Inequality, mobility and the financial accumulation process: A computational economic analysis
Supplementary Material

1 Additional Results

In order to further corroborate the results shown in the main text, we present here additional results for different indicators of wealth inequality and distribution as well as of social mobility.

Wealth Inequality. Concerning the wealth inequality, an alternative to the Gini index is the Theil Index (Theil 1967), which is defined as follows:

$$Th_t = \frac{\sum_{i=1}^{N} \frac{W_{i,t}}{\hat{W}_t} \log\left(\frac{W_i}{\hat{W}_t}\right)}{N \log(N)} \quad \text{with} \quad 0 \leq T_t \leq 1 \tag{1}$$

where $\hat{W}_t$ is the average wealth of the population of agents at that time step t. Results concerning the Theil Index and relative to the cases studied in the main text are shown in Figures 1, 2 and 3.

Our result on wealth inequality is further reinforced by visualising the share of wealth appropriated by the upper 1% of population, as well as the ratio of it over the share appropriated by the lower 50% (Figures 9, 10, 11, 12 and 13).

Finally we also reproduce in a Log-Log graph the wealth of individuals as function of their position in the population' wealth ranking (Figures 4, 5, 6, 7 and 8, Bottom Panels.) with the aim of reproducing the results of Levy and Levy (2003).

Social Mobility. Concerning the social mobility in the main text we introduced the Weighted Mobility index. To corroborate the results of this index, we here propose two alternative indexes (displayed in Figures 14, 15,16, 17, 18 and 19):

- The Mean Wealth Change Index V_t denotes the average of the relative change in wealth of each agent i between two adjacent time periods t and $t-1$, weighted by the aggregate range of wealth at time t (i.e., the difference between maximum and minimum wealth), as follows:

$$V_t = \frac{1}{N} \sum_{i=1}^{N} \frac{W_{i,t} - W_{i,t-1}}{\max_i W_{i,t} - \min_i W_{i,t}} \tag{2}$$

  This index captures the relative capacity by agent i to move across wealth positions relative to the maximum inequality that is present at time t. A positive index implies a relative improvement, and viceversa.

- The Absolute Wealth Change Index, which corresponds to the magnitude of V_t, defined as:

$$|V_t| = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{W_{i,t} - W_{i,t-1}}{\max W_t - \min W_t} \right| \tag{3}$$

1.1 Theil Index

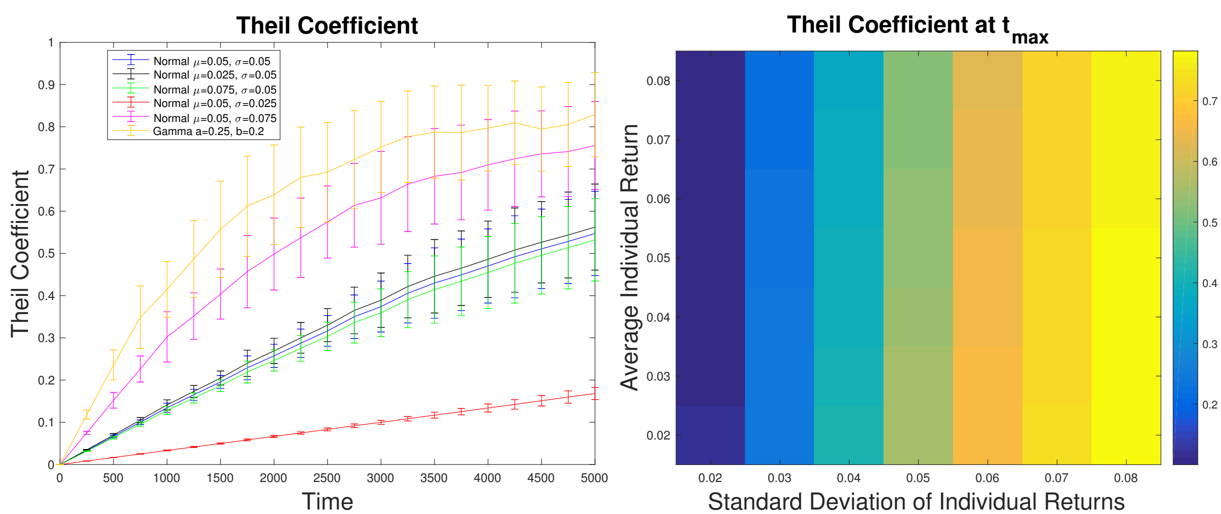

Figure 1: Left Panel: Theil index over time periods for wealth distribution of 5000 individuals under different return structures: $r_{i,t} \sim N(\mu_r, \sigma_r)$ and $gamma(a, b)$. Averages and standard deviations are computed out of 100 simulations. Right Panel: Theil Index value Th_t at time $t_{max} = 5000$ under various return structures $r_{i,t} \sim N(\mu_r, \sigma_r)$.

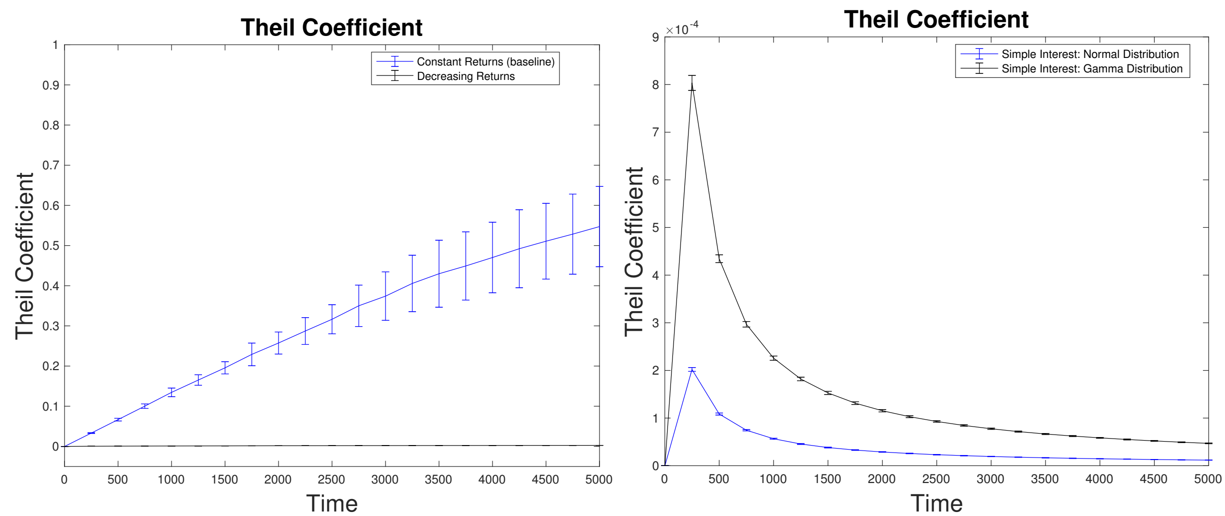

Figure 2: Left Panel: Comparison of Theil Index over time under constant and decreasing returns to aggregate wealth. Right Panel: Theil Index over time under simple return structure under normal distribution $N(\mu_r, \sigma_r)$, and simple return structure under Gamma distribution $gamma(a, b)$. Averages and standard deviations are computed out of 100 simulations.

