Appendix

Part 1: PMF of the Shipment Quantity

To evaluate the shipment costs for various parameters, we have to consider eleven cases in total. As stated in the paper, the shipment quantity is always the sum of demands/orders during t_{n-1} and t_n plus $K(t_{n-1})$ minus $K(t_n)$. However, the formulas for the remaining units differ from case to case. We present in the following the graphics and explanations for the cases, which where not discussed in the paper.

Case 2: $L_d \leq T, T < L_s \leq T + L_d$

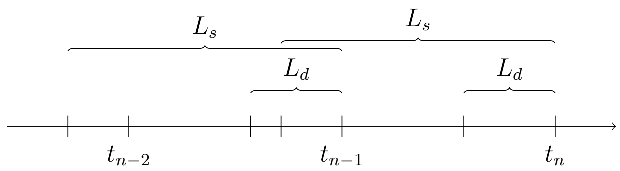

Fig. 1 Shipment cycle when $L_d \leq T, T < L_s \leq T + L_d$

The main difference to case 1 is that it cannot be assured that backorders are satisfied at the subsequent shipment day. Based on figure 1, the formula for $K(t_n)$ can be expressed by

$$\begin{aligned} K(t_n) = \max\Big(\Big(&\operatorname*{mod}_{(R,Q)} (IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-2}) - D(t_{n-2}, t_{n-1} - L_d) \\ &- D(t_{n-1} - L_d, t_n - L_s)) - D(t_n - L_s, t_{n-1}) - D(t_{n-1}, t_n - L_d) \\ &- D(t_n - L_d, t_n)\Big)^-, \min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n - L_d) \\ &- D(t_n - L_d, t_n) - K(t_{n-1}))^-\Big)\Big), \end{aligned} \tag{1}$$

whereas the remaining units at t_{n-1} can be computed by

$$\begin{aligned} K(t_{n-1}) = \max(&(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-2}) - D(t_{n-2}, t_{n-1} - L_d) \\ &- D(t_{n-1} - L_d, t_n - L_s) - D(t_n - L_s, t_{n-1}))^-, \\ &\min(D(t_{n-1} - L_d, t_n - L_s) + D(t_n - L_s, t_{n-1}), (Cap \\ &- D(t_{n-2}, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_s) - D(t_n - L_s, t_{n-1}) \\ &- K(t_{n-2}))^-)). \end{aligned} \tag{2}$$

Case 3: $L_d \leq T, T + L_d < L_s \leq 2T$

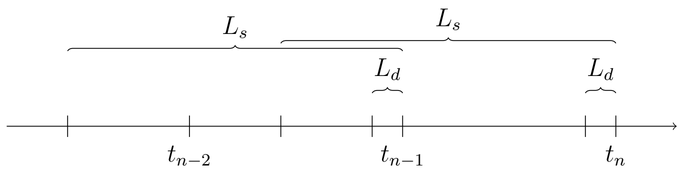

Fig. 2 Shipment cycle when $L_d \leq T, T + L_d < L_s \leq 2T$

Compared to case 2, $t_n - L_s$ is before $t_{n-1} - L_d$ which is illustrated in figure 2. Based on the given sequence of time points, the remaining units at t_n can be obtained by

$$\begin{aligned} K(t_n) = \max\Big(\Big(&\operatorname*{mod}_{R,Q} (IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-2}) - D(t_{n-2}, t_n - L_s)) \\ &- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}) - D(t_{n-1}, t_n - L_d) \\ &- D(t_n - L_d, t_n)\Big)^-, \min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n - L_d) \\ &- D(t_n - L_d, t_n) - K(t_{n-1}))^-)\Big) \end{aligned} \quad (3)$$

and the remaining units at t_{n-1} by

$$\begin{aligned} K(t_{n-1}) = \max(&(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-2}) - D(t_{n-2}, t_n - L_s) \\ &- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}))^-, \\ &\min(D(t_{n-1} - L_d, t_{n-1}), (Cap - D(t_{n-2}, t_n - L_s) \\ &- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}) - K(t_{n-2}))^-). \end{aligned} \quad (4)$$

Case 4: $L_d \leq T, L_s > 2T$

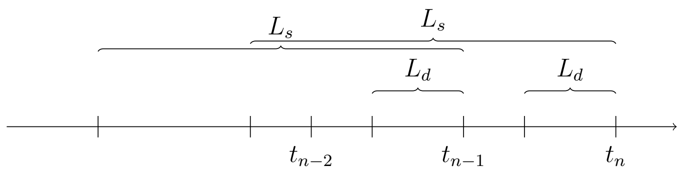

Fig. 3 Shipment cycle when $L_d \leq T, L_s > 2T$

Case 4 is the last case where $L_s \leq T$ because as soon as $L_s > 2T$, the sequence of events is the same for each length of L_s. In figure 3 an example is shown. The remaining units $K(t_n)$ can be computed by

