Valuation of Firms with Multiple Business Units

Technical Appendix

Proof of Lemma 1

Unit A is characterized by a constant return $ROIC^A$ and net investment rate n^A. Therefore, the expected invested capital, NOPLAT, and the net investments in this unit uniformly grow at the constant nominal rate g^A which yields the expressions in Lemma 1 a).

To determine the expected accounting measures in unit B, we must consider the effects of the cross-unit investments. At time t, a share n^{AB} of unit A's NOPLAT after intra-unit investments is diverted to unit B. This implies an expected increase in unit B's invested capital by

$$\overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A]. \tag{A.1}$$

In unit B, these cross-unit investments yield a return of $ROIC^B$ and are subject to periodic reinvestments according to the rate n^B. They increase unit B's NOPLAT in all subsequent periods. The NOPLAT generated in unit B grows at the rate g^B. Thus, the cross-unit investments at time t cause the following incremental NOPLAT at time $s > t$:

$$\begin{aligned} E[\widetilde{NOPLAT}_s^{B,(t)}] &= ROIC^B \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A] \cdot (1+g^B)^{s-(t+1)} \\ &= ROIC^B \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A] \cdot (1+g^A)^{t-1} \cdot (1+g^B)^{s-(t+1)}. \end{aligned} \tag{A.2}$$

To determine the total NOPLAT of unit B at time t, we consider the NOPLAT generated by the initial capital $E[\widetilde{IC}_0^B]$, as well as the NOPLAT from all previous cross-unit investments according to (A.2):

$$\begin{aligned} E[\widetilde{NOPLAT}_t^B] &= ROIC^B \cdot E[\widetilde{IC}_0^B] \cdot (1+g^B)^{t-1} + \sum_{s=1}^{t-1} E[\widetilde{NOPLAT}_t^{B,(s)}] \\ &= ROIC^B \cdot E[\widetilde{IC}_0^B] \cdot (1+g^B)^{t-1} \\ &\quad + \overline{n}^{AB} \cdot ROIC^B \cdot ROIC^A \cdot E[\widetilde{IC}_0^A] \cdot \frac{(1+g^B)^{t-1} - (1+g^A)^{t-1}}{g^B - g^A}. \end{aligned} \tag{A.3}$$

The net investments at time t are the sum of the intra-unit and cross-unit investments:

$$
\begin{aligned}
E[\widetilde{NI}_t^B] &= n^B \cdot E[\widetilde{NOPLAT}_t^B] + \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A] \\
&= n^B \cdot E[\widetilde{NOPLAT}_1^B] \cdot (1+g^B)^{t-1} \\
&\quad + \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A] \cdot \frac{g^B \cdot (1+g^B)^{t-1} - g^A \cdot (1+g^A)^{t-1}}{g^B - g^A}.
\end{aligned} \tag{A.4}
$$

Finally, the invested capital of unit B at time t is the sum of the initial capital and all subsequent net investments:

$$
\begin{aligned}
E[\widetilde{IC}_t^B] &= E[\widetilde{IC}_0^B] + \sum_{s=1}^{t} E[\widetilde{NI}_s^B] \\
&= E[\widetilde{IC}_0^B] \cdot (1+g^B)^t + \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A] \cdot \frac{(1+g^B)^t - (1+g^A)^t}{g^B - g^A}.
\end{aligned} \tag{A.5}
$$

Proof of Proposition 1

Based on the results of Lemma 1, we obtain the following solutions for the expected NOPLAT, net investments, invested capital, and free cash flow at the firm level:

$$
\begin{aligned}
E[\widetilde{NOPLAT}_t] &= E[\widetilde{NOPLAT}_t^A] + E[\widetilde{NOPLAT}_t^B] \\
&= \frac{(n^B - \overline{n}^{AB}) \cdot ROIC^B - n^A \cdot ROIC^A}{g^B - g^A} \cdot ROIC^A \cdot E[\widetilde{IC}_0^A] \cdot (1+g^A)^{t-1} \\
&\quad + ROIC^B \cdot \left(\frac{ROIC^A \cdot \overline{n}^{AB}}{g^B - g^A} \cdot E[\widetilde{IC}_0^A] + E[\widetilde{IC}_0^B] \right) \cdot (1+g^B)^{t-1},
\end{aligned} \tag{A.6}
$$