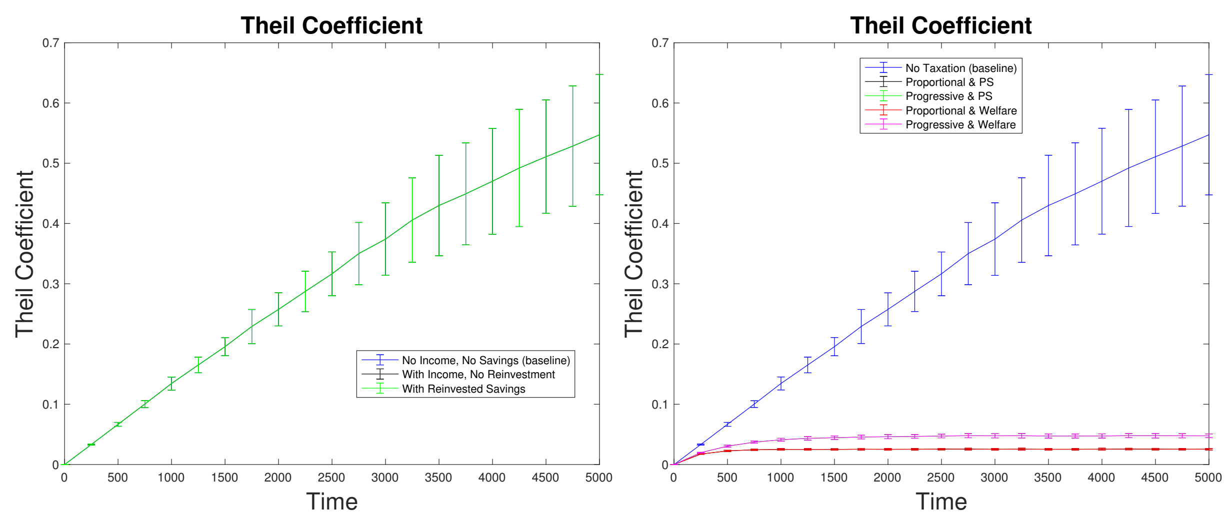

Figure 3: Left Panel: Theil Index value over time under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time. Right Panel: Theil Index over time under baseline case (no taxation), proportional taxation and public service model (**Proportional & PS**), proportional taxation and welfare model (**Proportional & Welfare**), progressive taxation and public service model (**Progressive & PS**) and progressive taxation and welfare model (**Progressive & Welfare**). Averages and standard deviations are computed out of 100 simulations.

1.2 Evolution of wealth and its distribution

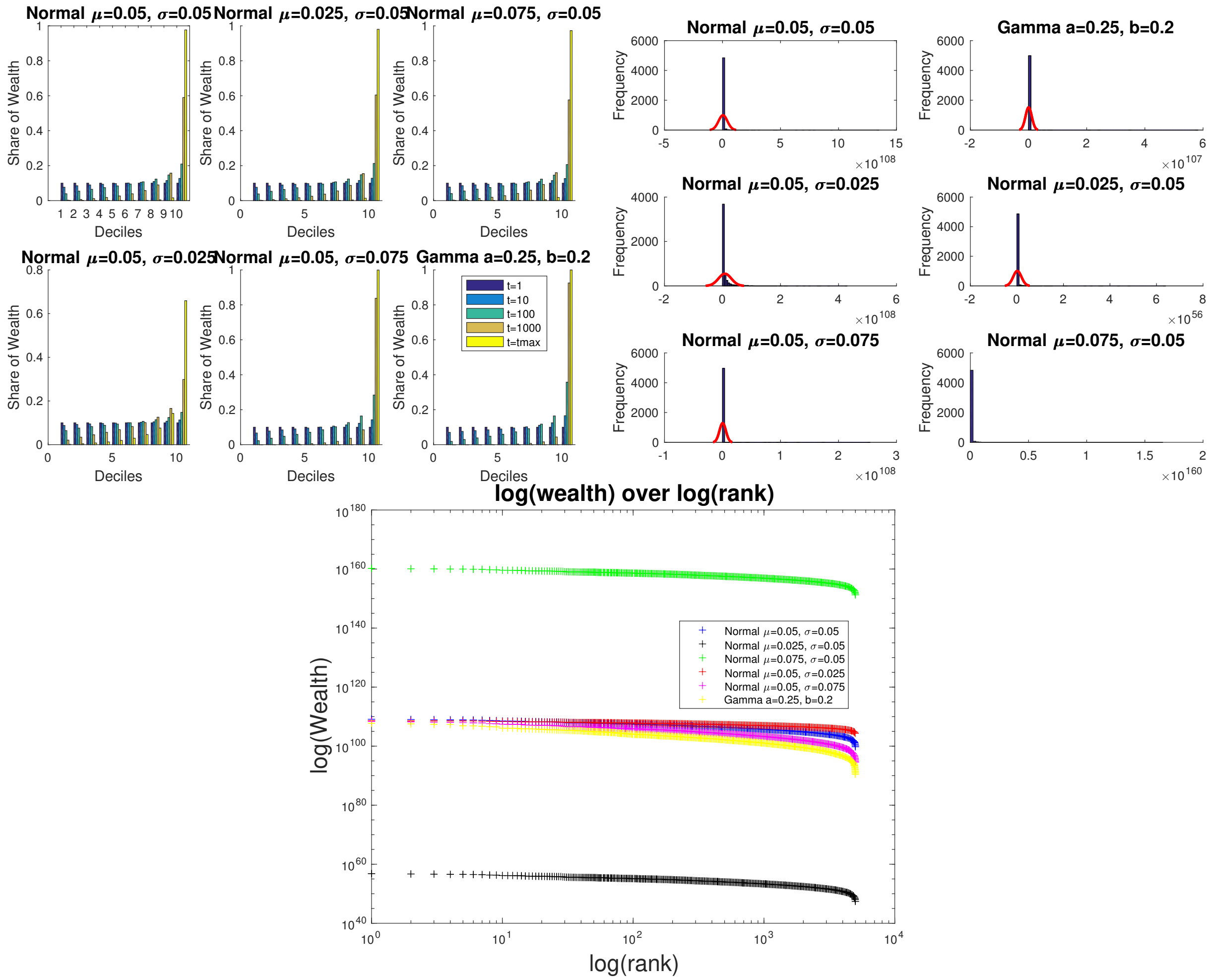

Figure 4: Top Left Panel: Decile-based distribution of wealth for 5000 individuals under different return structures: $r_{i,t} \sim N(\mu_r, \sigma_r)$ and $gamma(a, b)$. The red line shows a normal distribution with same mean and variance. Top Right Panel: Frequency based distribution of wealths at $t_{max} = 5000$, for the same return structures. Lower Panel: Log-log plot of wealth-rank relationship at $t_{max} = 5000$ for the same return structures.