$$K(t_n) = \max\Big(\Big(\underset{(R,Q)}{\mathrm{mod}}\,(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s)) - D(t_n - L_s, t_{n-2})$$
$$- D(t_{n-2}, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}) - D(t_{n-1}, t_n - L_d)$$
$$- D(t_n - L_d, t_n))^-, \min(D(t_n - L_d, t_n), (Cap - D(t_{n-1}, t_n - L_d)$$
$$- D(t_n - L_d, t_n) - K(t_{n-1}))^-\Big)\Big), \qquad (5)$$

whereas the remaining units at t_{n-1} can be obtained by

$$K(t_{n-1}) = \max((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s) - D(t_n - L_s, t_{n-2})$$
$$- D(t_{n-2}, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}))^-, \min(D(t_{n-1} - L_d, t_{n-1}),$$
$$(Cap - D(t_{n-2}, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-1}) - K(t_{n-2}))^-)). \qquad (6)$$

Case 6: $T < L_d \leq 2T, 2T < L_s \leq T + L_d$

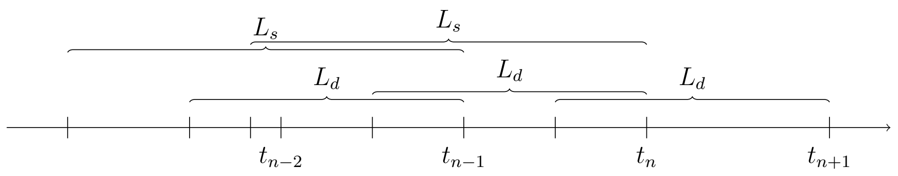

Fig. 4 Shipment cycle when $T < L_d \leq 2T, 2T < L_s \leq T + L_d$

Similar to case 5, we now investigate cases where $T < L_d \leq 2T$, which means orders are known more than one shipment interval earlier. However, the shipment policy only allows to ship units to the production facilities one shipment interval earlier. Therefore, all orders occurring during t_{n+1} and t_n cannot be shipped at t_n and are added to the remaining units directly. Considering the sequence of events given in figure 4, we get

$$K(t_n) = D(t_{n+1} - L_d, t_n) + \max\Big(\Big(\underset{R,Q}{\mathrm{mod}}\,(IP(t_{n-1} - L_s)$$
$$- D(t_{n-1} - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_s)) - D(t_n - L_s, t_{n-2})$$
$$- D(t_{n-2}, t_n - L_d) - D(t_n - L_d, t_{n-1}) - D(t_{n-1}, t_{n+1} - L_d)\Big)^-,$$
$$\min(D(t_n - L_d, t_{n-1}) + D(t_{n-1}, t_{n+1} - L_d), (Cap - D(t_{n-1}, t_{n+1} - L_d)$$
$$- K(t_{n-1}))^-)\Big), \qquad (7)$$

and

$$K(t_{n-1}) = D(t_n - L_d, t_{n-1}) + \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ - D(t_{n-1} - L_d, t_n - L_s) - D(t_n - L_s, t_{n-2}) - D(t_{n-2}, t_n - L_d))^-, \\ \min(D(t_{n-1} - L_d, t_n - L_s) + D(t_n - L_s, t_{n-2}) + D(t_{n-2}, t_n - L_d), \\ (Cap - D(t_{n-2}, t_n - L_d) - K(t_{n-2}))^-\Big). \tag{8}$$

Case 7: $T < L_d \leq 2T, L_s > T + L_d$

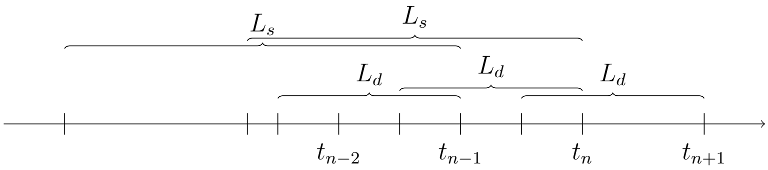

Fig. 5 Shipment cycle when $T < L_d \leq 2T, L_s > T + L_d$

Case 7 is close to case 6, however, L_s has to be larger than $T + L_d$. In figure 5 we can observe that $t_{n-1} - L_s$ and $t_n - L_s$ are the earliest time points which is why an increase of L_s will not change the sequence of time points anymore. Therefore, we can obtain the remaining units at t_n by

$$K(t_n) = D(t_{n+1} - L_d, t_n) + \max\Big(\Big(\underset{R,Q}{\text{mod}}\,(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s)) \\ - D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-2}) - D(t_{n-2}, t_n - L_d) \\ - D(t_n - L_d, t_{n-1}) - D(t_{n-1}, t_{n+1} - L_d)\Big)^-, \min(D(t_n - L_d, t_{n-1}) \\ + D(t_{n-1}, t_{n+1} - L_d), (Cap - D(t_{n-1}, t_{n+1} - L_d) - K(t_{n-1}))^-)\Big). \tag{9}$$

and the remaining units at t_{n-1} by

$$K(t_{n-1}) = D(t_n - L_d, t_{n-1}) + \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s) \\ - D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_{n-2}) - D(t_{n-2}, t_n - L_d))^-, \\ \min(D(t_{n-1} - L_d, t_{n-2}) + D(t_{n-2}, t_n - L_d), \\ (Cap - D(t_{n-2}, t_n - L_d) - K(t_{n-2}))^-\Big) \tag{10}$$

