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Assessing differences in household needs: A comparison of approaches for the estimation of equivalence scales using German expenditure data

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Supplementary material

A: Calculation of inequality and poverty indices based on different equivalence scales

Table 1 Measurement of inequality and poverty.

Approach	Gini coefficient	At-risk-poverty rate (%)	Interquartile range (Euro)
Quadratic expenditure system	0.23	14.6	3623
Quadratic almost ideal demand system	0.24	14.6	3900
Semiparametric (modified)*	0.23	14.9	3886
Matching	0.23	14.4	3760
Modified OECD scale	0.25	14.8	3964
Square root scale	0.24	14.8	4221

Note: This observation is based on income data from the EVS 2013, while using the more plausible equivalence scales from Table 6. The scales are partly based on 2003, 2008, and 2013 data.

In Table 1, we present our findings on the degree to which different equivalence scale estimates influence the measurement of inequality and poverty. To calculate equivalence income, each of the more plausible equivalence scales was applied to EVS household income data in 2013 (matching; QAI), with the exception of the nonparametric approach, as its interval estimates were not well-suited for this exercise. We also added two equivalence scales that were less plausible (QES, Stengos et al., 2006), but that displayed equivalence scale elasticities close to those of the plausible estimates. The modified OECD scale and the square root scale were also applied. In a second step, equivalence income was used to calculate three commonly used indicators: the Gini coefficient, the at-risk-of-poverty rate (ARP), and the interquartile range (IQR). As the estimation was done without additional weighting, these values indicated the household level.

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When using the modified OECD scale for this calculation, we obtained values of 0.25 for the Gini coefficient, 14.8% for the ARP rate, and around EUR $4,000$ for the IQR.[1] The equivalence scales of all of the methods shown in Table 1 generated very similar findings. For instance, for the equivalence scale obtained from the Quadratic Almost Ideal Demand System, we calculated values of 0.24 for the Gini coefficient; 14.9% for the ARP rate, and around EUR $4,200$ for the IQR. In contrast, the methods with implausible equivalence scales led to deviating results (not shown in Table 1). For instance, the equivalence scale of the AI demand system generated a Gini coefficient of 0.27, an at-risk-poverty rate of 19%, and an IQR of EUR 6,300.

Overall, we conclude that applying our plausible equivalence scales leads to consistent assessments of inequality and poverty. We also observe that applying less plausible scales seems to lead to similar results if their equivalence scale elasticities are similar.

[1] When applied, the weights for children differ. This is considered in the application. The household-specific OECD scale can vary by 0.2 per child depending on the ages of the household members.

B: The QAI demand system and income independence

Fig. 1 Income independence test based on the QAI (Banks et al., 1997), Household type single (A)