$$
\begin{aligned}
E[\widetilde{NI}_t] &= (n^A + \overline{n}^{AB}) \cdot E[\widetilde{NOPLAT}_t^A] + n^B \cdot E[\widetilde{NOPLAT}_t^B] \\
&= \frac{n^A \cdot g^B - (n^A + \overline{n}^{AB}) \cdot g^A}{g^B - g^A} \cdot ROIC^A \cdot E[\widetilde{IC}_0^A] \cdot (1+g^A)^{t-1} \\
&\quad + g^B \cdot \left(\frac{\overline{n}^{AB} \cdot ROIC^A}{g^B - g^A} \cdot E[\widetilde{IC}_0^A] + E[\widetilde{IC}_0^B] \right) \cdot (1+g^B)^{t-1},
\end{aligned} \tag{A.7}
$$

$$
\begin{aligned}
E[\widetilde{IC}_t] &= E[\widetilde{IC}_0^A] + E[\widetilde{IC}_0^B] + \sum_{s=1}^{t} E[\widetilde{NI}_s] \\
&= E[\widetilde{IC}_0^A] + E[\widetilde{IC}_0^B] + \frac{n^A \cdot g^B - (n^A + \overline{n}^{AB}) \cdot g^A}{(g^B - g^A) \cdot g^A} \cdot ROIC^A \cdot E[\widetilde{IC}_0^A] \cdot ((1+g^A)^t - 1) \\
&\quad + \left(\frac{\overline{n}^{AB} \cdot ROIC^A}{g^B - g^A} \cdot E[\widetilde{IC}_0^A] + E[\widetilde{IC}_0^B] \right) \cdot ((1+g^B)^t - 1),
\end{aligned} \tag{A.8}
$$

(A.9)

$$
\begin{aligned}
E[\widetilde{FCF}_t] &= E[\widetilde{NOPLAT}_t] - E[\widetilde{NI}_t] \\
&= (1-n^A)\cdot\frac{(n^B-n^{AB})\cdot ROIC^B-(1-n^{AB})\cdot g^A}{g^B-g^A}\cdot ROIC^A\cdot E[\widetilde{IC}_0^A]\cdot(1+g^A)^{t-1} \\
&\quad +ROIC^B\cdot(1-n^B)\cdot\left(\frac{ROIC^A\cdot\overline{n}^{AB}\cdot E[\widetilde{IC}_0^A]}{g^B-g^A}+E[\widetilde{IC}_0^B]\right)\cdot(1+g^B)^{t-1}.
\end{aligned}
\tag{A.10}
$$

These results can be used to determine the expected absolute increases of the expected accounting measures:

$$
\begin{aligned}
&E[\widetilde{NOPLAT}_{t+1}] - E[\widetilde{NOPLAT}_t] \\
&\quad = \frac{(n^B-\overline{n}^{AB})\cdot ROIC^B - n^A\cdot ROIC^A}{g^B-g^A}\cdot g^A\cdot ROIC^A\cdot E[\widetilde{IC}_0^A]\cdot(1+g^A)^{t-1} \\
&\qquad + ROIC^B\cdot\left(\frac{ROIC^A\cdot\overline{n}^{AB}}{g^B-g^A}\cdot E[\widetilde{IC}_0^A]+E[\widetilde{IC}_0^B]\right)\cdot g^B\cdot(1+g^B)^{t-1},
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
E[\widetilde{NI}_{t+1}]-E[\widetilde{NI}_t] &= \frac{n^A\cdot g^B-(n^A+\overline{n}^{AB})\cdot g^A}{g^B-g^A}\cdot g^A\cdot ROIC^A\cdot E[\widetilde{IC}_0^A]\cdot(1+g^A)^{t-1} \\
&\quad +g^B\cdot\left(\frac{\overline{n}^{AB}\cdot ROIC^A}{g^B-g^A}\cdot E[\widetilde{IC}_0^A]+E[\widetilde{IC}_0^B]\right)\cdot g^B\cdot(1+g^B)^{t-1},
\end{aligned}
\tag{A.12}
$$

$$
E[\widetilde{IC}_{t+1}] - E[\widetilde{IC}_t] = E[\widetilde{NI}_{t+1}],
\tag{A.13}
$$

