WATER RESOURCES MANAGEMENT

Joint Entropy Based Multi-objective Evolutionary Optimization of Water Distribution Networks: Supplementary Data

Tiku T. Tanyimboh[1,2*] and Anna M. Czajkowska[2,3]

Tiku T. Tanyimboh[1,2], PhD ([*]Corresponding author)
[1]School of Civil and Environmental Engineering, University of the Witwatersrand, Johannesburg, Private Bag 3, WITS 2050 South Africa
[2]Department of Civil and Environmental Engineering, University of Strathclyde, Glasgow, James Weir Building, 75 Montrose Street, Glasgow G1 1XJ, UK

Email: tiku.tanyimboh@wits.ac.za; Tel: + 27 (0) 11 717 7105
ORCID ID: https://orcid.org/0000-0003-3741-7689

Anna M. Czajkowska[2,3], PhD
[2]Department of Civil and Environmental Engineering, University of Strathclyde, Glasgow, James Weir Building, 75 Montrose Street, Glasgow G1 1XJ, UK
[3]RPS Group, Merchantile Building, 53 Bothwell Street, Glasgow G2 6TS, UK

Cite this article as: Tanyimboh TT, Czajkowska AM (2018) Joint entropy based multi-objective evolutionary optimization of water distribution networks. Water Resources Management, DOI: 10.1007/s11269-017-1888-y

Funding: This project was funded by the UK Engineering and Physical Sciences Research Council (EPSRC) (EPSRC Grant Number EP/G055564/1).

SUPPLEMENTARY DATA

Appendix A: Informational Entropy Function

The informational entropy of the pipe flow rates for the k^{th} loading condition is (Tanyimboh and Templeman 1993a, 1993b)

$$S_k = S_{0,k} + \sum_{n=1}^{nn} p_{n,k} S_{n,k} \;;\forall k \qquad \text{(A1)}$$

where S_k is the entropy; $S_{0,k}$ is the entropy due to the contributions of the supply nodes; $S_{n,k}$ is the entropy at node *n;* and *nn* is the number of nodes in the network. $p_{n,k} \equiv T_{n,k} / T_k$ is the fraction of the total flow through the network that reaches node *n;* $T_{n,k}$ is the total flow that reaches node *n*; and T_k is the sum of the nodal demands, for the k^{th} loading condition.

The entropy due to the contributions of the supply nodes is

$$S_{0,k} = -\sum_{n \in I_k} \frac{Q_{0n,k}}{T_k} \ln\left(\frac{Q_{0n,k}}{T_k}\right); \;\; \forall k \qquad \text{(A2)}$$

$Q_{0n,k}$ is the flow at supply node *n* and I_k represents the set of supply nodes for the k^{th} loading condition. Similarly, the entropy at demand node *n* is

$$S_{n,k} = -\frac{Q_{n0,k}}{T_{n,k}} \ln\left(\frac{Q_{n0,k}}{T_{n,k}}\right) - \sum_{ij \in ND_{n,k}} \frac{Q_{ij,k}}{T_{n,k}} \ln\left(\frac{Q_{ij,k}}{T_{n,k}}\right); \qquad n = 1, \;\; \ldots, \;\; nn \,, \forall k \qquad \text{(A3)}$$

$Q_{n0,k}$ is the demand at node *n*; $Q_{ij,k}$ is the flow rate in pipe *ij* with nodes *i* and *j* as the upstream and downstream nodes, respectively. The set $ND_{n,k}$ represents the pipe flows from node *n.*

A comparison of the entropy models proposed in Awumah *et al.* (1990, 1991) and Tanyimboh and Templeman (1993a, 1993b) is available in Tanyimboh (1993) where, *inter alia,* it was observed that there were inconsistencies in the probability models used in Awumah *et al.* (1990, 1991).

Appendix B: Network Performance Evaluation

Two of the criteria used to assess the solutions achieved by the proposed maximum entropy approach were the hydraulic capacity reliability and failure tolerance. By definition, the hydraulic capacity reliability incorporates the mechanical reliability. Pressure-driven analysis was used to simulate the effects of pipe failures, based on the logistic nodal pressure-discharge function (Tanyimboh and Templeman 2010).