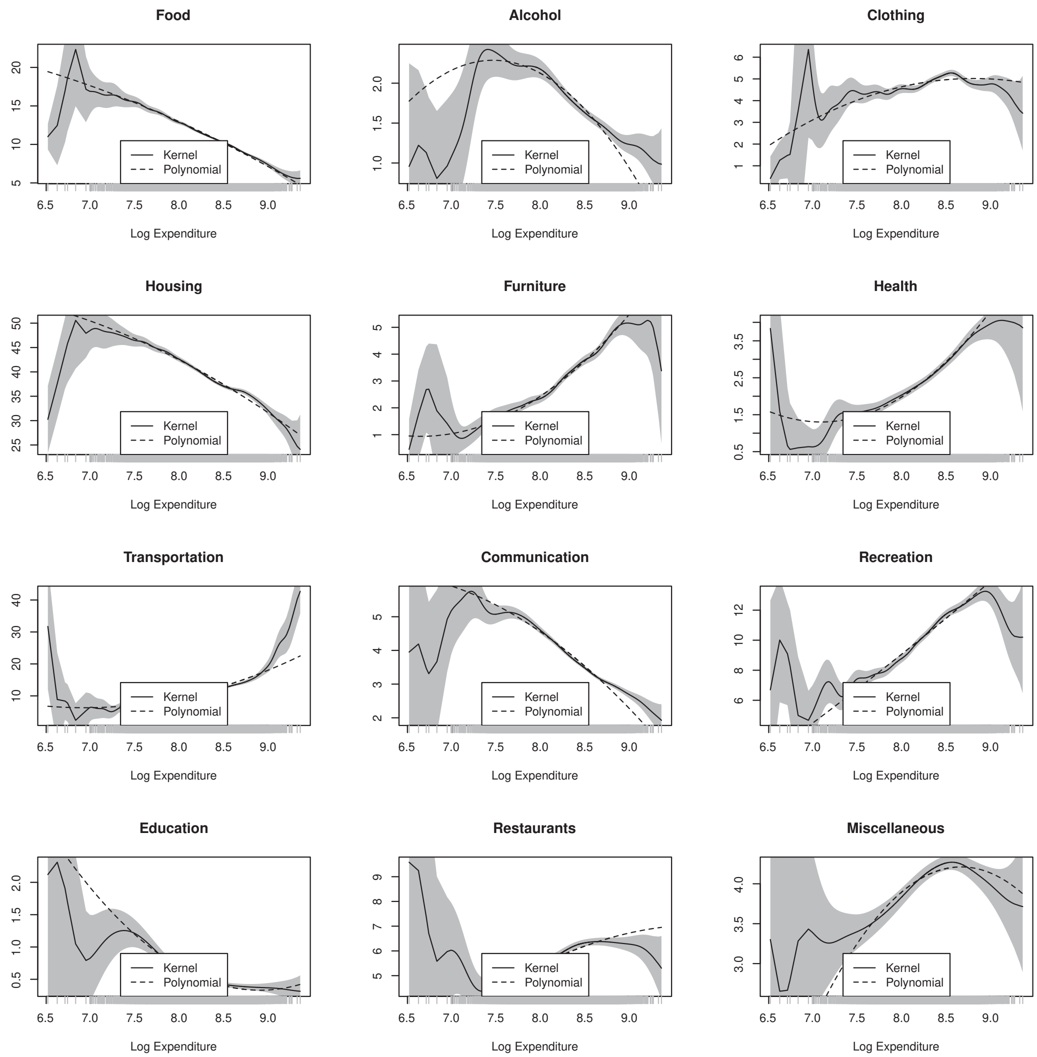


Fig. 2 Income independence test based on the QAI (Banks et al., 1997), Household type couple (AA)