$$
\begin{aligned}
&E[\widetilde{FCF}_{t+1}]-E[\widetilde{FCF}_t] \\
&\quad = ROIC^A\cdot(1-n^A)\cdot E[\widetilde{IC}_0^A]\cdot\frac{(n^B-n^{AB})\cdot ROIC^B-(1-n^{AB})\cdot g^A}{g^B-g^A}\cdot g^A\cdot(1+g^A)^{t-1} \\
&\qquad + ROIC^B\cdot(1-n^B)\cdot\left(\frac{ROIC^A\cdot n^{AB}\cdot(1-n^A)\cdot E[\widetilde{IC}_0^A]}{g^B-g^A}+E[\widetilde{IC}_0^B]\right)\cdot g^B\cdot(1+g^B)^{t-1}.
\end{aligned}
\tag{A.14}
$$

These results can be used to determine the growth rates (7) and the firm's ROIC and payout ratio (8) and to identify the limits according to Proposition 1.

Proof of Corollary 1

The derivatives of the weights λ^{ROIC} and λ^q are given by:

$$
\frac{\partial\lambda^{ROIC}}{\partial n^{AB}} = \frac{(g^B-g^A)\cdot(1-n^A)\cdot ROIC^A}{(g^A-g^B+\overline{n}^{AB}\cdot ROIC^A)^2} \leq 0 \quad\Leftrightarrow\quad g^A \geq g^B,
\tag{A.15}
$$

$$
\frac{\partial\lambda^{ROIC}}{\partial ROIC^A} = \frac{\overline{n}^{AB}\cdot g^B}{(g^A-g^B+\overline{n}^{AB}\cdot ROIC^A)^2} \geq 0,
\tag{A.16}
$$

$$\frac{\partial \lambda^{ROIC}}{\partial ROIC^B} = -\frac{\overline{n}^{AB} \cdot (1 - n^A) \cdot n^B \cdot ROIC^A}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^A)^2} \leq 0, \tag{A.17}$$

$$\frac{\partial \lambda^{ROIC}}{\partial n^A} = \frac{(ROIC^A - g^B) \cdot n^{AB} \cdot ROIC^A}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^A)^2} \geq 0 \quad \Leftrightarrow \quad ROIC^A \geq g^B, \tag{A.18}$$

$$\frac{\partial \lambda^{ROIC}}{\partial n^B} = -\frac{\overline{n}^{AB} \cdot ROIC^A \cdot ROIC^B}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^A)^2} \leq 0, \tag{A.19}$$

$$\frac{\partial \lambda^q}{\partial n^{AB}} = \frac{(g^B - g^A) \cdot (1 - n^A) \cdot ROIC^B}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^B)^2} \leq 0 \quad \Leftrightarrow \quad g^A \geq g^B, \tag{A.20}$$

$$\frac{\partial \lambda^q}{\partial ROIC^A} = \frac{n^A \cdot \overline{n}^{AB} \cdot ROIC^B}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^B)^2} \geq 0, \tag{A.21}$$

$$\frac{\partial \lambda^q}{\partial ROIC^B} = -\frac{\overline{n}^{AB} \cdot g^A}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^B)^2} \leq 0, \tag{A.22}$$

$$\frac{\partial \lambda^q}{\partial n^A} = \frac{(ROIC^A - g^B) \cdot ROIC^B \cdot n^{AB}}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^B)^2} \geq 0 \quad \Leftrightarrow \quad ROIC^A \geq g^B, \tag{A.23}$$

$$\frac{\partial \lambda^q}{\partial n^B} = -\frac{n^{AB} \cdot (1 - n^A) \cdot ROIC^{B2}}{(g^A - g^B + \overline{n}^{AB} \cdot ROIC^B)^2} \leq 0. \tag{A.24}$$

As we study the case $g^B < g^A$, the expressions confirm the results of Corollary 1.