Case 8: $2T < L_d \leq 3T, 2T < L_s \leq T + L_d$

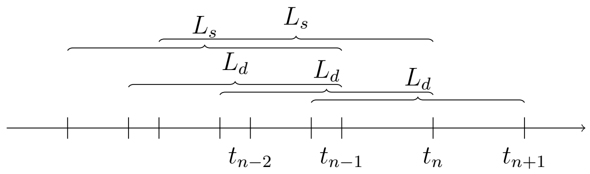

Fig. 6 Shipment cycle when $L_d > 2T, 2T < L_s \leq T + L_d$

Now, we consider situations where $2T < L_d \leq 3T$. Case 8 is illustrated in figure 6 which helps us to determine $K(t_n)$ and $K(t_{n-1})$. The remaining units at t_n can be computed by

$$\begin{aligned} K(t_n) = {} & D(t_{n+1} - L_d, t_{n-1}) + D(t_{n-1}, t_n) + \max\Bigg(\Big(\operatorname*{mod}_{R,Q}(IP(t_{n-1} - L_s) \\ & - D(t_{n-1} - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_s)) \\ & - D(t_n - L_s, t_n - L_d) - D(t_n - L_d, t_{n-2}) - D(t_{n-2}, t_{n+1} - L_d)\Big)^{-}, \\ & \min(D(t_n - L_d, t_{n-2}) + D(t_{n-2}, t_{n+1} - L_d), (Cap \\ & + D(t_{n+1} - L_d, t_{n-1}) - K(t_{n-1}))^{-})\Bigg), \end{aligned} \tag{11}$$

whereas the remaining units at t_n can be computed by

$$\begin{aligned} K(t_{n-1}) = {} & D(t_n - L_d, t_{n-2}) + D(t_{n-2}, t_{n+1} - L_d) + D(t_{n+1} - L_d, t_{n-1}) \\ & + \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ & - D(t_{n-1} - L_d, t_n - L_s) - D(t_n - L_s, t_n - L_d))^{-}, \\ & \min(D(t_{n-1} - L_d, t_n - L_s) + D(t_n - L_s, t_n - L_d), (Cap - K(t_{n-2}))^{-}\Big). \end{aligned} \tag{12}$$

Case 9: $2T < L_d \leq 3T, L_s > T + L_d$

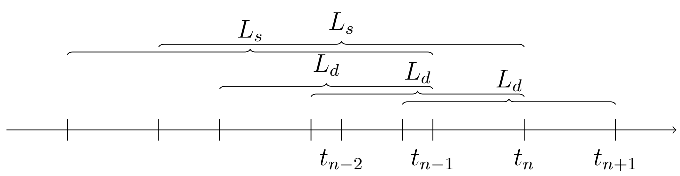

Fig. 7 Shipment cycle when $L_d > 2T, L_s > T + L_d$;

Compared to case 8, $t_n - L_s$ is before $t_{n-2} - L_d$ which is shown in figure 7. Summarizing, the remaining units at t_n for case 9 can be obtained by

$$\begin{aligned} K(t_n) = D(t_{n+1} - L_d, t_{n-1}) + D(t_{n-1}, t_n) + \max\Big(\Big(&\operatorname*{mod}_{R,Q}(IP(t_{n-1} - L_s) \\ &- D(t_{n-1} - L_s, t_n - L_s)) - D(t_n - L_s, t_{n-1} - L_d) \\ &- D(t_{n-1} - L_d, t_n - L_d) - D(t_n - L_d, t_{n-2}) - D(t_{n-2}, t_{n+1} - L_d)\Big)^{-}, \\ &\min(D(t_n - L_d, t_{n-2}) + D(t_{n-2}, t_{n+1} - L_d), (Cap \\ &+ D(t_{n+1} - L_d, t_{n-1}) - K(t_{n-1}))^{-})\Big), \end{aligned} \tag{13}$$

and the remaining units at t_{n-1} by

$$\begin{aligned} K(t_{n-1}) = D(t_n - L_d, t_{n-2}) &+ D(t_{n-2}, t_{n+1} - L_d) + D(t_{n+1} - L_d, t_{n-1}) \\ &+ \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s) \\ &- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_d))^{-}, \\ &\min(D(t_{n-1} - L_d, t_n - L_d), (Cap - K(t_{n-2}))^{-}\Big). \end{aligned} \tag{14}$$