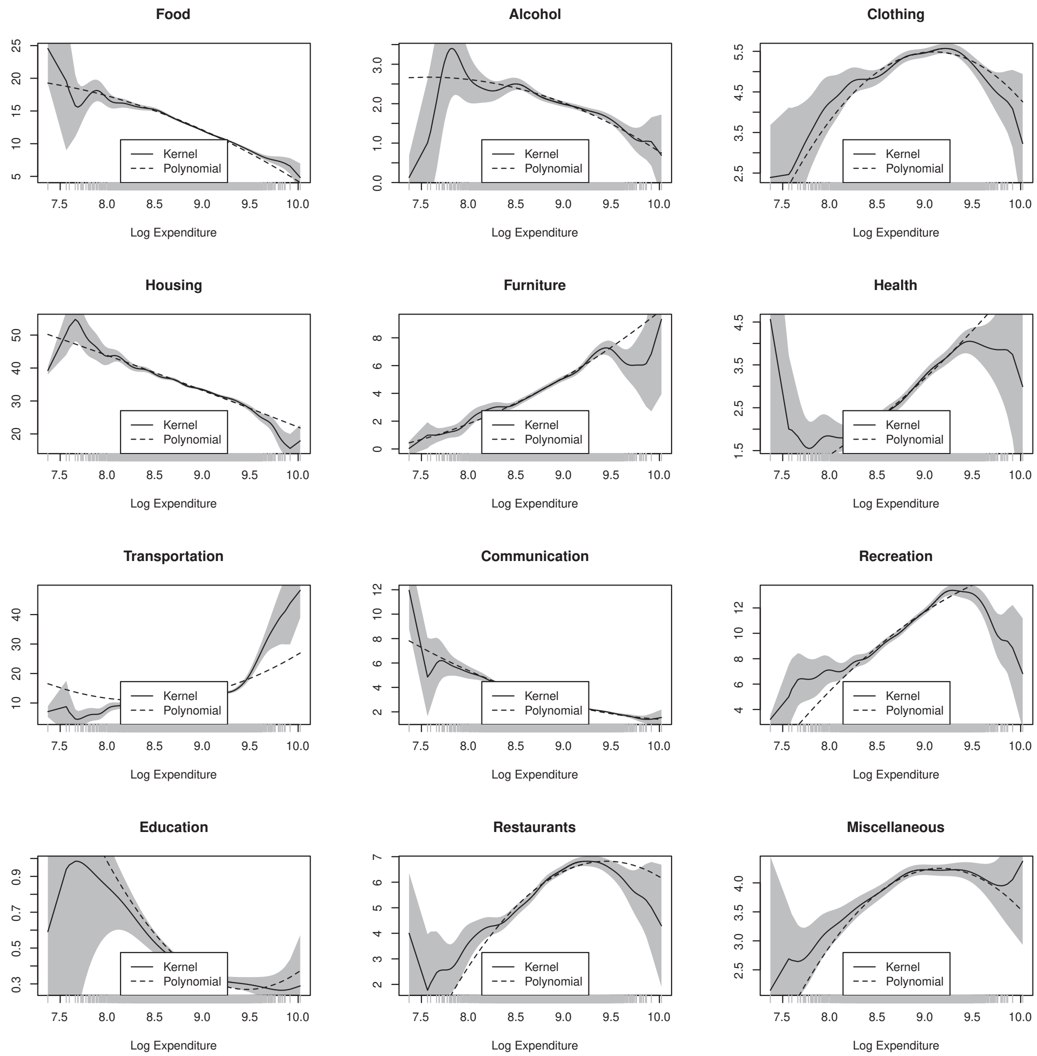


Fig. 3 Income independence test based on the QAI (Banks et al., 1997), Household type couple with one child (AAC)

