

Online Resource 1

Lifespan Disparity as an Additional Indicator for Evaluating Mortality Forecasts

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A Extending the age range beyond 110+

The data used are death counts and exposures by single age, 0 to 110+, from the Human Mortality Database (2015). To enable forecasting approaches to shift deaths to ages beyond 110+, we extend the age range of mortality data, like Ševčíková et al. (2016), with the model of Kannisto:

$$\mu_x = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}} \quad (2)$$

as it is described in Thatcher et al. (1998, p. 16). We fit the model with the function *optim* in R (2015) to mortality at ages 80 to 110 using a Poisson log-likelihood for women in Italy, Denmark, and Japan for each year between 1950 and 2009. We then use the fitted Kannisto models to smooth mortality for ages 80 to 110, and to predict mortality for the ages above 110 in each year. Figure S1 depicts such mortality data for ages 80 to 130 for Italian, Danish, and Japanese women in the year 1950. The model of Kannisto is particularly suitable for populations with low mortality. It is also applied by, e.g., the Human Mortality Database and the United Nations (2014) to fit old-age mortality.

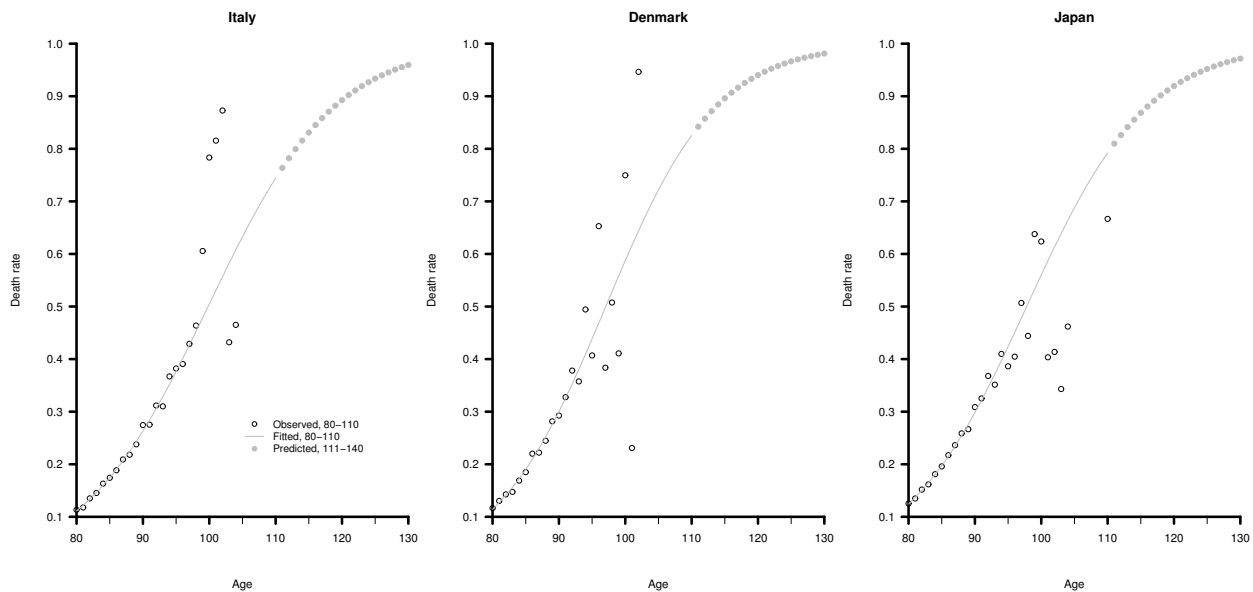


Fig. S1 Extended age range of mortality data of Italian (left), Danish (center), and Japanese (right) women in 1950. We used the model of Kannisto to fit (gray line) observed mortality (black circles) at ages 80 to 110, and to predict it at ages 111 to 130 (gray dots).