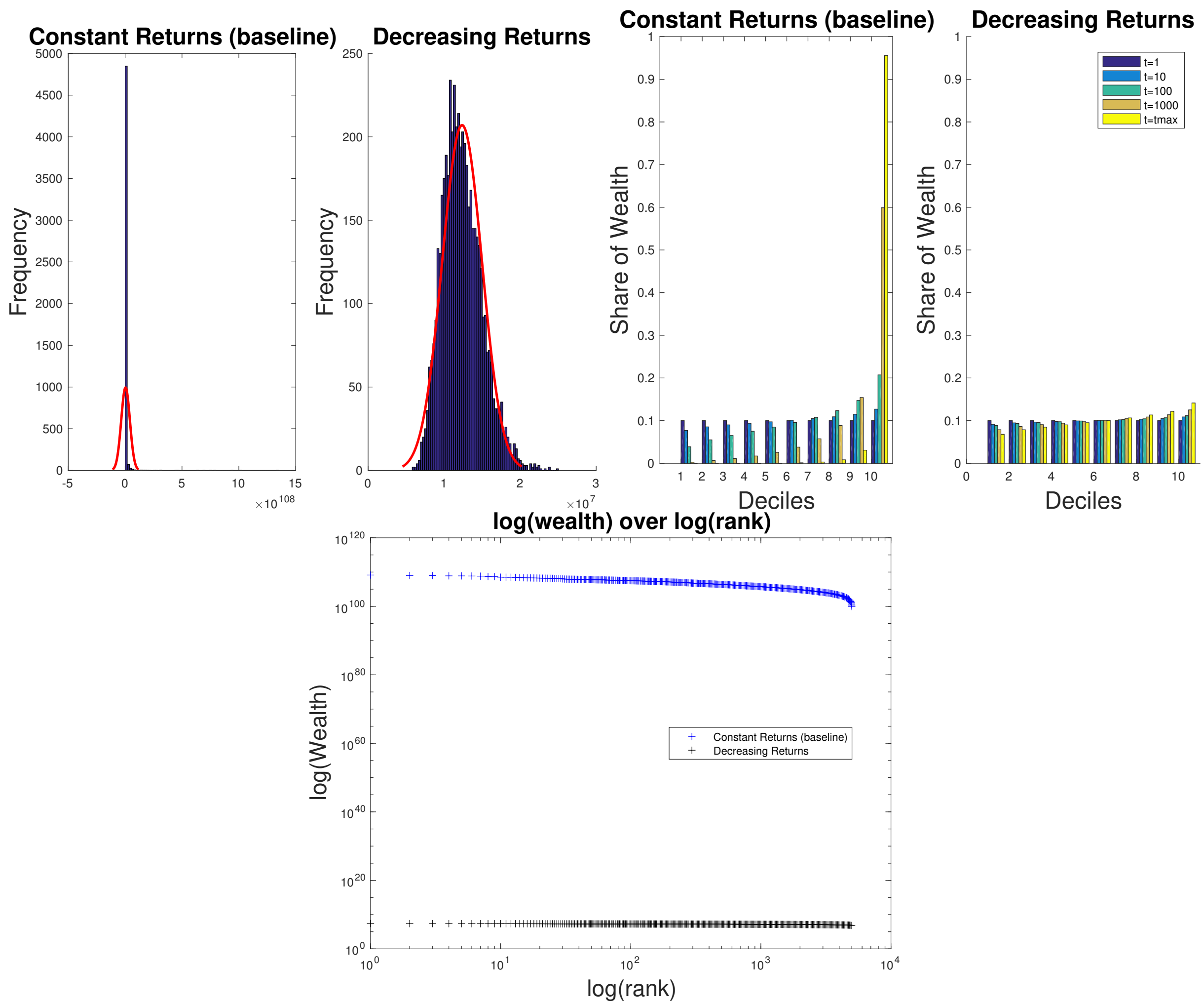

Figure 5: Top Left Panel: Comparison of wealth distribution at $t_{max} = 5000$ under constant returns (baseline) and decreasing returns to aggregate wealth. The red line shows a normal distribution with same mean and variance. Top Right Panel: Comparison of decile-based representations of wealth distribution at $t_{max} = 5000$ under constant and decreasing returns to aggregate wealth. Bottom Panel: Log-log plot of wealth-rank relationship at $t_{max} = 5000$ under constant and decreasing returns to aggregate wealth.

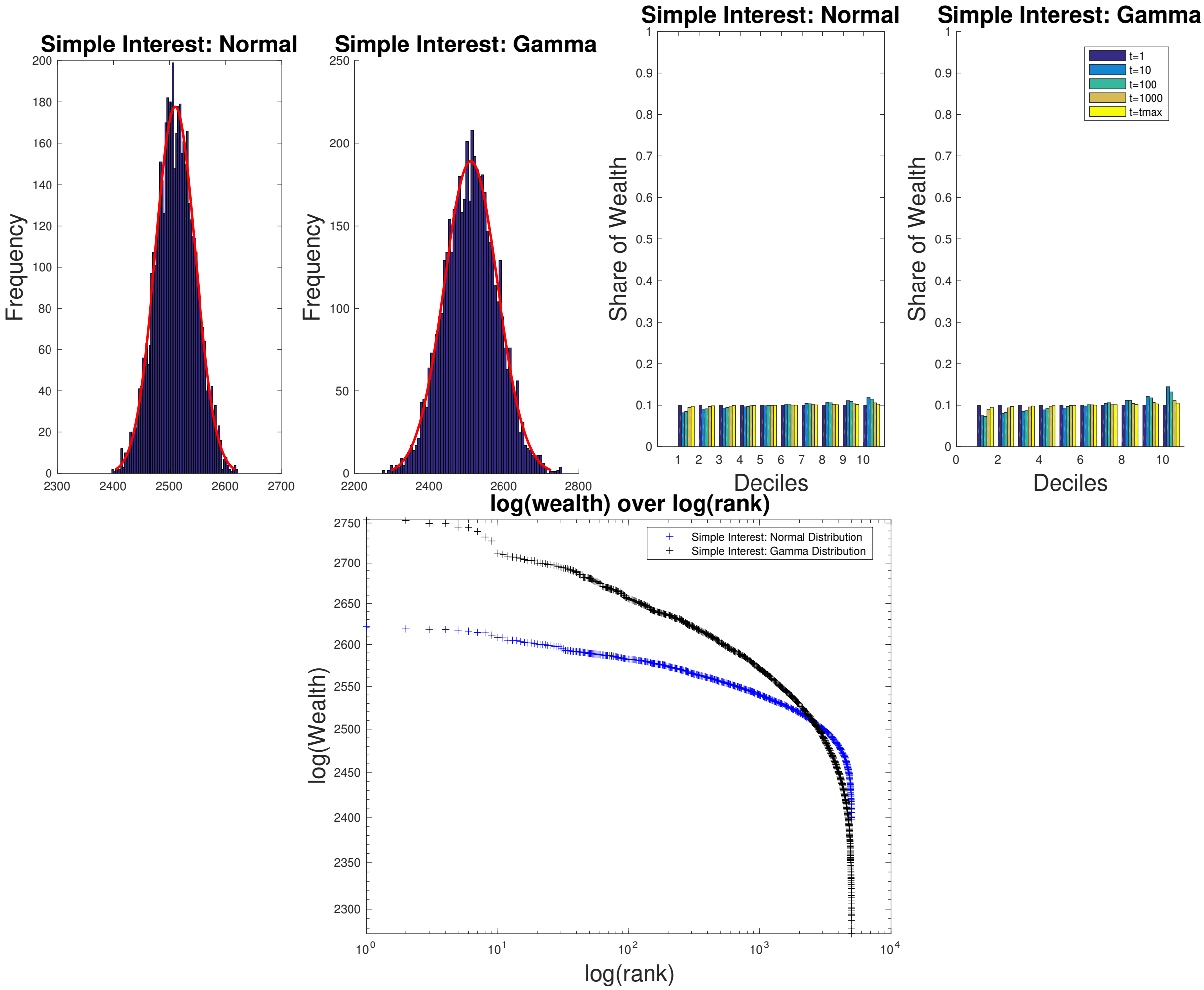

Figure 6: Top Left Panel: Comparison of wealth distributions under simple return structure with normal distribution $N(\mu_r, \sigma_r)$, and under simple return with Gamma distribution $gamma(a, b)$. The red line shows a normal distribution with same mean and variance. Top Right Panel: Decile-based distributions under simple return structure with normal distribution $N(\mu_r, \sigma_r)$, and under simple return with Gamma distribution $gamma(a, b)$. Bottom Panel: Log-log plots of wealth-rank relationship under simple return structure with normal distribution $N(\mu_r, \sigma_r)$, and under simple return with Gamma distribution $gamma(a, b)$