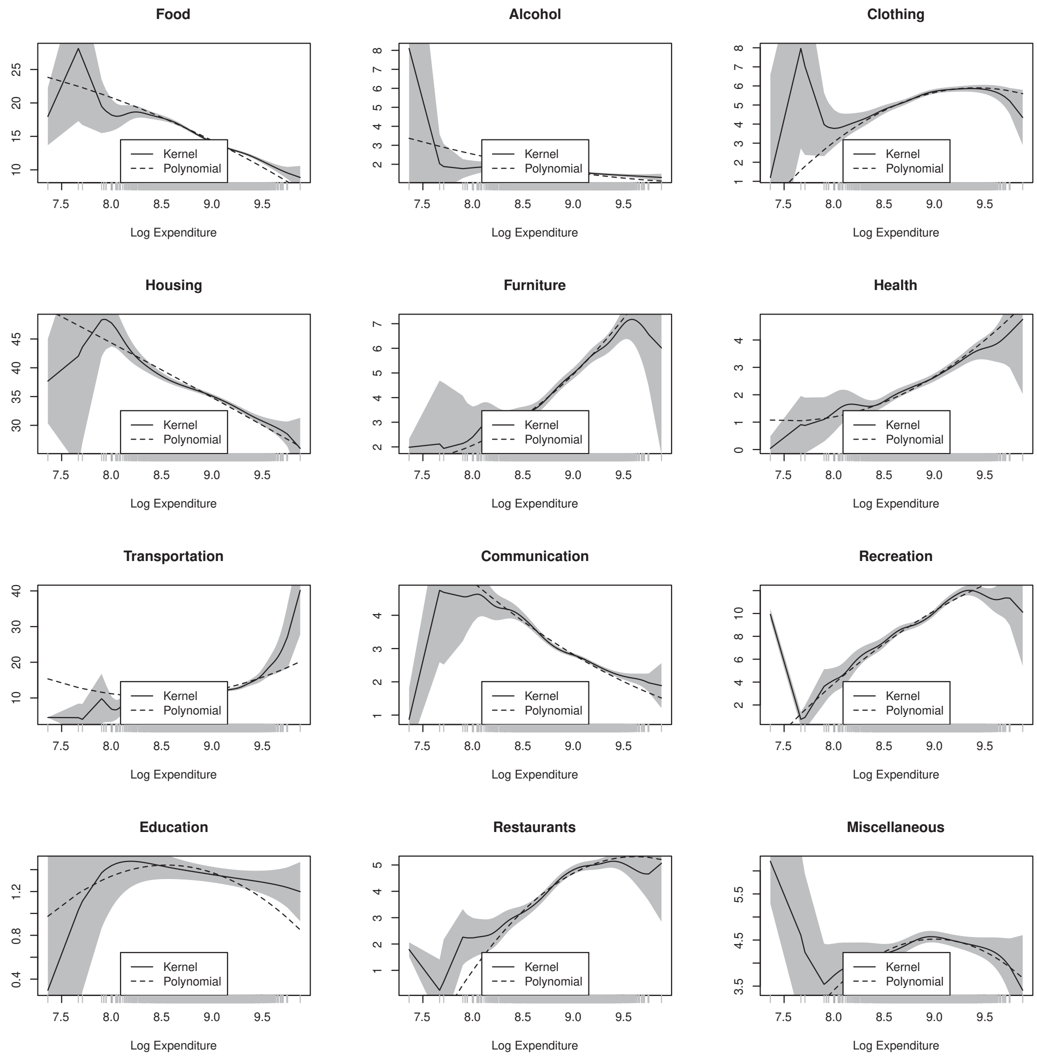


Fig. 4 Income independence test based on the QAI (Banks et al., 1997), Household type couple with two children (AACC)