The definition used for the hydraulic (capacity) reliability is the ability to fulfil the nodal demands at adequate pressure under both normal and abnormal operating conditions (Tanyimboh and Templeman 2000). The nodal demands usually refer to the peak loading condition, and the reliability refers to the fraction of the demands satisfied at adequate pressure.

$$\begin{aligned} R = \frac{1}{T}\left(p(0)T(0) + \sum_{m=1}^{np} p(m)T(m) + \sum_{m=1}^{np-1}\sum_{n=m+1}^{np} p(m,n)T(m,n) + \ldots \right) \\ + \frac{1}{2}\left(1 - p(0) - \sum_{m=1}^{np} p(m) - \sum_{m=1}^{np-1}\sum_{n=m+1}^{np} p(m,n) - \ldots \right) \end{aligned} \quad \text{(B1)}$$

where *R* represents the hydraulic reliability; *np* is the number of pipes or links in the network; *p(0)* is the probability that all links are in service; *p(m)* is the probability that only link *m* is not in service; *p(m, n)* is the probability that only links *m* and *n* are not in service. *T* is the sum of the nodal demands; *T(0)*, *T(m)* and *T(m, n*) represent the respective total flows supplied with zero link, only link *m*, and only links *m* and *n* out of service. The probabilistic pipe failure model developed by Cullinane *et al.* (1992) was used.

The pipe or link failure tolerance (Tanyimboh *et al.* 2001) provides an estimate of the total demand that the water distribution network is capable of satisfying on average when one or more components are out of service. The need to include the failure tolerance when assessing the hydraulic properties and resilience of water distribution networks has been discussed previously in the literature (Gheisi and Naser 2013, 2015).

The pipe failure tolerance, *FT*, and hydraulic reliability, *R*, are related as follows (Tanyimboh *et al.* 2001).

$$FT = \frac{R - p(0)T(0)/T}{1 - p(0)} \tag{B2}$$

The failure tolerance is usually evaluated at the same time as the hydraulic reliability and it is a simple calculation once *R* and *p(0)* are available. To calculate the hydraulic reliability, the *minimum* number of pressure-driven simulations required for each solution under consideration is $np+1$; *np* is the number of links or pipes. Generally, the procedure is highly expensive computationally, especially if there are many Pareto sets with large populations to analyse.

REFERENCES

Awumah K, Goulter I, Bhatt SK (1990) Assessment of reliability in water distribution networks using entropy based measures. Stoch. Hydrol. Hydraul. 4(4): 309-320

Awumah K, Goulter I, Bhatt SK (1991) Entropy-based redundancy measures in water distribution network design. J. Hydraulic Engineering 117(5): 595-614

Cullinane MJ, Lansey KE, Mays LW (1992) Optimization-availability-based design of water distribution networks. J. Hydraulic Engineering 118: 420-441

Gheisi A, Naser G (2013) On the significance of maximum number of component failures in reliability analysis of water distribution systems. Urban Water 10(1): 10-25

Gheisi A, Naser G (2015) Multistate reliability of water-distribution systems: comparison of surrogate measures. J. Water Res. Pl.-ASCE 141(10)

Tanyimboh TT (1993) An entropy based approach to the optimum design of reliable water distribution networks. PhD Thesis, University of Liverpool, UK

Tanyimboh T, Tabesh M, Burrows R (2001) An appraisal of source head methods for calculating the reliability of water distribution networks. J. Water Res. Pl.-ASCE 127(4):

206-213

Tanyimboh TT, Templeman AB (1993a) Calculating maximum entropy flows in networks. J. Operational Research Society 44(4): 383-396

Tanyimboh TT, Templeman AB (1993b) Optimum design of flexible water distribution networks. Civil Engineering Systems 10(3): 243-258

Tanyimboh TT, Templeman AB (2000) A quantified assessment of the relationship between the reliability and entropy of water distribution systems. Eng. Optim. 33(2): 179-199