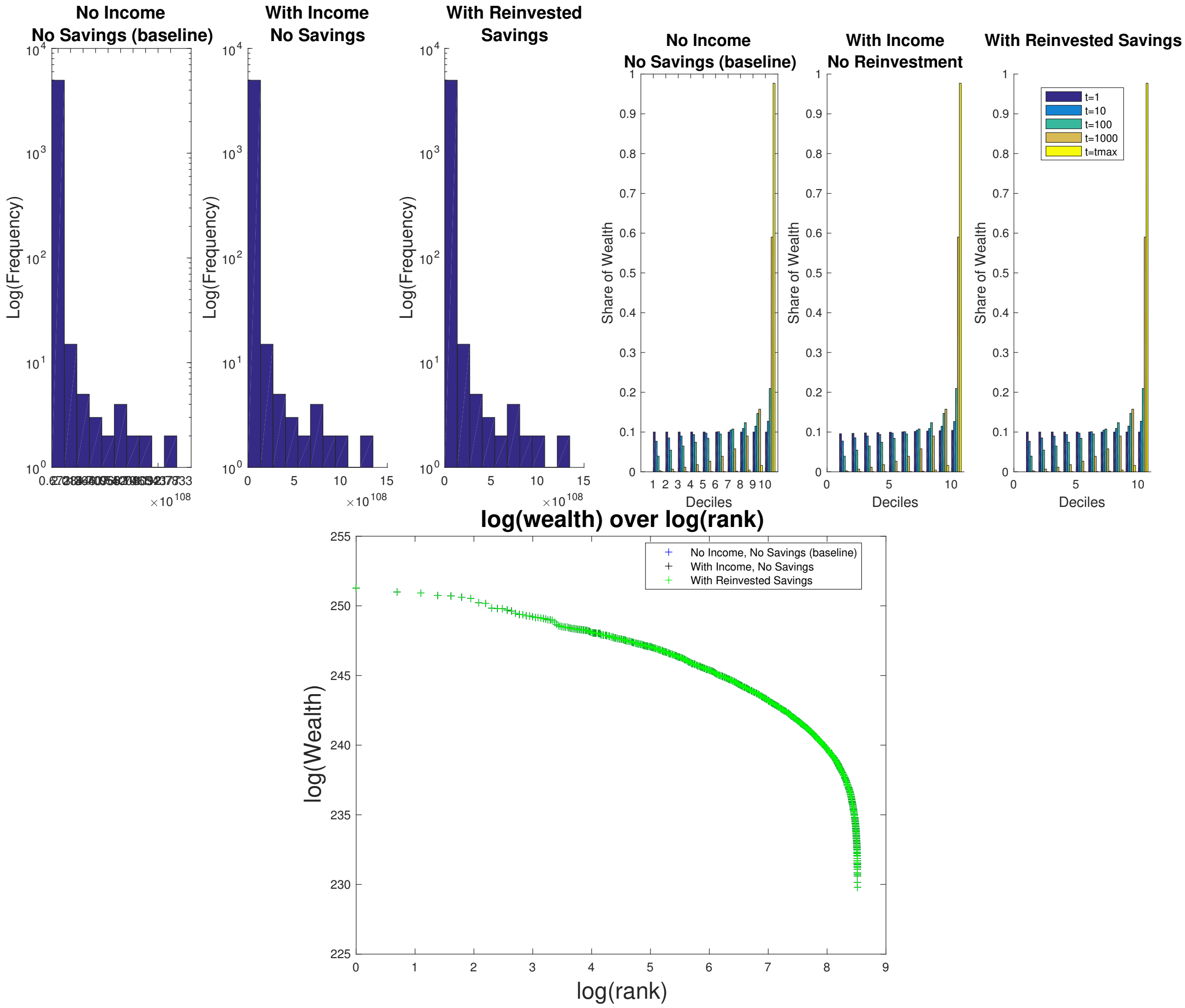

Figure 7: Top Left Panel: Comparison of wealth distributions under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time. Top Right Panel: Decile-based wealth distributions under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time Bottom Panel: Log-log plots of wealth-rank relationship under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time.

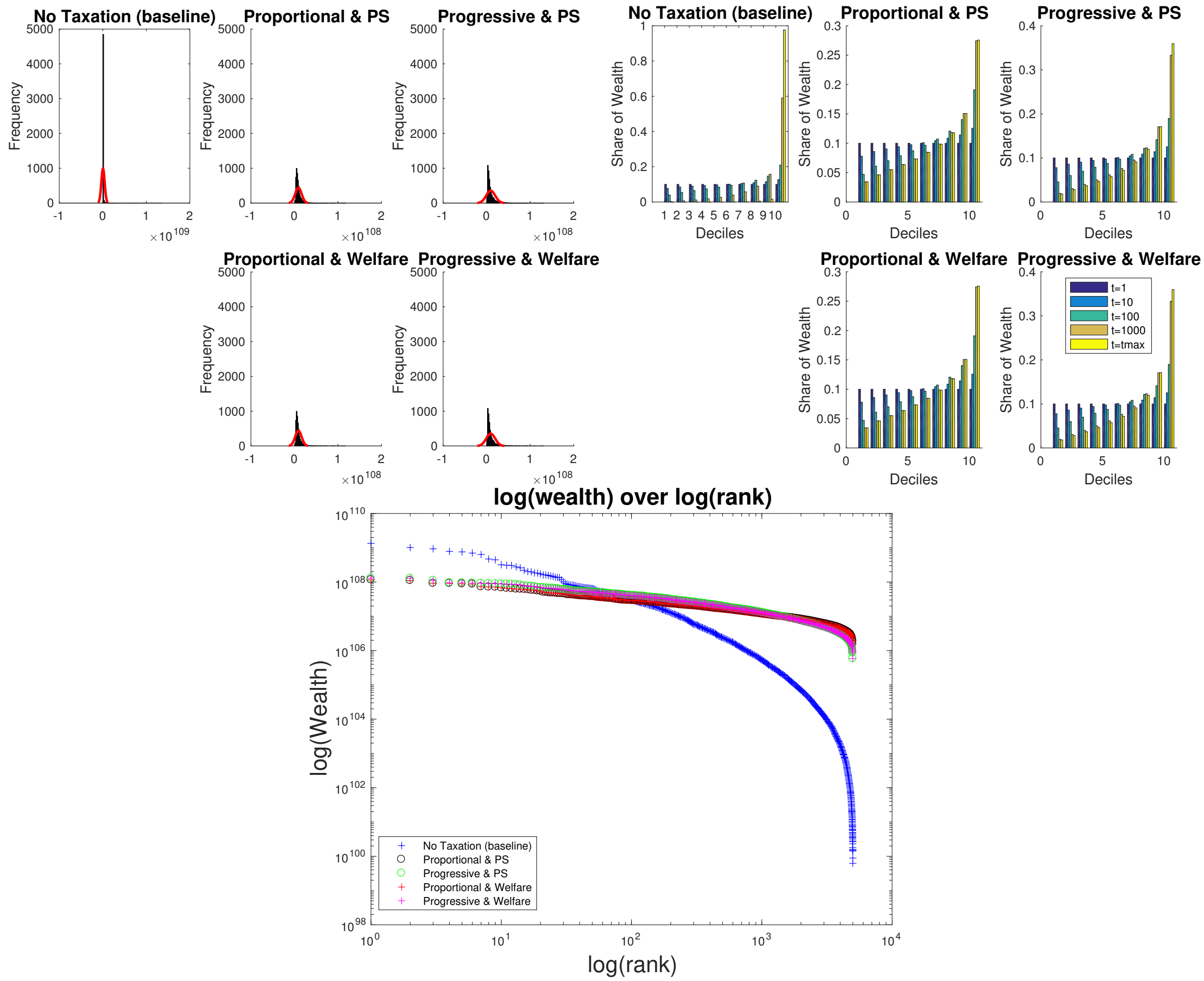

Figure 8: Top Left Panel: Wealth distributions under baseline case (no taxation), proportional taxation and public service model (**Proportional & PS**), proportional taxation and welfare model (**Proportional & Welfare**), progressive taxation and public service model (**Progressive & PS**) and progressive taxation and welfare model (**Progressive & Welfare**). The red line shows a normal distribution with same mean and variance. Top Right Panel: Decile-based distributions under the same cases. Bottom Panel: Log-log plots of wealth-rank relationship under the same cases.