B Forecasts until 2009 with reference periods 1960-1985, 1955-1980, and 1950-1975

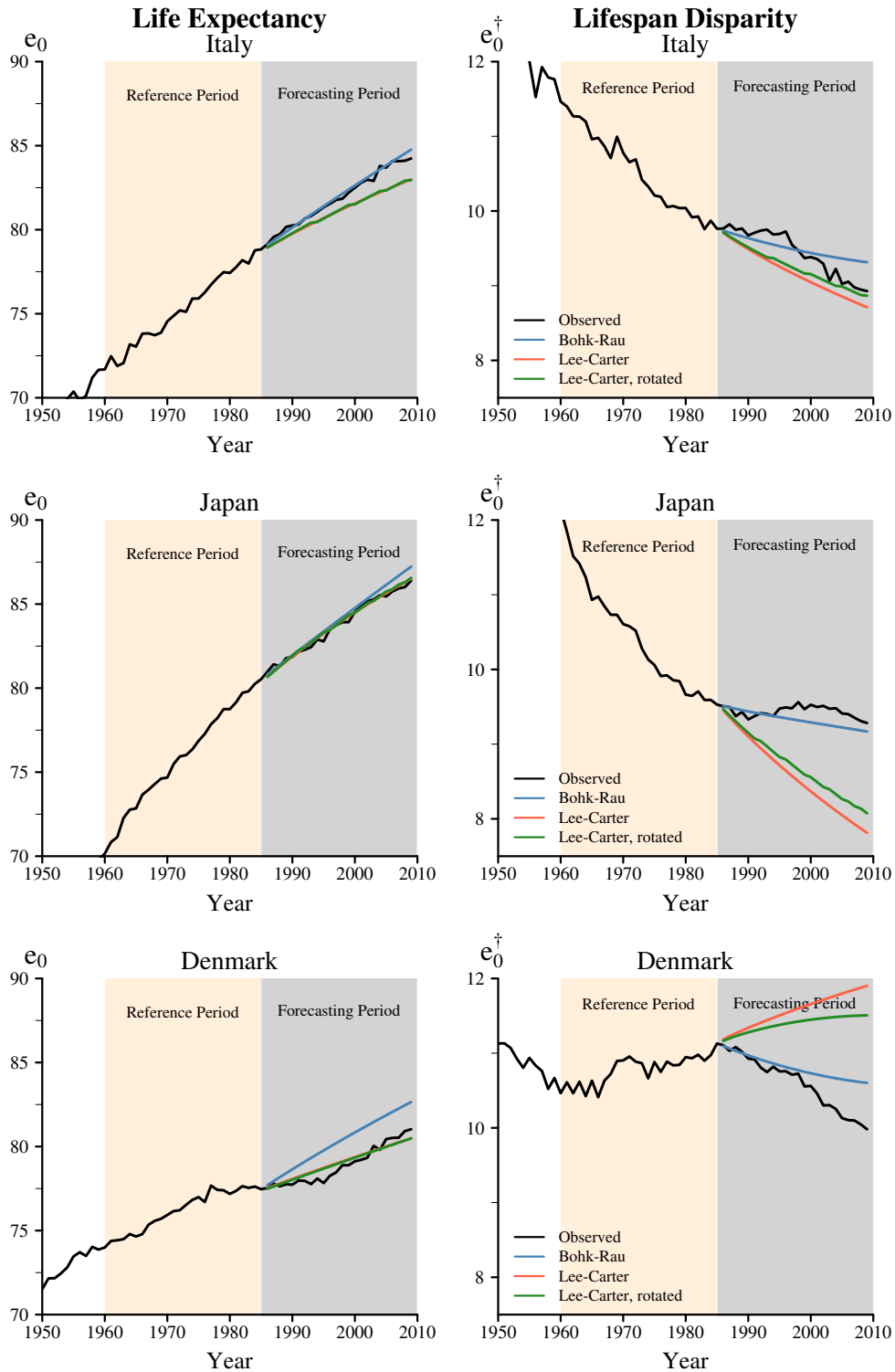


Fig. S2 Life expectancy at birth (left panels) and life years lost at birth (right panels) for women in Italy (top), Japan (center), and Denmark (bottom); observed data are in black, forecasted data are in red (Lee-Carter model), green (rotating variant), and blue (Bohk-Rau model); reference period: 1960-1985.