Proof of Proposition 2

The stand-alone values of the units are determined according to benchmark case (a). To determine the net value contribution of the cross-unit investments, consider their effects on the free cash flow. The cross-unit investments at time t reduce the free cash flow by the retained amount (A.1) but generate an incremental NOPLAT according to (A.2). A share $1 - n^B$ of this NOPLAT is distributed to shareholders and debtors. Thus, the cross-unit investments at time t imply the following value $E[\tilde{V}_t^{AB,(t)}]$ of future cash flow increases:

$$\begin{aligned} E[\tilde{V}_t^{AB,(t)}] &= \sum_{s=t+1}^{\infty} \frac{(1 - n^B) \cdot E[\widetilde{NOPLAT}_s^{B,(t)}]}{(1 + k)^{s-t}} \\ &= \frac{(1 - n^B) \cdot ROIC^B \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A]}{k - g^B} \\ &= E[\tilde{V}_1^{AB,(1)}] \cdot (1 + g^A)^{t-1}. \end{aligned} \tag{A.25}$$

The net contribution $E[\tilde{V}_0^{AB}]$ of the cross-unit investments combines the value-reducing effect (A.1) and the value-enhancing effect (A.24):

$$
\begin{aligned}
E[\tilde{V}_0^{AB}] &= \sum_{t=1}^{\infty} \frac{(E[\tilde{V}_1^{AB,(1)}] - \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A]) \cdot (1+g^A)^{t-1}}{(1+k)^t} \\
&= \frac{(ROIC^B - k) \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A]}{(k-g^A) \cdot (k-g^B)}.
\end{aligned} \tag{A.26}
$$

Proof of Corollaries 2

The derivatives of the net value contribution of cross-unit investments $E[\tilde{V}_0^{AB}]$ are

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial n^{AB}} = \frac{(ROIC^B - k) \cdot (1 - n^A) \cdot ROIC^A \cdot E[\widetilde{IC}_0^A]}{(k-g^A) \cdot (k-g^B)} \geq 0, \tag{A.27}
$$

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial ROIC^A} = \frac{(ROIC^B - k) \cdot k \cdot \overline{n}^{AB} \cdot E[\widetilde{IC}_0^A]}{(k-g^A)^2 \cdot (k-g^B)} \geq 0, \tag{A.28}
$$

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial ROIC^B} = \frac{k \cdot \overline{n}^{AB} \cdot (1 - n^B) \cdot ROIC^A \cdot E[\widetilde{IC}_0^A]}{(k-g^A) \cdot (k-g^B)^2} \geq 0, \tag{A.29}
$$

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial n^A} = \frac{(ROIC^B - k) \cdot (ROIC^A - k) \cdot n^{AB} \cdot ROIC^A \cdot E[\widetilde{IC}_0^A]}{(k-g^A)^2 \cdot (k-g^B)} \geq 0, \tag{A.30}
$$

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial n^B} = \frac{(ROIC^B - k) \cdot \overline{n}^{AB} \cdot ROIC^A \cdot ROIC^B \cdot E[\widetilde{IC}_0^A]}{(k-g^A) \cdot (k-g^B)^2} \geq 0, \tag{A.31}
$$

$$
\frac{\partial E[\tilde{V}_0^{AB}]}{\partial k} = -\frac{(k-g^A)\cdot(k-g^B) + (ROIC^B - k)\cdot(2\cdot k - g^A - g^B)}{(k-g^A) \cdot (k-g^B)} \cdot \overline{n}^{AB} \cdot ROIC^A \cdot E[\widetilde{IC}_0^A] \leq 0. \tag{A.32}
$$

Proof of Corollary 3

The results in a) are obtained from Lemma 1 using definition (9):

$$
\begin{aligned}
E[\widetilde{RI}_t^A] &= E[\widetilde{NOPLAT}_t^A] - k \cdot E[\widetilde{IC}_{t-1}^A] \\
&= (ROIC^i - k) \cdot E[\widetilde{IC}_0^i].
\end{aligned} \tag{A.33}
$$

$$
\begin{aligned}
E[\widetilde{RI}_t^B] &= E[\widetilde{NOPLAT}_t^B] - k \cdot E[\widetilde{IC}_{t-1}^B] \\
&= (ROIC^B - k) \cdot E[\widetilde{IC}_{t-1}^B] \\
&= (ROIC^B - k) \\
&\quad \cdot \left(E[\widetilde{IC}_0^B] \cdot (1+g^B)^{t-1} + \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A] \cdot \frac{(1+g^B)^{t-1} - (1+g^A)^{t-1}}{g^B - g^A} \right).
\end{aligned} \tag{A.34}
$$