Case 10: $L_d > 3T, 3T < L_s \leq T + L_d$

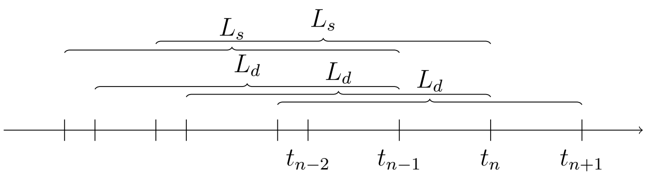

Fig. 8 Shipment cycle when $L_d > 2T, L_s > T + L_d$;

As soon as $L_s > 3T$, $t_{n+1} - L_d$ will always be before t_{n-2}, why this is the last range fore L_d. Again we have to consider the length of L_s compared to $T + L_d$. In figure 8 the considered case is illustrated, which leads to

$$\begin{aligned} K(t_n) = D(t_{n+1} - L_d, t_{n-2}) &+ D(t_{n-2}, t_{n-1}) + D(t_{n-1}, t_n) \\ &+ \max\Big(\Big(\operatorname*{mod}_{R,Q}(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d) \\ &- D(t_{n-1} - L_d, t_n - L_s)) - D(t_n - L_s, t_n - L_d) \\ &- D(t_n - L_d, t_{n+1} - L_d)\Big)^{-}, \min(D(t_n - L_d, t_{n+1} - L_d), \\ &(Cap + D(t_{n+1} - L_d, t_{n-2}) + D(t_{n-2}, t_{n-1}) - K(t_{n-1}))^{-})\Big). \end{aligned} \tag{15}$$

and

$$K(t_{n-1}) = D(t_n - L_d, t_{n+1} - L_d) + D(t_{n+1} - L_d, t_{n-2}) + D(t_{n-2}, t_{n-1})$$
$$+ \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_{n-1} - L_d)$$
$$- D(t_{n-1} - L_d, t_n - L_s) - D(t_n - L_s, t_n - L_d))^-,$$
$$\min(D(t_{n-1} - L_d, t_n - L_s) + D(t_n - L_s, t_n - L_d), (Cap - K(t_{n-2}))^-\Big). \tag{16}$$

Case 11: $L_d > 3T, L_s > T + L_d$

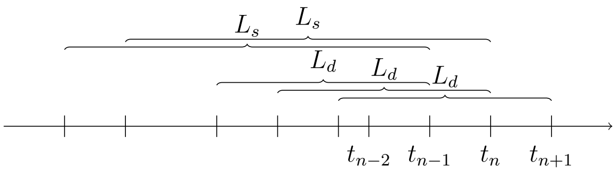

Fig. 9 Shipment cycle when $L_d > 2T, L_s > T + L_d$;

Case 11 is shown in figure 9 and represents the last case for the determination of the pmf of the shipment quantity. The remaining units at t_n can be computed by

$$K(t_n) = D(t_{n+1} - L_d, t_{n-2}) + D(t_{n-2}, t_{n-1}) + D(t_{n-1}, t_n)$$
$$+ \max\Big(\Big(\operatorname*{mod}_{R,Q}(IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s))$$
$$- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_d)$$
$$- D(t_n - L_d, t_{n+1} - L_d)\Big)^-, \min(D(t_n - L_d, t_{n+1} - L_d), (Cap$$
$$+ D(t_{n+1} - L_d, t_{n-2}) + D(t_{n-2}, t_{n-1}) - K(t_{n-1}))^-)\Big), \tag{17}$$

and the remaining units at t_{n-1} by

$$K(t_{n-1}) = D(t_n - L_d, t_{n+1} - L_d) + D(t_{n+1} - L_d, t_{n-2}) + D(t_{n-2}, t_{n-1})$$
$$+ \max\Big((IP(t_{n-1} - L_s) - D(t_{n-1} - L_s, t_n - L_s)$$
$$- D(t_n - L_s, t_{n-1} - L_d) - D(t_{n-1} - L_d, t_n - L_d))^-,$$
$$\min(D(t_{n-1} - L_d, t_n - L_d), (Cap - K(t_{n-2}))^-\Big). \tag{18}$$

Part 2: Derivation of the expected inventory cost when $S > 0$

In this subsection we derive the cost expressions the for inventory cost for all situations. Therefore, we first determine the joint distribution function $f_i(x, y), i \in \{A, B, C, D, E, F, G\}$.