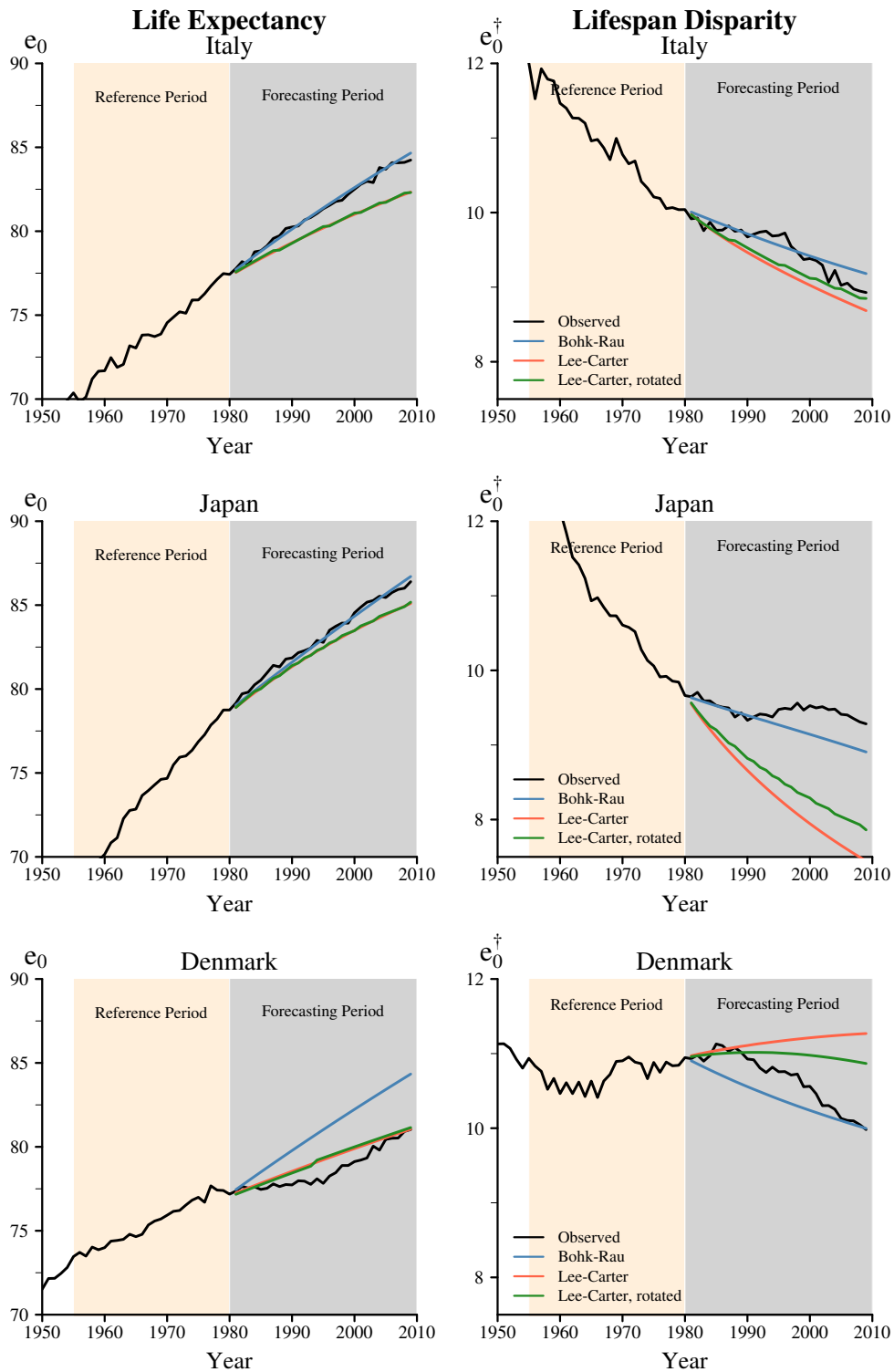


Fig. S3 Life expectancy at birth (left panels) and life years lost at birth (right panels) for women in Italy (top), Japan (center), and Denmark (bottom); observed data are in black, forecasted data are in red (Lee-Carter model), green (rotating variant), and blue (Bohk-Rau model); reference period: 1955-1980.