1.3 Absolute and Relative wealth of Top 1%

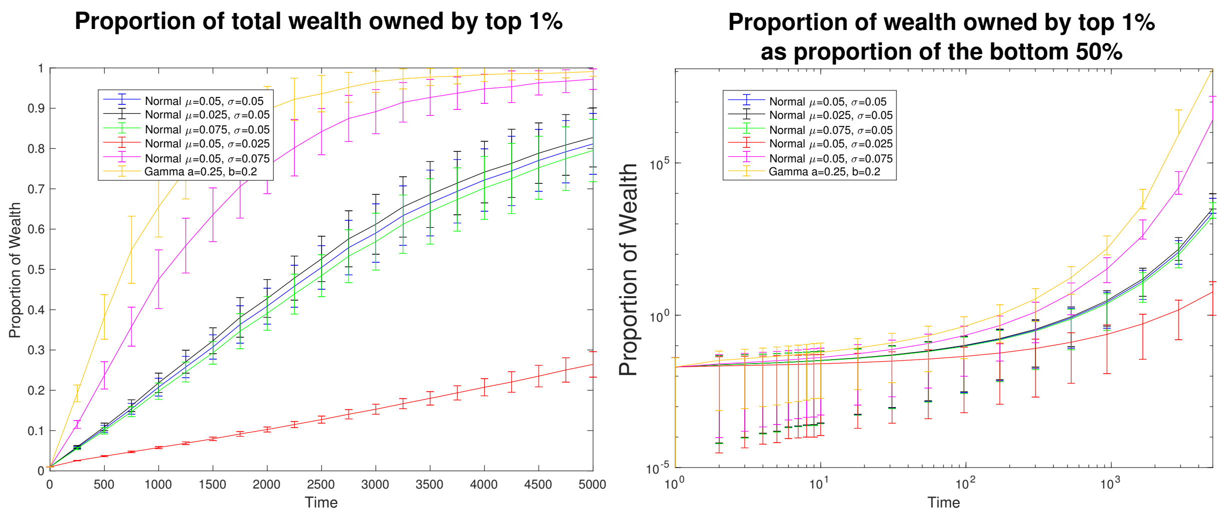

Figure 9: Left Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the total wealth (minimum theoretical value of 0.01). Right Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the wealth owned by the bottom 50% (minimum theoretical value of 0.02). Through time, mean value increases while its variance decreases. Averages and standard deviations are computed out of 100 simulations.

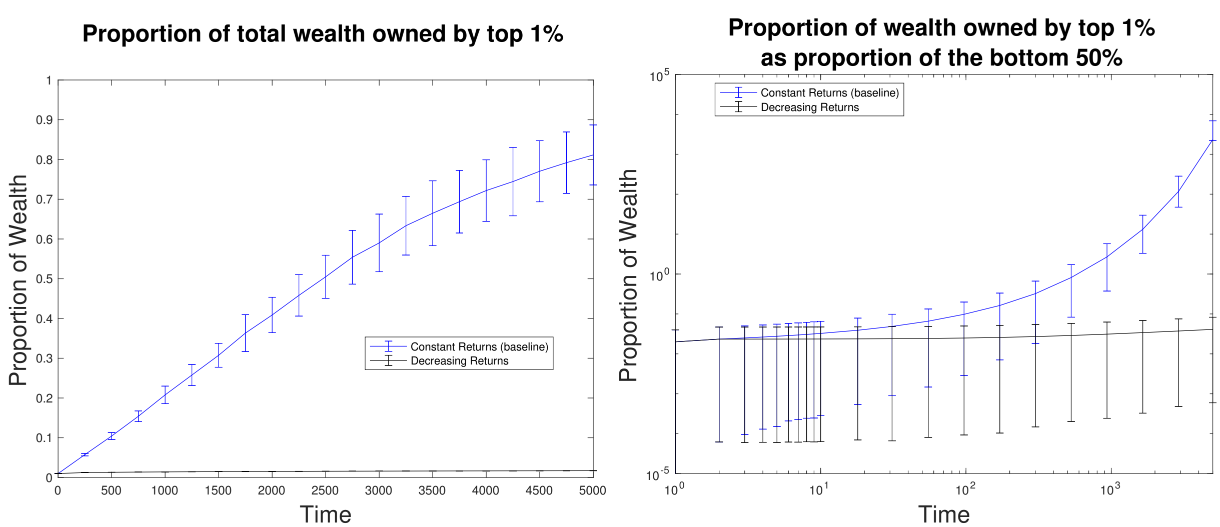

Figure 10: Left Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the total wealth (minimum theoretical value of 0.01). Right Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the wealth owned by the bottom 50% (minimum theoretical value of 0.02). Averages and standard deviations are computed out of 100 simulations.

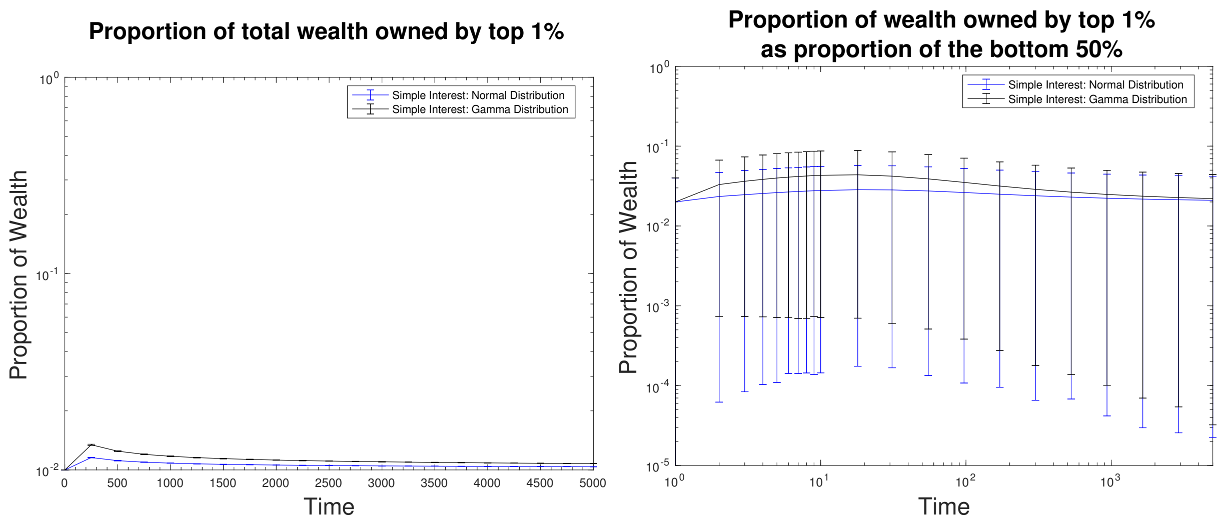

Figure 11: Left Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the total wealth (minimum theoretical value of 0.01). Right Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the wealth owned by the bottom 50% (minimum theoretical value of 0.02). Averages and standard deviations are computed out of 100 simulations.