Situation A

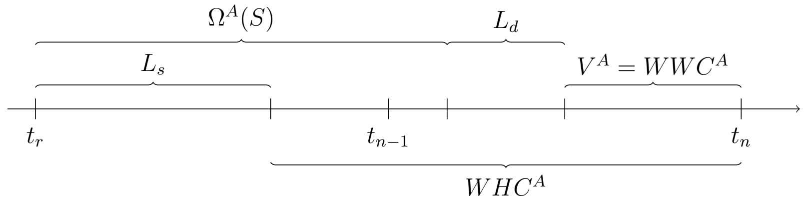

Fig. 10 Situation A

The derivation for $f_A(x, y)$ is already given in the paper. Based on that, the inventory cost can be derived.

$$\begin{aligned}
E[C_A(\Omega(S), V)] &= \int_0^\infty \int_0^T C_A(x,y) f_A(x,y)\, dy\, dx \\
&= \int_{L_s}^\infty \int_0^{(T-L_d)^+} \Big(h(x - L_s + L_d + y) + wy\Big) g^S(x) u(y)\, dy\, dx \\
&= \int_{L_s}^\infty \frac{(T-L_d)^+}{T} \Big(h \frac{S}{\lambda} g^{S+1}(x) + h(L_d - L_s) g^S(x)\Big) \\
&\quad + (h+w) \frac{((T-L_d)^+)^2}{2T} g^S(x) dx \\
&= h \frac{(T-L_d)^+}{T} \Big((L_d - L_s)(1 - G^S(x) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big)\Big) \\
&\quad + (h+w) \frac{((T-L_d)^+)^2}{2T} \Big(1 - G^S(L_s)\Big) \\
&= h \frac{(T-L_d)^+}{T} \Big(\Big(L_d - L_s + \frac{(T-L_d)^+}{2}\Big)\Big(1 - G^S(L_s)\Big) \\
&\quad + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big)\Big) + w \frac{((T-L_d)^+)^2}{2T}\Big(1 - G^S(L_s)\Big)
\end{aligned}$$

The further derivation depends on the length of T and L_d, why we separate in $L_d \leq T$ and $L_d > T$. For $L_d \leq T$ we get

$$\begin{aligned}
E[C_A(\Omega(S), V)] &= h \frac{T - L_d}{T} \Big(\Big(L_d - L_s + \frac{T - L_d}{2}\Big)\Big(1 - G^S(L_s)\Big) \\
&\quad + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big) + w \frac{(T-L_d)^2}{2T}\Big(1 - G^S(L_s)\Big)\Big) \\
&= h \frac{T - L_d}{T} \Big(\Big(\frac{T + L_d}{2} - L_s\Big)\Big(1 - G^S(L_s)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big)\Big) \\
&\quad + w \frac{(T-L_d)^2}{2T}\Big(1 - G^S(L_s)\Big),
\end{aligned} \tag{19}$$

whereas $L_d > T$ lead to

$$\begin{aligned} E[C_A(\Omega(S),V)] &= h\frac{0}{T}\Big((L_d - L_s + \frac{0}{2})\Big(1-G^S(L_s)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big)\Big) \\ &\quad + w\frac{(0)^2}{2T}\Big(1-G^S(L_s)\Big) \\ &= 0. \end{aligned} \tag{20}$$

Summarizing we get

$$E[C_A(\Omega(S),V)] = \begin{cases} h\frac{T-L_d}{T}\Big(\Big(\frac{T+L_d}{2} - L_s\Big)\Big(1-G^S(L_s)\Big) \\ +\frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big)\Big) \\ +w\frac{(T-L_d)^2}{2T}\Big(1-G^S(L_s)\Big), & \text{if } L_d \le T \\ \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Situation B

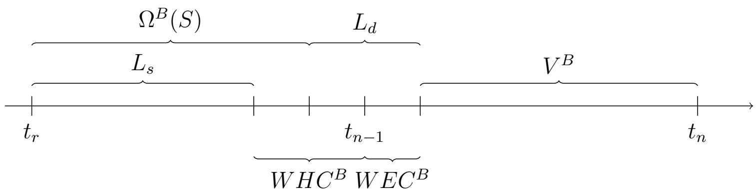

Fig. 11 Situation B

To obtain the joint probability function $f_B(x,y)$, we determine the cumulative distribution function for situation B by

$$F_B(x,y) = P(\Omega(S) \le x, V \le y, t_a < t_o, t_o < t_{n-1} < t_d < t_n, t_s = t_{n-1}).$$

Thus,

$$\begin{aligned} F_B(x,y) &= P(\Omega(S) \le x, V \le y, t_r + L_s < t_r + \Omega(S), \\ &\quad t_r + \Omega(S) < t_{n-1} < t_r + \Omega(S) + L_d < t_n) \\ &= P(\Omega(S) \le x, V \le y, L_s < \Omega(S), \\ &\quad \Omega(S) < t_{n-1} - t_r < \Omega(S) + L_d < t_n - t_r). \end{aligned} \tag{22}$$

It holds $T = t_r + \Omega(S) + L_d - t_{n-1} + V$, which leads to

$$\begin{aligned} F_B(x,y) &= P(L_s < \Omega(S) \le x, V \le y, \\ &\quad \Omega(S) < \Omega(S) + L_d + V - T < \Omega(S) + L_d < t_n - t_{n-1} + t_{n-1} - t_r) \\ &= P(L_s < \Omega(S) \le x, V \le y, \end{aligned}$$

$$
\begin{aligned}
&\Omega(S) < \Omega(S) + L_d + V - T < \Omega(S) + L_d < \Omega(S) + L_d + V) \\
&= P(L_s < \Omega(S) \leq x, V \leq y, 0 < L_d + V - T < L_d < L_d + V) \\
&= P(L_s < \Omega(S) \leq x, (T - L_d)^+ < V \leq y).
\end{aligned}
$$