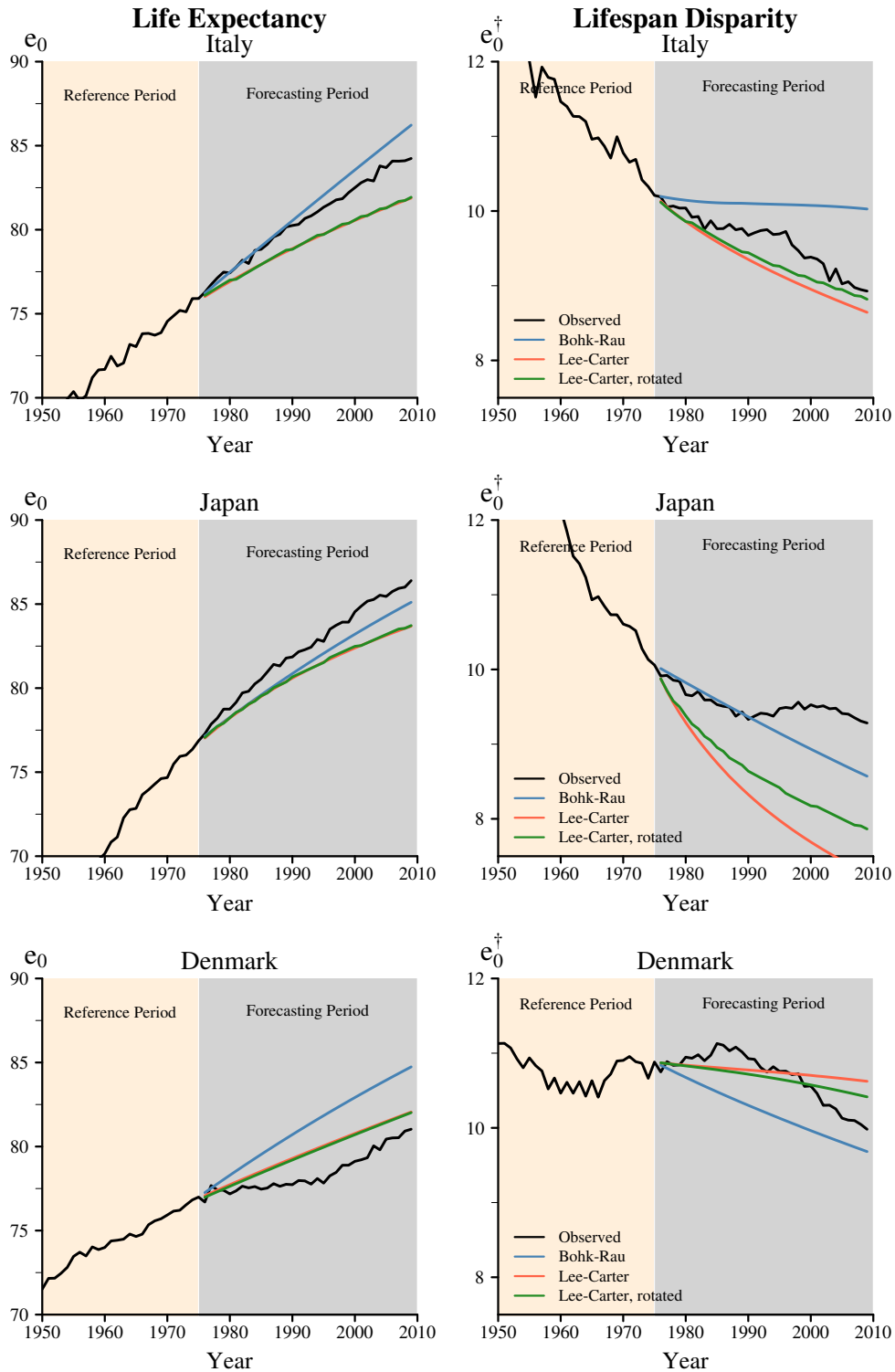


Fig. S4 Life expectancy at birth (left panels) and life years lost at birth (right panels) for women in Italy (top), Japan (center), and Denmark (bottom); observed data are in black, forecasted data are in red (Lee-Carter model), green (rotating variant), and blue (Bohk-Rau model); reference period: 1950-1975.

C MAPE estimates for e_{65} and e_{65}^+

Country	Measure	LC	LC, rotated	Bohk–Rau
Validation 1 (Ref.years: 1965–1990; Forecast years: 1991–2009)				
Italy	e_{65}	0.016	0.013	0.014
	e_{65}^{\dagger}	0.009	0.008	0.021
Japan	e_{65}	0.015	0.012	0.008
	e_{65}^{\dagger}	0.056	0.050	0.012
Denmark	e_{65}	0.020	0.018	0.026
	e_{65}^{\dagger}	0.016	0.013	0.010
Validation 2 (Ref.years: 1960–1985; Forecast years: 1986–2009)				
Italy	e_{65}	0.046	0.040	0.008
	e_{65}^{\dagger}	0.016	0.012	0.034
Japan	e_{65}	0.025	0.018	0.007
	e_{65}^{\dagger}	0.050	0.041	0.020
Denmark	e_{65}	0.054	0.046	0.062
	e_{65}^{\dagger}	0.051	0.045	0.044
Validation 3 (Ref.years: 1955–1980; Forecast years: 1981–2009)				
Italy	e_{65}	0.055	0.049	0.006
	e_{65}^{\dagger}	0.027	0.022	0.020