Tanyimboh TT, Templeman AB (2010) Seamless pressure-deficient water distribution system model. J. Water Management 163(8): 389-396

Appendix C: Additional Results

Table C1 Non-dominated feasible solutions achieved

Design	Cost (€10^6)	Surplus Head (m) Peak Demand	Fire Flow 1	Fire Flow 2	Entropy Peak Demand	Fire Flow 1	Fire Flow 2	Total Entropy
1	8.074	0.030	13.619	13.828	5.125	*5.033*	**5.134**	15.292
2	8.150	0.030	13.768	13.528	5.175	**5.205**	*5.171*	15.551
3	8.155	0.066	13.632	13.422	5.214	*5.140*	**5.277**	15.631
4	8.168	0.087	13.279	13.872	*5.247*	**5.342**	5.267	15.856
5	8.193	0.010	13.479	13.914	5.277	**5.341**	*5.248*	15.866
6	8.215	0.106	13.596	14.007	5.322	**5.372**	*5.286*	15.980
7	8.253	0.173	13.857	13.771	*5.316*	**5.366**	5.337	16.019
8	8.288	0.048	13.484	11.647	5.413	**5.487**	*5.410*	16.310
9	8.319	0.064	13.761	13.636	**5.521**	5.515	*5.504*	16.540
10	8.335	0.104	13.722	13.898	5.586	*5.537*	**5.606**	16.729
11	8.355	0.054	13.788	11.609	**5.612**	*5.579*	5.596	16.787
12	8.362	0.006	13.583	13.423	5.660	*5.639*	**5.737**	17.036
13	8.408	0.123	13.798	13.295	5.942	*5.917*	**5.952**	17.811
14	8.464	0.007	13.637	13.474	5.935	*5.905*	**6.004**	17.844
15	8.473	0.037	13.711	13.555	6.034	*6.021*	**6.104**	18.159
16	8.492	0.050	13.797	12.840	**6.181**	*6.150*	6.180	18.511
17	8.509	0.007	13.697	13.418	6.232	*6.207*	**6.295**	18.734
18	8.561	0.025	13.737	13.042	6.259	*6.231*	**6.311**	18.801
19	8.583	0.044	13.729	13.517	6.270	*6.234*	**6.346**	18.850
20	8.610	0.019	13.701	13.507	6.301	*6.267*	**6.368**	18.936
21	8.619	0.019	13.676	13.555	6.334	*6.278*	**6.402**	19.014
22	8.642	0.002	13.683	13.521	6.358	*6.317*	**6.411**	19.086
23	8.668	0.039	13.720	13.544	6.400	*6.355*	**6.468**	19.223
24	8.684	0.022	13.699	13.506	6.414	*6.370*	**6.486**	19.270
25	8.686	0.032	13.770	13.617	6.438	*6.392*	**6.506**	19.336
26	8.718	0.024	13.714	13.533	6.433	*6.408*	**6.534**	19.375
27	8.744	0.030	13.724	13.492	6.456	*6.428*	**6.537**	19.421
28	8.750	0.010	13.711	13.515	6.485	*6.454*	**6.539**	19.478
29	8.755	0.015	13.666	13.660	6.496	*6.453*	**6.571**	19.520
30	8.776	0.039	13.707	13.510	6.508	*6.452*	**6.577**	19.537
31	8.793	0.022	13.709	13.498	6.513	*6.464*	**6.574**	19.551
32	8.803	0.006	13.663	13.456	6.522	*6.488*	**6.591**	19.601
33	8.809	0.021	13.697	13.486	6.541	*6.522*	**6.616**	19.679
34	8.825	0.005	13.681	13.470	6.566	*6.548*	**6.631**	19.745
35	8.841	0.027	13.711	13.625	6.569	*6.544*	**6.646**	19.759
36	8.856	0.016	13.687	13.613	6.606	*6.550*	**6.679**	19.835
37	8.862	0.028	13.704	13.664	6.598	*6.567*	**6.676**	19.841
38	8.867	0.013	13.701	13.637	6.627	*6.570*	**6.699**	19.896
39	8.872	0.006	13.704	13.383	6.644	*6.602*	**6.703**	19.949
40	8.901	0.014	13.668	13.512	6.643	*6.610*	**6.719**	19.972

The highest entropy value in each row is in bold and the lowest is in italics and shaded.