We get for $x \geq L_s$ and $y \geq (T - L_d)^+$ we get

$$
\begin{aligned}
F_B(x, y) &= P(L_s < \Omega(S) \leq x, V \leq y) \\
&= \Big(G^s(x) - G^S(L_s)\Big)\Big(U(y) - U(T - L_d)\Big).
\end{aligned}
$$

Thus, the partial derivative with respect to both variables is given as

$$
f_B(x, y) = \begin{cases} g^S(x)u(y) & , \quad L_s < x < \infty, (T - L_d)^+ < y \leq T \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{23}
$$

We get for the expected costs

$$
\begin{aligned}
&E[C_B(\Omega(S), V)] \\
&= \int_0^\infty \int_0^T C_B(x, y) f_B(x, y) \, dy \, dx \\
&= \int_{L_s}^\infty \int_{(T-L_d)^+}^T \Big(hx + h(L_d - T - L_s) + eT + (h - e)y\Big) g^S(x) u(y) \, dy \, dx \\
&= \int_{L_s}^\infty \frac{T - (T - L_d)^+}{T} \Big(h \frac{S}{\lambda} g^{S+1}(x) + (h(L_d - T - L_s) + eT) g^S(x) \\
&\quad + (h - e) \frac{T^2 - ((T - L_d)^+)^2}{2T} g^S(x) \, dx \\
&= \frac{T - (T - L_d)^+}{T} \Big(h \frac{S}{\lambda} \Big(1 - G^{S+1}(L_s)\Big) + \Big(h(L_d - T - L_s) + eT\Big) \cdot \\
&\quad (1 - G^s(L_s)\Big) + (h - e) \frac{T^2 - ((T - L_d)^+)^2}{2T} \Big) \Big(1 - G^S(L_s)\Big)
\end{aligned}
$$

Assuming $L_d \leq T$, we get

$$
\begin{aligned}
&E[C_B(\Omega(S), V)] \\
&= \frac{L_d}{T} \Big(h \frac{S}{\lambda} \Big(1 - G^{S+1}(L_s)\Big) + \Big(h(L_d - T - L_s) + eT\Big)(1 - G^s(L_s)\Big) \\
&\quad + (h - e) \frac{T^2 - (T - L_d)^2}{2T} \Big) \Big(1 - G^S(L_s)\Big) \\
&= h \frac{L_d}{T} \Big(\Big(L_d - T - L_s + \frac{2T - L_d}{2} \Big) \Big(1 - G^s(L_s)\Big) + \frac{S}{\lambda} \Big(1 - G^{S+1}(L_s)\Big) \Big) \\
&\quad + e \frac{L_d}{T} \Big(T - \frac{2T - L_d}{2}\Big) \Big(1 - G^S(L_s)\Big) \\
&= h \frac{L_d}{T} \Big(\Big(\frac{L_d}{2} - L_s \Big) \Big(1 - G^S(L_s)\Big) + \frac{S}{\lambda} \Big(1 - G^{S+1}(L_s)\Big) \Big) \\
&\quad + e \frac{L_d^2}{2T} \Big(1 - G^S(L_s)\Big),
\end{aligned} \tag{24}
$$

whereas for $L_d > T$, we get

$$
\begin{aligned}
&E[C_B(\Omega(S), V)] \\
&= \frac{T-0}{T}\Big(h\frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big) + \Big(h(L_d - T - L_s) + eT\Big)\Big(1 - G^S(L_s)\Big)\Big) \\
&\quad + (h-e)\frac{T^2 - 0^2}{2T}\Big(1 - G^S(L_s)\Big) \\
&= h\Big(\Big(L_d - T - L_s + \frac{T}{2}\Big)\Big(1 - G^S(x)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big)\Big) + \\
&\quad + e\Big(T - \frac{T}{2}\Big)\Big(1 - G^S(L_s)\Big) \\
&= h\Big(\Big(L_d - \frac{T}{2} - L_s\Big)\Big(1 - G^S(L_s)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s)\Big) \\
&\quad + e\frac{T}{2}\Big(1 - G^S(L_s)\Big).
\end{aligned}
\tag{25}
$$

$$
E[C_B(\Omega(S), V)] = \begin{cases} h\frac{L_d}{T}\Big(\frac{L_d}{2} - L_s\Big)\Big(1 - G^S(L_s)\Big)\frac{S}{\lambda}\cdot \\ \Big(1 - G^{S+1}(L_s)\Big)\Big) + e\frac{L_d^2}{2T}\Big(1 - G^S(L_s)\Big), & \text{if } L_d \le T \\ h\Big((L_d - \frac{T}{2} - L_s)\Big(1 - G^S(L_s)\Big) + \frac{S}{\lambda}\cdot \\ \Big(1 - G^{S+1}(L_s)\Big) + e\frac{T}{2}\Big(1 - G^S(L_s)\Big), & \text{otherwise} \end{cases}
\tag{26}
$$