Japan	e_{65}	0.068	0.056	0.018
	e_{65}^{\dagger}	0.067	0.055	0.008
Denmark	e_{65}	0.063	0.057	0.099
	e_{65}^{\dagger}	0.009	0.010	0.012
Validation 4 (Ref.years: 1950–1975; Forecast years: 1976–2009)				
Italy	e_{65}	0.065	0.057	0.041
	e_{65}^{\dagger}	0.026	0.020	0.076
Japan	e_{65}	0.105	0.085	0.050
	e_{65}^{\dagger}	0.071	0.054	0.029
Denmark	e_{65}	0.066	0.060	0.102
	e_{65}^{\dagger}	0.011	0.011	0.009

Table S1 Mean of the Absolute Percentage Errors (MAPE) for e_{65} and e_{65}^{\dagger} over the forecast years by country and method. MAPEs are shown for four validating settings that all forecast mortality until 2009, but use different historical periods.

Country	Measure	LC	LC, rotated	Bohk-Rau
Average across all validation settings				
Italy	e_{65}	0.046	0.040	0.017
	e_{65}^{\dagger}	0.020	0.016	0.038
Japan	e_{65}	0.053	0.043	0.021
	e_{65}^{\dagger}	0.061	0.050	0.017
Denmark	e_{65}	0.051	0.045	0.072
	e_{65}^{\dagger}	0.021	0.019	0.019

Table S2 Mean of the Absolute Percentage Errors (MAPE) for e_{65} and e_{65}^{\dagger} over all validation settings by country and method.