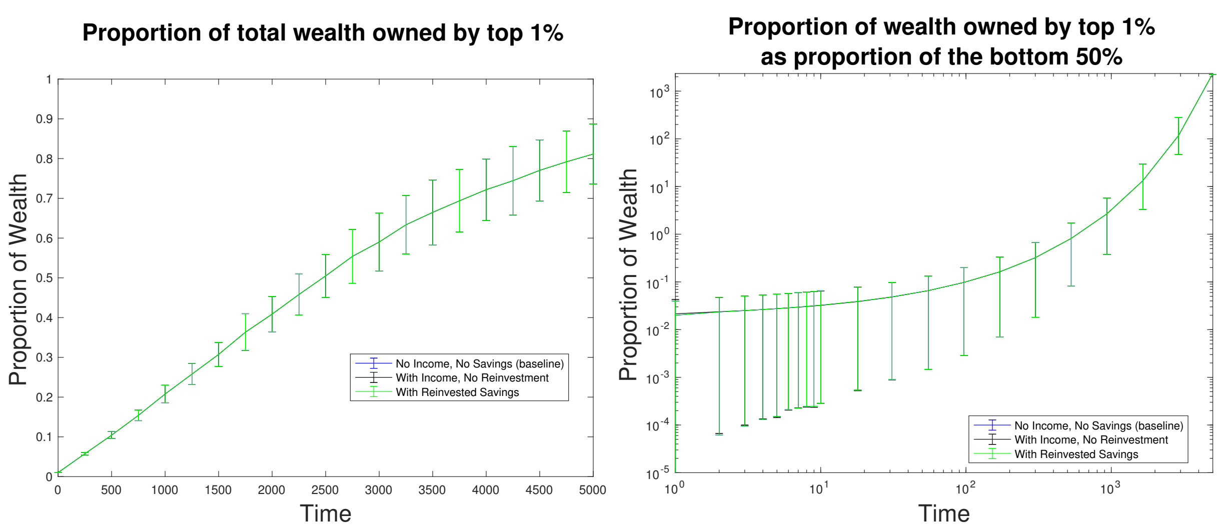

Figure 12: Left Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the total wealth (minimum theoretical value of 0.01). Right Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the wealth owned by the bottom 50% (minimum theoretical value of 0.02). Through time, mean value increases while its variance decreases. Averages and standard deviations are computed out of 100 simulations.

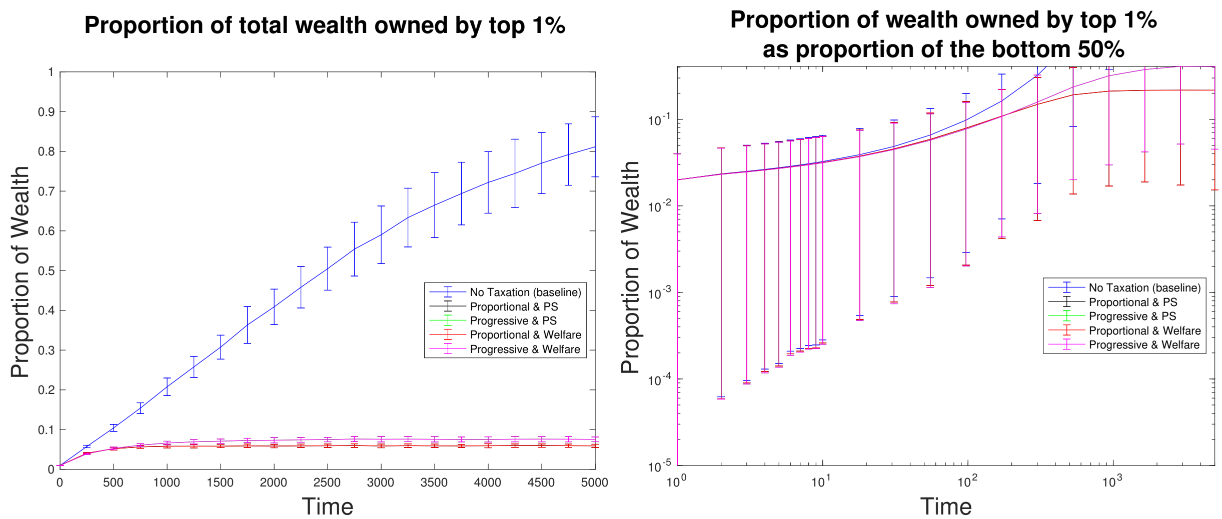

Figure 13: Left Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the total wealth (minimum theoretical value of 0.01). Right Panel: Proportion of wealth owned at each time by the richest 1% of the population as proportion of the wealth owned by the bottom 50% (minimum theoretical value of 0.02). Averages and standard deviations are computed out of 100 simulations.

1.4 Wealth Change Indexes

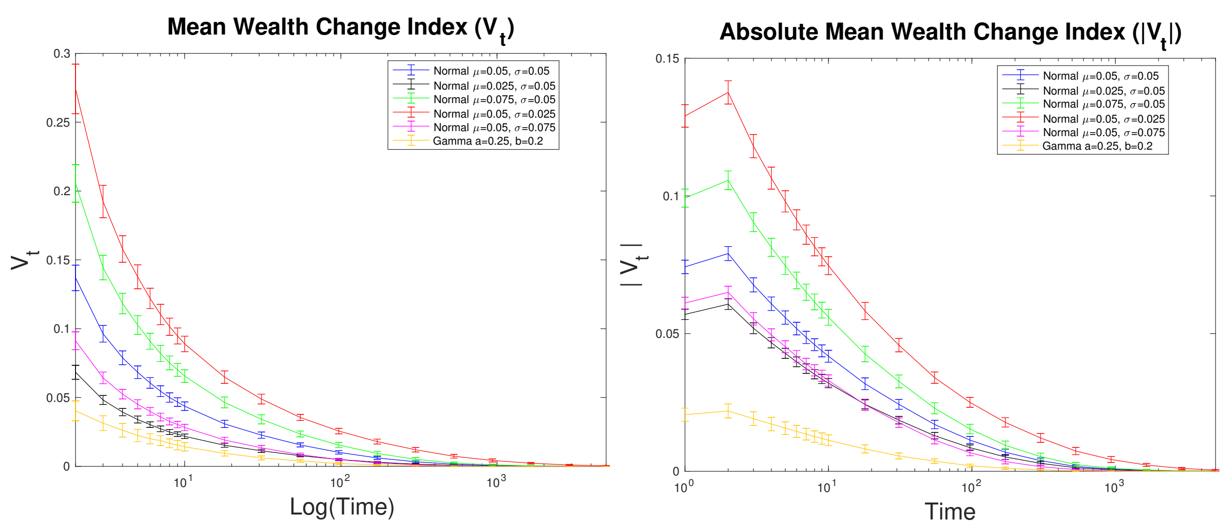

Figure 14: Left Panel: Log plot over time of Mean Wealth Change Index, defined in Equation 2. The index is computed under various return structures : $r_{i,t} \sim N(\mu_r, \sigma_r)$ and $gamma(a, b)$. Right Panel: Log plot over time of Absolute Mean Wealth Change Index defined in Equation 3. The index is computed under various return structures : $r_{i,t} \sim N(\mu_r, \sigma_r)$ and $gamma(a, b)$. Averages and standard deviations are computed out of 100 simulations.