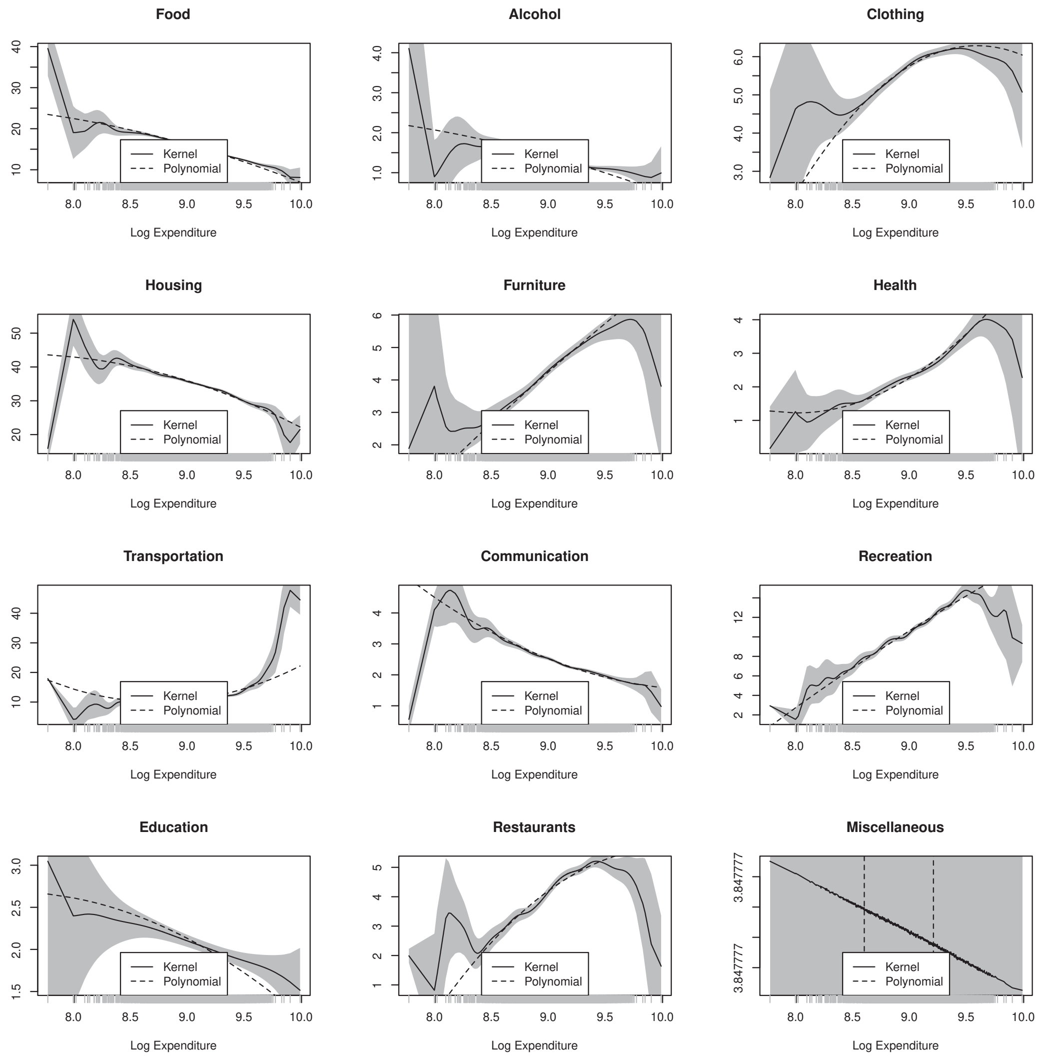
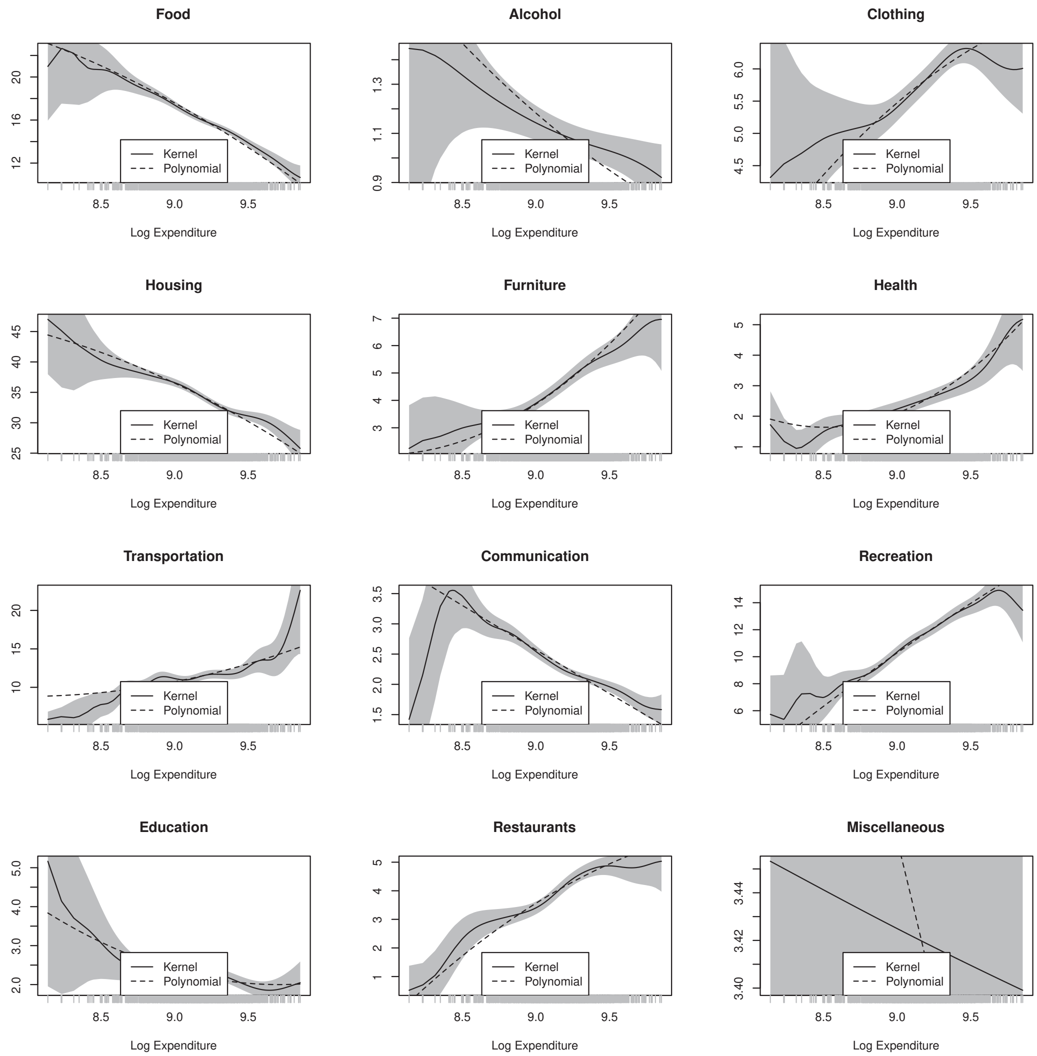