D Empirical frequencies for e_0 and e_0^\dagger

Country	Measure	LC	LC, rotated	Bohk–Rau
Validaton 1 (Ref.years: 1965–1990; Forecast years: 1991–2009)				
Italy	e_0	100.00	100.00	94.74
	e_0^\dagger	10.53	31.58	36.84
Japan	e_0	100.00	100.00	94.74
	e_0^\dagger	0.00	0.00	0.00
Denmark	e_0	100.00	100.00	89.47
	e_0^\dagger	0.00	0.00	42.11
Validation 2 (Ref.years: 1960–1985; Forecast years: 1986–2009)				
Italy	e_0	79.17	91.67	91.67
	e_0^\dagger	0.00	20.83	41.67
Japan	e_0	100.00	100.00	87.50
	e_0^\dagger	0.00	0.00	50.00
Denmark	e_0	100.00	100.00	54.17
	e_0^\dagger	0.00	0.00	37.50
Validation 3 (Ref.years: 1955–1980; Forecast years: 1981–2009)				
Italy	e_0	55.17	72.41	89.66
	e_0^\dagger	13.79	34.48	48.28
Japan	e_0	100.00	100.00	48.28
	e_0^\dagger	0.00	0.00	96.55
Denmark	e_0	100.00	100.00	55.17
	e_0^\dagger	13.79	13.79	37.93
Validation 4 (Ref.years: 1950–1975; Forecast years: 1976–2009)				
Italy	e_0	64.71	79.41	88.24
	e_0^\dagger	5.88	26.47	2.94
Japan	e_0	64.71	100.00	26.47
	e_0^\dagger	2.94	2.94	82.35
Denmark	e_0	100.00	100.00	41.18
	e_0^\dagger	20.59	17.65	32.35

Table S3 Empirical frequencies, in %, for the 95% prediction intervals of e_0 and e_0^\dagger over the forecast years by country and method. The empirical frequencies are shown for four validating settings that all forecast mortality until 2009, but use different historical periods.

E Empirical frequencies for e_{65} and e_{65}^{\dagger}

Country	Measure	LC	LC, rotated	Bohk–Rau
Validaton 1 (Ref.years: 1965–1990; Forecast years: 1991–2009)				
Italy	e_{65}	94.74	100.00	100.00
	e_{65}^{\dagger}	21.05	10.53	57.89
Japan	e_{65}	100.00	100.00	47.37
	e_{65}^{\dagger}	0.00	0.00	63.16
Denmark	e_{65}	100.00	100.00	57.89
	e_{65}^{\dagger}	31.58	36.84	73.68
Validation 2 (Ref.years: 1960–1985; Forecast years: 1986–2009)				
Italy	e_{65}	4.17	29.17	83.33
	e_{65}^{\dagger}	4.17	12.50	66.67
Japan	e_{65}	70.83	95.83	50.00
	e_{65}^{\dagger}	4.17	4.17	91.67
Denmark	e_{65}	100.00	100.00	45.83
	e_{65}^{\dagger}	0.00	0.00	8.33
Validation 3 (Ref.years: 1955–1980; Forecast years: 1981–2009)				
Italy	e_{65}	13.79	20.69	93.10
	e_{65}^{\dagger}	0.00	0.00	75.86
Japan	e_{65}	6.90	41.38	24.14
	e_{65}^{\dagger}	0.00	0.00	93.10
Denmark	e_{65}	100.00	100.00	55.17
	e_{65}^{\dagger}	48.28	41.38	89.66
Validation 4 (Ref.years: 1950–1975; Forecast years: 1976–2009)				
Italy	e_{65}	20.59	35.29	94.12
	e_{65}^{\dagger}	2.94	2.94	61.76
Japan	e_{65}	5.88	38.24	23.53
	e_{65}^{\dagger}	2.94	0.00	94.12
Denmark	e_{65}	100.00	100.00	61.76
	e_{65}^{\dagger}	35.29	41.18	88.24

Table S4 Empirical frequencies, in %, for the 95% prediction intervals of e_{65} and e_{65}^{\dagger} over the forecast years by country and method. The empirical frequencies are shown for four validating settings that all forecast mortality until 2009, but use different historical periods.