SUPPLEMENTARY MATERIALS: FAMILIARIZING STUDENTS WITH DEFINITION OF LEBESGUE OUTER MEASURE USING MATHEMATICA - SOME EXAMPLES OF CALCULATION DIRECTLY FROM ITS DEFINITION

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The following Figures 1, 2, 3 and 4 are dynamic versions of the Figures 1, 2, 3 and 4 in the article: **Familiarizing students with definition of Lebesgue outer measure using Mathematica - some examples of calculation directly from its definition**. We would like to stress the fact that the controls at the bottom of the Figures 1, 2, 3 and 4 work.

FIGURE 1. Approximation of Lebesgue outer measure of $A = \big\{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq x^2, 0 \leq x \leq 1\big\}$ by rectangles $\bigcup_{j=1}^{n} \underline{R}_j^n$, $n \in \{20, 2, 4, 6, 10, 40, 100\}$ for **Example 1.** The controls at the bottom of the Figure work.

FIGURE 2. Approximation of Lebesgue outer measure of $A = \left\{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq x^2, 0 \leq x \leq 1 \right\}$ by rectangles $\bigcup_{j=1}^{n} \bar{R}_j^n$, $n \in \{20, 2, 4, 6, 10, 40, 100\}$ for **Example 1.** The controls at the bottom of the Figure work.

FIGURE 3. Approximation of Lebesgue outer measure of $A = \left\{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq \exp(-x), x \geq 0\right\}$ by rectangles $\bigcup_{j=0}^{n2^n-1} \underline{R}_j^n$, $n \in \{3,1,2,4,5\}$ for **Example 2.** The controls at the bottom of the Figure work.

FIGURE 4. Approximation of Lebesgue outer measure of $A = \left\{ (x,y) \in \mathbb{R}^2 : 0 \leq y \leq \exp(-x), x \geq 0 \right\}$ by rectangles $\bigcup_{j=0}^{2^n-1} \bigcup_{k=0}^{\infty} \bar{R}_{jk}^n$, $n \in \{3,1,2,3,4,8\}$ for **Example 2.** The controls at the bottom of the Figure work.

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