Situation C

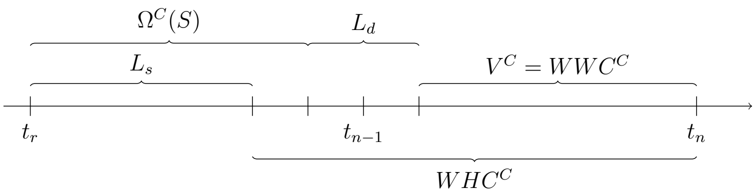

Fig. 12 Situation C

Case C is similar as Case B except the shipment point.

$$F_C(x, y) = P(\Omega(S) \le x, V \le y, t_a < t_o, t_o < t_{n-1} < t_d < t_n, t_s = t_n)$$

This is the case with no early delivery.

$$
\begin{aligned}
F_C(x, y) = P(&\Omega(S) \le x, V \le y, t_r + L_s < t_r + \Omega(S), \\
&t_r + \Omega(S) < t_{n-1} < t_r + \Omega(S) + L_d < t_n)
\end{aligned}
$$

This yields

$$f_C(x,y) = \begin{cases} g^S(x)u(y) & , \quad L_s < x < \infty, (T-L_d)^+ \le y \le T, \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{27}$$

The expected inventory cost for situation C can be obtained by

$$\begin{aligned}
E[C_C(\Omega(S),V)] &= \int_0^\infty \int_0^T C_C(x,y) f_C(x,y)\; dy\; dx \\
&= \int_{L_s}^\infty \int_{(T-L_d)^+}^T \Big(h(x-L_s+L_d+y)+wy\Big) g^S(x)u(y)\; dy\; dx \\
&= \int_{L_s}^\infty h\frac{T-(T-L_d)^+}{T}\frac{S}{\lambda}g^{S+1}(x) + (L_d-L_s)g^S(x)\Big) \\
&\quad + (h+w)\frac{T^2-((T-L_d)^+)^2}{2T} g^S(x)\; dy\; dx \\
&= h\frac{T-(T-L_d)^+}{T}\Big((L_d-L_s)\Big(1-G^S(x)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big) \\
&\quad + (h+w)\frac{T^2-((T-L_d)^+)^2}{2T}\Big(1-G^S(x)\Big).
\end{aligned}$$

The length of T and L_d define if the further cost derivation. For $L_d \le T$ we get

$$\begin{aligned}
E[C_C(\Omega(S),V)] &= h\frac{T-(T-L_d)}{T}\Big((L_d-L_s)\Big(1-G^S(x)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big) \\
&\quad + (h+w)\frac{T^2-(T-L_d)^2}{2T}\Big(1-G^S(x)\Big) \\
&= h\frac{L_d}{T}\Big((L_d-L_s)\Big(1-G^S(x)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big) \\
&\quad + (h+w)\frac{2TL_d-L_d^2}{2T}\Big(1-G^S(x)\Big) \\
&= h\frac{L_d}{T}\Big(\Big(\frac{L_d}{2}-L_s+T\Big)\Big(1-G^S(L_s)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big) \\
&\quad + w\frac{2TL_d-L_d^2}{2T}\Big(1-G^S(L_s)\Big),
\end{aligned}$$

whereas $L_d > T$ occurs inventory cost of

$$\begin{aligned}
E[C_C(\Omega(S),V)] &= h\frac{T-0}{T}\Big((L_d-L_s)\Big(1-G^S(x)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big)\Big) \\
&\quad + (h+w)\frac{T^2-0^2}{2T}\Big(1-G^S(x)\Big) \\
&= h\Big(\Big(L_d-L_s+\frac{T}{2}\Big)\Big(1-G^S(x)\Big) + \frac{S}{\lambda}\Big(1-G^{S+1}(L_s)\Big) \\
&\quad + w\frac{T}{2}\Big(1-G^S(x)\Big).
\end{aligned}$$

Finally, the expected inventory cost for situation C are defined as follows.

$$E[C_C(\Omega(S),V)] = \begin{cases} h\frac{L_d}{T}\Big(\frac{L_d}{2} - L_S + T)\Big(1 - G^S(x)\Big) + \frac{S}{\lambda}\cdot \\ \Big(1 - G^{S+1}(L_s)\Big)\Big) + w\frac{2TL_d - L_d^2}{2T}\Big(1 - G^S(L_s)\Big), & \text{if } L_d \leq T \\ h\Big((L_d - L_s + \frac{T}{2})\Big(1 - G^S(x)\Big) + \frac{S}{\lambda}\cdot \\ \Big(1 - G^{S+1}(L_s)\Big)\Big) + w\frac{T}{2}\Big(1 - G^S(L_s)\Big), & \text{otherwise} \end{cases} \tag{28}$$