Table C1 (continued) Non-dominated feasible solutions achieved

Design	Cost (€10^6)	Surplus Head (m) Peak Demand	Fire Flow 1	Fire Flow 2	Entropy Peak Demand	Fire Flow 1	Fire Flow 2	Total Entropy
41	8.905	0.044	13.744	13.635	6.658	*6.602*	**6.732**	19.992
42	8.910	0.067	13.744	13.555	6.660	*6.626*	**6.736**	20.022
43	8.937	0.001	13.646	13.389	6.669	*6.625*	**6.763**	20.057
44	8.961	0.019	13.669	13.644	6.689	*6.644*	**6.772**	20.105
45	8.982	0.001	13.671	13.278	6.705	*6.653*	**6.795**	20.153
46	9.016	0.021	13.698	13.307	6.711	*6.657*	**6.807**	20.175
47	9.019	0.001	13.676	13.284	6.721	*6.665*	**6.815**	20.201
48	9.041	0.003	13.674	13.408	6.727	*6.669*	**6.832**	20.228
49	9.048	0.020	13.693	13.427	6.730	*6.672*	**6.835**	20.237
50	9.067	0.033	13.708	13.440	6.733	*6.676*	**6.838**	20.247
51	9.100	0.001	13.727	13.519	6.768	*6.715*	**6.837**	20.320
52	9.159	1.3E-4	13.743	13.058	6.862	*6.799*	**6.903**	20.564
53	9.170	0.011	13.756	13.071	6.870	*6.806*	**6.912**	20.588
54	9.211	0.015	13.721	13.695	6.892	*6.826*	**6.949**	20.667
55	9.284	0.032	13.712	13.695	6.906	*6.812*	**6.970**	20.688
56	9.332	0.003	13.700	13.590	6.901	*6.844*	**7.005**	20.750
57	9.420	0.005	13.705	13.594	6.925	*6.869*	**7.030**	20.824
58	9.458	0.006	13.638	13.453	6.930	*6.897*	**7.013**	20.840
59	9.522	0.003	13.682	13.762	6.970	*6.900*	**7.032**	20.902
60	9.536	0.011	13.692	13.705	6.970	*6.900*	**7.056**	20.926
61	9.578	0.027	13.646	13.607	6.953	*6.916*	**7.069**	20.938
62	9.624	0.009	13.690	13.705	6.982	*6.610*	**7.068**	20.960
63	9.629	0.059	13.672	13.769	6.961	*6.922*	**7.085**	20.968
64	9.641	0.039	13.727	13.740	6.990	*6.916*	**7.076**	20.982
65	9.691	0.026	13.651	13.764	7.003	*6.969*	**7.099**	21.071
66	9.747	2.0E-6	13.659	13.690	7.010	*6.959*	**7.123**	21.092
67	9.804	0.002	13.656	13.727	7.038	*6.984*	**7.156**	21.178
68	9.832	0.005	13.663	13.716	7.052	*6.995*	**7.168**	21.215
69	9.872	0.008	13.670	13.721	7.070	*7.016*	**7.186**	21.272
70	9.919	0.002	13.665	13.694	7.103	*7.046*	**7.218**	21.367
71	9.945	0.010	13.672	13.724	7.104	*7.048*	**7.221**	21.373
72	10.013	0.006	13.662	13.728	7.111	*7.055*	**7.229**	21.395
73	10.037	0.016	13.678	13.731	7.119	*7.060*	**7.237**	21.416
74	10.108	0.009	13.667	13.724	7.127	*7.069*	**7.244**	21.440

The highest entropy value in each row is in bold and the lowest is in italics and shaded.