The residual income approach in b) follows the same logic as the proof of Proposition 2. First, consider a setting without cross-unit investments. In this case, the expected residual income of business unit $i \in \{A, B\}$ at time $t = 1$ is

$$\begin{aligned} E[\widetilde{RI}_1^i] &= E[\widetilde{NOPLAT}_1^i] - k \cdot E[\widetilde{IC}_0^i] \\ &= (ROIC^i - k) \cdot E[\widetilde{IC}_0^i]. \end{aligned} \tag{A.35}$$

The residual income grows at the constant rate g^i. This immediately implies the expression $E[\widetilde{MVA}_0^i]$ for the stand-alone MVA of unit i. The component $E[\widetilde{MVA}_0^{AB}]$ measures the adjustment of the residual incomes attributable to the cross-unit investments. Consider the expected cross-unit investments (A.1). The investments in business unit B are subject to the constant return $ROIC^B$ and the intra-unit investment rate n^B. They generate the following additional invested capital at time $s \geq t$:

$$E[\widetilde{IC}_s^{B,(t)}] = \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A] \cdot (1 + g^B)^{s-t}. \tag{A.36}$$

Moreover, the incremental NOPLAT resulting from the cross-unit investment at time t is given by (A.2). Using (A.2) and (A.35), we obtain the following expected residual income in period s as a result of the cross-unit investment at time t:

$$\begin{aligned} E[\widetilde{RI}_s^{B,(t)}] &= E[\widetilde{NOPLAT}_s^{B,(t)}] - k \cdot E[\widetilde{IC}_{s-1}^{B,(t)}] \\ &= (ROIC^B - k) \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_t^A] \cdot (1 + g^B)^{s-(t+1)}. \end{aligned} \tag{A.37}$$

Thus, the MVA of the cross-unit investments at time t is given by:

$$\begin{aligned} E[\widetilde{MVA}_t^{B,(t)}] &= \sum_{s=t+1}^{\infty} \frac{E[\widetilde{RI}_s^{B,(t)}]}{(1+k)^{s-t}} \\ &= \frac{ROIC^B - k}{k - g^B} \cdot \overline{n}^{AB} \cdot E[\widetilde{NOPLAT}_1^A] \cdot (1 + g^A)^{t-1}. \end{aligned} \tag{A.38}$$

The MVA of the cross-unit investments $E[\widetilde{MVA}_0^{AB}]$ is the present value of these components:

$$\begin{aligned} E[\widetilde{MVA}_0^{AB}] &= \sum_{t=1}^{\infty} \frac{E[\widetilde{MVA}_t^{B,(t)}]}{(1+k)^t} \\ &= \frac{\overline{n}^{AB} \cdot ROIC^A \cdot (ROIC^B - k)}{(k - g^A) \cdot (k - g^B)} \cdot E[\widetilde{IC}_0^A]. \end{aligned} \tag{A.39}$$

Proof of Proposition 3

The total ROIC at time t can be determined from the NOPLAT (A.6) and the invested capital (A.8). This expression is time-invariant if

$$(1 - n^{AB}) \cdot (1 - n^A) = 1 - n^B. \tag{A.40}$$

Analogously, we find that the growth rate of the free cash flow (A.9) is constant if

$$(1 - n^{AB}) \cdot \left(1 - \frac{g^A}{ROIC^B}\right) = 1 - n^B. \tag{A.41}$$

Proof of Corollary 4

With homogenous rentability $ROIC = ROIC^A = ROIC^B$, the conditions for a constant payout and constant growth strategy according to Proposition 3 coincide:

$$n^{AB} = \frac{n^B - n^A}{1 - n^A} = \frac{g^B - g^A}{ROIC - g^A}. \tag{A.42}$$

The simplified formula in Corollary 4 is obtained by substituting this result into the general valuation formula in Proposition 2.

References

Koller T, Goedhart M, Wessels D (2015) Valuation: measuring and managing the value of companies. 6th edn. John Wiley & Sons, Hoboken

Meitner M (2013) Multi-period asset lifetimes and accounting-based equity valuation: take care with constant-growth terminal value models! Abacus 49(3):340–366