Fig. 5 Income independence test based on the QAI (Banks et al., 1997), Household type couple with three children (AACCC)



C: Calculation of confidence intervals for nonparametric bounds

If (L_s, U_s) denote the bounds resulting for the sth bootstrap replication, then we choose the bounds l and u of the confidence interval, such that $l < L_s$ and $U_s < u$ for 95% of the bootstrap replications. As l and u usually will not be unique, we choose the values of l and u for which the interval width, $u - l$, is smallest. To calculate the smallest interval, an iterative procedure is used and re-run several times. Each run takes the 5% percentile of the lower bound L and the 95% percentile of the upper bound U as starting values, perturbed with noise $e_L \sim \mathcal{N}(0, \mathrm{sd}(L))$ or $e_U \sim \mathcal{N}(0, \mathrm{sd}(U))$, respectively. Let the resulting bounds be denoted by $L^{(0)}$ and $U^{(0)}$. The coverage achieved with these values is equal to $\rho^{(0)}$. If $\rho^{(0)}$ is smaller than $1-\alpha$, $L^{(0)}$ and $U^{(0)}$ are decreased and increased, respectively, by a stepsize $\lambda_L = 0.1\mathrm{sd}(L)$ or $\lambda_U = 0.1\mathrm{sd}(U)$ to get new values: $L^{(1)} = L^{(0)} - \lambda \varepsilon_L^{(0)}$ and $U^{(1)} = U^{(0)} + \lambda \varepsilon_U^{(0)}$, where $\varepsilon_U^{(0)}$ and $\varepsilon_L^{(0)}$ follow a uniform distribution. If $\rho^{(0)}$ is larger than $1-\alpha$, the signs for λ are instead changed to decrease the interval width. $\rho^{(1)}$ is the coverage achieved after these adjustments. Depending on whether it is above or below $1-\alpha$, the adjustments are applied to obtain updated values $L^{(2)}$ and $U^{(2)}$; $\rho^{(2)}$ is checked against $1-\alpha$ again, etc.; until $\rho^{(k)} = 1-\alpha$. This procedure is re-run for 100 different starting values and the interval with the smallest width is reported.

References

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