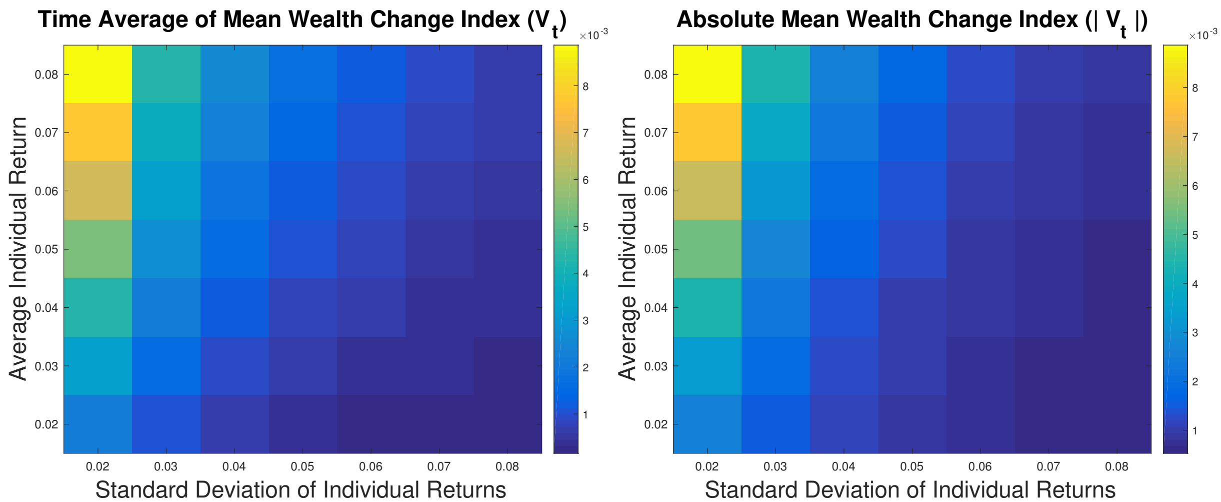

Figure 15: Left Panel: Mean Wealth Change Index V_t under various return structures $r_{i,t} \sim N(\mu_r, \sigma_r)$. Right Panel: Mean Absolute Wealth Change Index $|V_t|$ under various return structures $r_{i,t} \sim N(\mu_r, \sigma_r)$.

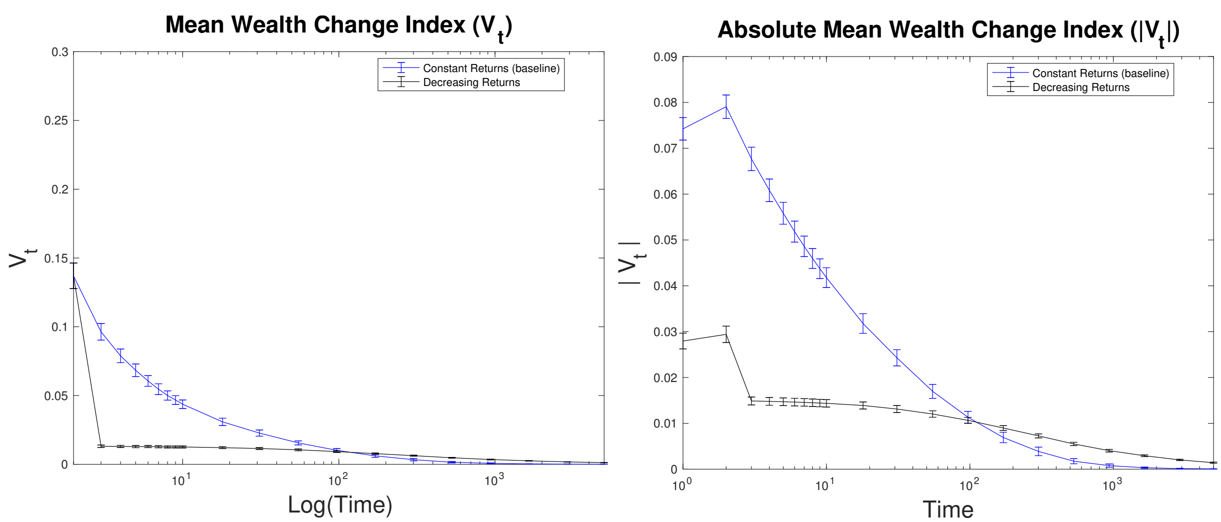

Figure 16: Top Right Panel: Comparison of Wealth Change Index V_t over time under constant and decreasing returns to aggregate wealth. Bottom Panel: Comparison of absolute Wealth Change Index $|V_t|$ over time under constant and decreasing returns to aggregate wealth. Averages and standard deviations are computed out of 100 simulations.

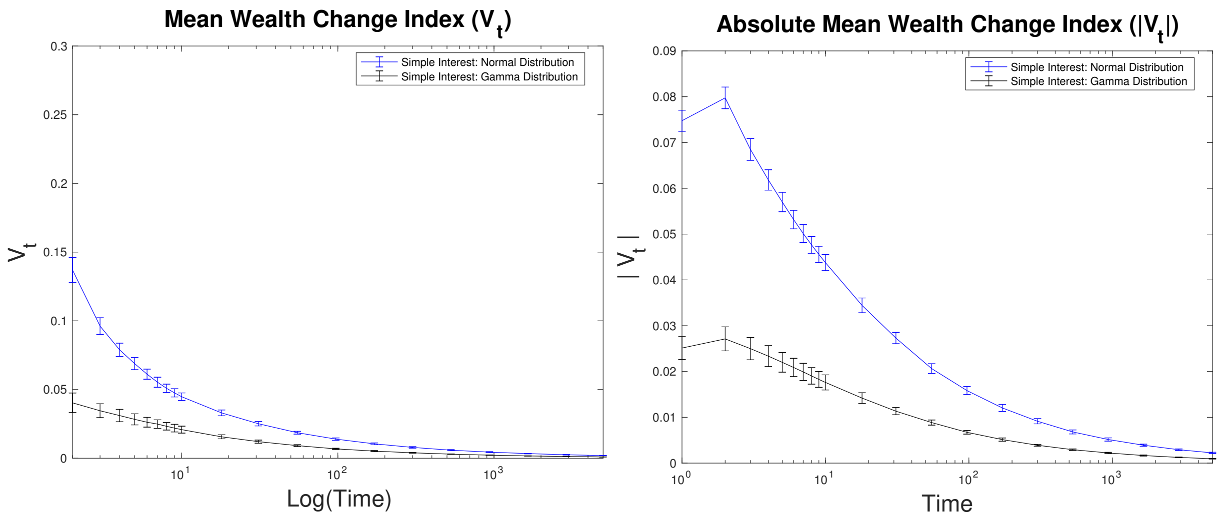

Figure 17: Left Panel:Mean Wealth Change Index under compound return structure (baseline case), simple return structure under normal distribution $N(\mu_r, \sigma_r)$, and simple return structure under Gamma distribution $gamma(a, b)$. Right Panel: Mean Absolute Wealth Change Index under under the same cases. Averages and standard deviations are computed out of 100 simulations.

