ imply

$$1-\lambda\left(1-\delta_k\frac{(1-p(v_k))}{p(v_k)}\right) = 1-\lambda\left(1-\delta_j\frac{(1-p(v_j))}{p(v_j)}\right)$$

because both sides are zero. Rearranging this equation then implies

$$\pi_k(0)(1-p(v_k))\, s_{kH} = \pi_i(0)(1-p(v_j))\, \frac{\pi_k(1)p(v_k)}{\pi_j(1)p(v_j)} s_{kH},$$

and the agent's expected utility also remains unchanged when not choosing high effort. But then the agent's incentive constraint is still binding, but the principal saves verification costs of $\pi_k(1)p(v_k)$. As a consequence, whenever it is optimal to verify the agent's behavior, auditing takes place only for outcome x_n. The equilibrium $s_n > 0$ and $v_n > 0$ is then characterized by the agent's incentive compatibility constraint as well as by (P21) and (P23) in the form

$$\begin{aligned} p'(v_n^*)\, s_{nH}^* \left(\lambda\left(1+\delta_n\right)-1\right) &= c, \\ \lambda\left(1-\delta_n \frac{\left(1-p\left(v_n^*\right)\right)}{p\left(v_n^*\right)}\right) &= 1, \\ s_{nH}^* \pi_n(1)\left(p\left(v_n^*\right)-\delta_n\left(1-p\left(v_n^*\right)\right)\right) &= c_H. \end{aligned}$$

Q.E.D. ■

Proof of Proposition 3 (Monitoring). Suppose that $(s_{1L}^*, s_{1H}^*, ..., s_{nL}^*, s_{nH}^*, v^*)$ is a solution of the principal's problem (P3):

$$\sum_{i=1}^{n} \pi_i(1)\left(p(v)\, s_{iH} + (1-p(v)) s_{iL}\right) + cv \text{ such that} \tag{P3}$$

$$\sum_{i=1}^{n} \pi_i(1)\left(p(v)\, s_{iH} + (1-p(v)) s_{iL}\right) - c_H \geq \tag{ICA}$$

$$\sum_{i=1}^{n} \pi_i(0)\left(p(v)\, s_{iL} + (1-p(v)) s_{iH}\right)$$

$$\sum_{i=1}^{n} \pi_i(1)\left(p(v)\, s_{iH} + (1-p(v)) s_{iL}\right) - c_H \geq 0. \tag{IRA}$$

Then the Lagriangian reads as

$$\begin{aligned} \mathcal{L}_3 = & \sum_{i=1}^{n} \pi_i(1)\left(p(v)\, s_{iH} + (1-p(v)) s_{iL} + cv\right) \\ & -\lambda\left(\sum_{i=1}^{n} \pi_i(1)\left(p(v)\, s_{iH} + (1-p(v)) s_{iL}\right) - c_H\right) \\ & +\lambda\left(\sum_{i=1}^{n} \pi_i(0)\left(p(v)\, s_{iL} + (1-p(v)) s_{iH}\right)\right) \\ & -\sum_{i=1}^{n} \xi_{iH} s_{iH} - \sum_{i=1}^{n} \xi_{iL} s_{iL} - \xi_v v. \end{aligned}$$

Then the FOCs read as:

$$\frac{\partial}{\partial s_{iH}}\mathcal{L}_3 = \pi_i(1)p(v)\left(1-\lambda\left(1-\delta_i\frac{(1-p(v))}{p(v)}\right)\right)-\xi_{iH}=0 \tag{P31}$$

$$\frac{\partial}{\partial s_{iL}}\mathcal{L}_3 = \pi_i(1)(1-p(v))\left(1-\lambda\left(1-\delta_i\frac{p(v)}{(1-p(v))}\right)\right) -\xi_{iL}=0 \tag{P32}$$

$$\frac{\partial}{\partial v}\mathcal{L}_3 = p'(v)\sum_{i=1}^{n}\pi_i(1)(s_{iH}-s_{iL})(1- \lambda\left(1+\frac{\sum_{i=1}^{n}\pi_i(0)(s_{iH}-s_{iL})}{\sum_{i=1}^{n}\pi_i(1)(s_{iH}-s_{iL})}\right)\Bigg)+c-\xi_v=0. \tag{P33}$$

The proposition is shown as follows. First, I show that the agent's incentive constraint is binding. Second, assuming monitoring takes place, I argue that the principal never pays a reward when the signal indicates normal effort. And third, a positive reward if the signal shows $e=1$ is only paid in the highest outcome.

Step 1: Suppose that the incentive constraint would not be binding; that is, $\lambda=0$. Then (P31) and (P32) imply that $\xi_{iH}>0$ and $\xi_{iL}>0$; hence $s_{iH}=s_{iL}=0$ for all $i=1,...,n$. Then, however, the agent's incentive constraint cannot be satisfied, a contradiction. As a result, $\lambda>0$.

Step 2: Moreover, whenever $v>0$

$$\frac{p(v)}{(1-p(v))}>1>\frac{(1-p(v))}{p(v)}.$$

Hence the RHS of condition (P31) is always greater than the RHS of (P32), implying that $s_{iH}\geq s_{iL}$. Moreover, $v>0$ implies $\xi_v>0$, and condition (P33) implies $s_{iH}>s_{iL}$ as well as $1<\lambda(1+\delta_i)$. But then $\xi_{iH}=0$ and (P31) imply

$$0=1-\lambda\left(1-\delta_i\frac{(1-p(v))}{p(v)}\right)<1-\lambda\left(1-\delta_i\frac{p(v)}{(1-p(v))}\right);$$

hence $\xi_{iL}>0$ and $s_{iL}=0$.

Step 3: Suppose finally that $s_{kH}>0$ for an outcome with δ_k with $k<n$. Then $\xi_{kH}=0$; hence

$$\lambda=1/\left(1-\delta_k\frac{(1-p(v))}{p(v)}\right).$$

Then, for all outcomes with $\delta_i<\delta_k$, the LHS of (P31) is strictly negative,

$$\left(1-\lambda\left(1-\delta_i\frac{(1-p(v))}{p(v)}\right)\right)=\left(1-\frac{\left(1-\delta_i\frac{(1-p(v))}{p(v)}\right)}{\left(1-\delta_k\frac{(1-p(v))}{p(v)}\right)}\right)<0;$$

hence condition (P31) cannot be satisfied, a contradiction. Consequently, only $s_{nH} > 0$. The equilibrium with $v > 0$ then is characterized by

$$\begin{aligned}
\lambda\left(1-\delta_n\frac{(1-p(v))}{p(v)}\right) &= 1,\\
p'(v^*)\, s_{nH}\,(\lambda(1+\delta_n)-1) &= \frac{c}{\pi_n(1)},\\
s_{nH}\pi_n(1)\,(p(v)-\delta_n(1-p(v))) &= c_H.
\end{aligned}$$

Note that the characterization of v^* reads as

$$\frac{c_H}{\frac{c}{\pi_n(1)}} = \frac{\pi_n(1)^2\,(p(v^*)-\delta_n(1-p(v^*)))^2}{\pi_n(0)p'(v^*)}.$$

The RHS as a function of v^* then is identical to the characterization of v_n^* in the case of auditing, but the LHS indicates higher costs of verification compared to the case of auditing. Since the optimal verification effort under auditing is decreasing in c, the optimal effort v^* under monitoring is lower than the optimal effort v_n^* under auditing.

Q.E.D. ■