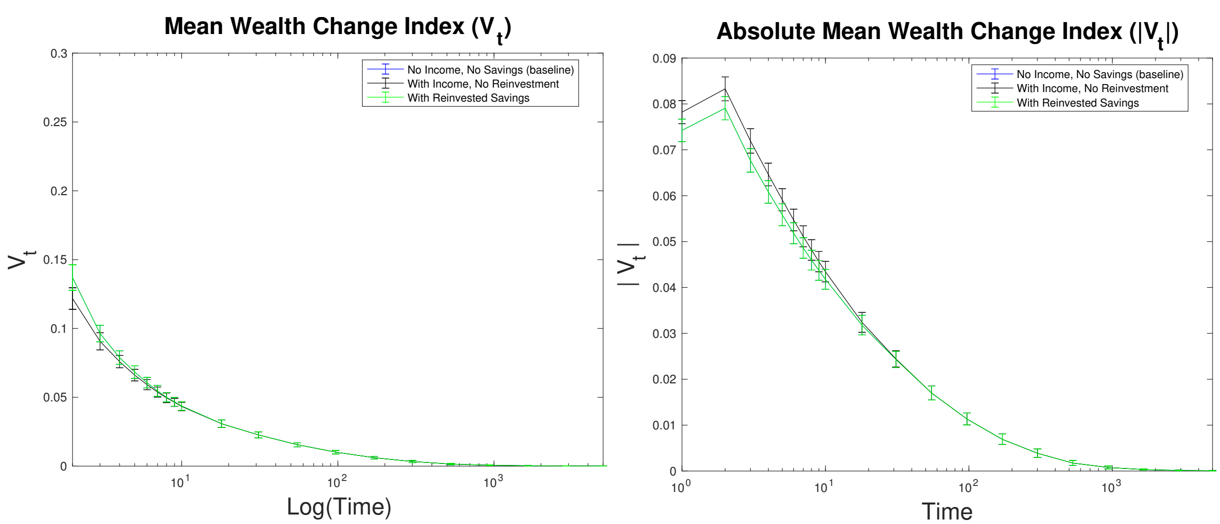

Figure 18: Left Panel: Mean Wealth Change Index under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time. Right Panel: Mean Absolute Wealth Change Index under baseline case (one-factor model), two-factors model without labor income savings, and two-factors model with reinvested savings over time. Averages and standard deviations are computed out of 100 simulations.

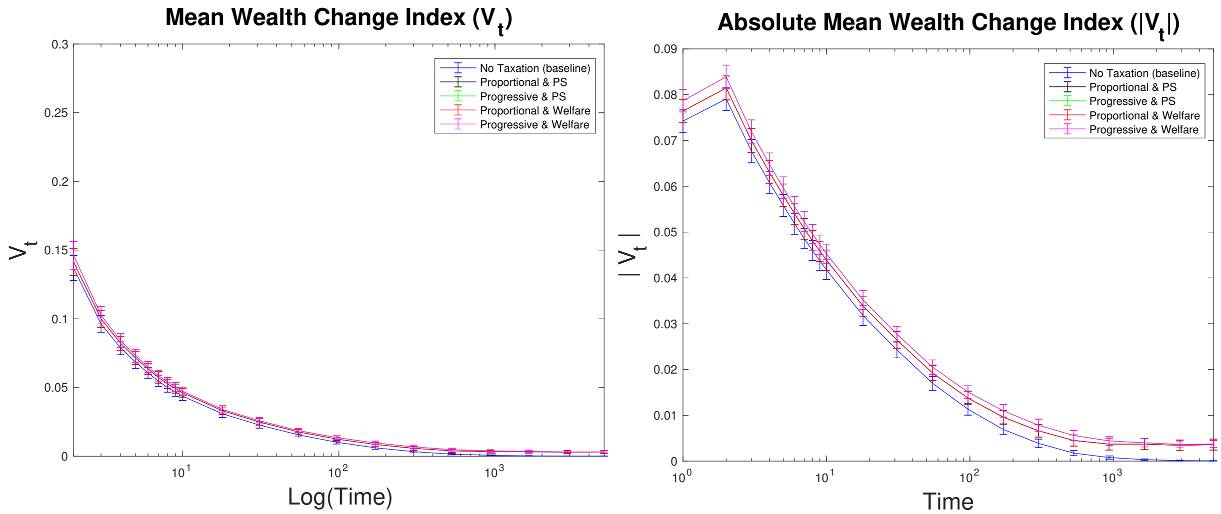

Figure 19: Left Panel: Mean Wealth Change Index over time under baseline case (no taxation), proportional taxation and public service model (**Proportional & PS**), proportional taxation and welfare model (**Proportional & Welfare**), progressive taxation and public service model (**Progressive & PS**) and progressive taxation and welfare model (**Progressive & Welfare**). Right Panel: Mean Absolute Wealth Change Index over time under the same cases. Averages and standard deviations are computed out of 100 simulations.

1.5 Median Redistribution and Tax Rates

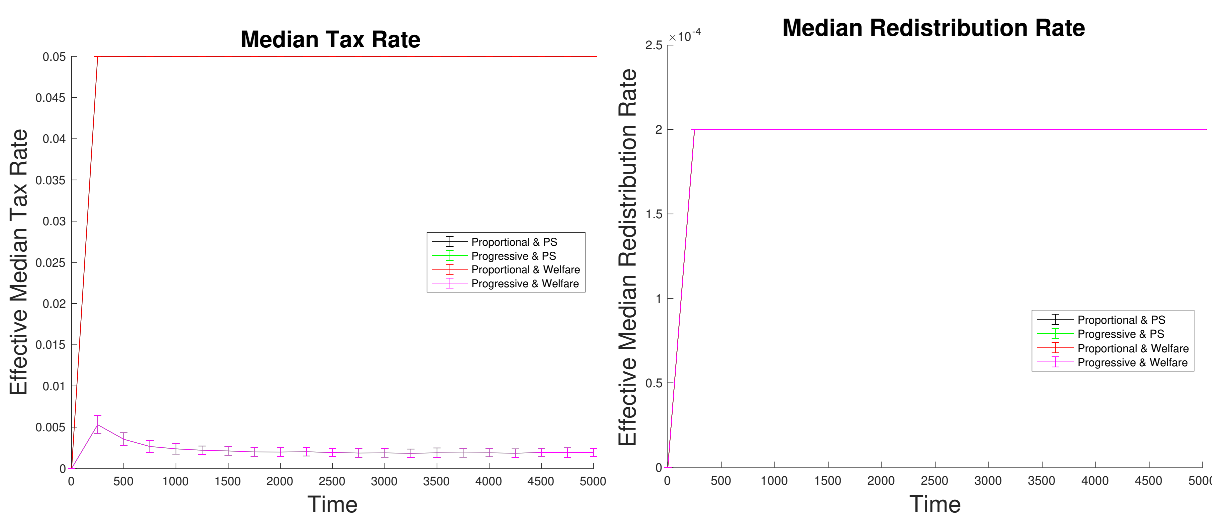

Figure 20: Left Panel: : Median Tax Rate over time under baseline case (no taxation), proportional taxation and public service model (**Proportional & PS**), proportional taxation and welfare model (**Proportional & Welfare**), progressive taxation and public service model (**Progressive & PS**) and progressive taxation and welfare model (**Progressive & Welfare**). Right Panel: Median Redistribution rate over time under the same cases. Averages and standard deviations are computed out of 100 simulations.

References

Levy, M. and Levy, H. (2003). Investment talent and the pareto wealth distribution: Theoretical and experimental analysis. *Review of Economics and Statistics*, 85(3):709–725.

Theil, H. (1967). *Economics and information theory*, volume 7. North-Holland Amsterdam.