Situation D

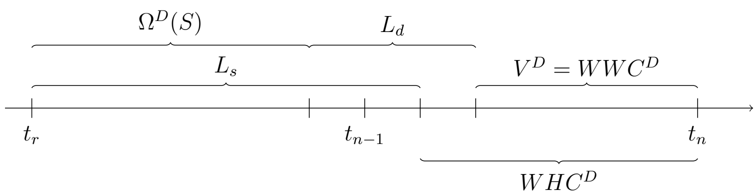

Fig. 13 Situation D

Now the unit is first ordered, than available and then demanded.

$$\begin{aligned} F_D(x,y) &= P(\Omega(S) \leq x, V \leq y, t_o \leq t_a, \\ &\quad t_{n-1} < t_a < t_d < t_n, t_s = t_n) \\ &= P(\Omega(S) \leq x, V \leq y, t_r + \Omega(S) \leq t_r + L_s, \\ &\quad t_{n-1} < t_r + L_s < t_r + \Omega(S) + L_d < t_n) \\ &= P(\Omega(S) \leq x, V \leq y, \Omega(S) \leq L_s, \\ &\quad 0 < t_r + L_s - t_{n-1} < t_r + \Omega(S) + L_d - t_{n-1} < T) \end{aligned}$$

Since $T = V + L_d + t_r + \Omega(S) - t_{n-1}$ we get

$$\begin{aligned} F_D(x,y) &= P(\Omega(S) \leq x, \Omega(S) \leq L_s, V \leq y, \\ &\quad 0 < T - V - L_d - \Omega(S) + L_s < T - V < T) \\ &= P(L_s - L_d \leq \Omega(S) \leq x, \Omega(S) < L_s, V \leq y, \\ &\quad V < T - L_d - \Omega(S) + L_s) \end{aligned}$$

For $L_s - L_d \leq x \leq L_s$ and $y \leq T - L_d + L_s - x$ we get

$$\begin{aligned}
F_D(x,y) &= P(L_s - L_d \leq \Omega(S) \leq x, V \leq y, V < T - L_d - \Omega(S) + L_s) \\
&= \int_{L_s-L_d}^{x} P(V \leq y, V < T - L_d - t + L_s \mid \Omega(S) = t) g^S(t)\, dt \\
&= \int_{L_s-L_d}^{x} P(V \leq \min\{y, T - L_d + L_s - t\}) g^S(t)\, dt \\
&= \int_{L_s-L_d}^{x} \frac{1}{T} y g^S(t)\, dt \\
&= \frac{y}{T}(G^S(x) - G^s(L_s - L_d)) \qquad (29)
\end{aligned}$$

To obtain $f_D(x,y)$ we have to compute partial derivative with respect to x and y For $L_s - L_d \leq x < L_s$ and $y \leq T - L_d + L_s - x$ we get

$$\begin{aligned}
f_D(x,y) &= \frac{\partial}{\partial x \partial y} F_D(x,y) \\
&= \frac{\partial}{\partial x \partial y}\Big(\frac{y}{T}(G^S(x) - G^s(L_s - L_d))\Big) \\
&= \frac{\partial}{\partial y}\Big(\frac{y}{T} g^S(x)\Big) \\
&= \frac{1}{T} g^S(x) = g^S(x) u(y) \qquad (30)
\end{aligned}$$

Thus, we obtain

$$f_D(x,y) = \begin{cases} g^S(x)u(y) & , \quad L_s - L_d < x \leq L_s, 0 \leq y \leq (T - L_d + L_s - x)^+ \\ 0 & , \quad \text{otherwise.} \end{cases} \qquad (31)$$

The inventory cost can be derived by

$$\begin{aligned}
&E[C_D(\Omega(S), V)] \\
&= \int_0^{\infty} \int_0^{T} C_D(x,y) f_D(x,y)\, dy\, dx \\
&= \int_{L_s-L_d}^{L_s} \int_0^{(T-L_d+L_s-x)^+} \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x) u(y)\, dy\, dx
\end{aligned}$$

Now, we have to separate the interval $L_s - L_d \leq \Omega(S) < L_s$ in $L_s - L_d \leq \Omega(S) \leq y \leq L_s - (L_d - T)^+$ and $L_s - (L_d - T)^+ < \Omega(S) < L_s$ to divide $0 \leq y \leq (T - L_d + L_s - x)^+$ into $0 \leq y \leq T - L_d + L_s - x$ and $0 \leq y \leq 0$. We get

$$\begin{aligned}
&E[C_D(\Omega(S), V)] \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} \int_0^{T-L_d+L_s-x} \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x) u(y)\, dy\, dx \\
&\quad + \int_{L_s-(L_d-T)^+}^{L_s} \int_0^{0} \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x) u(y)\, dy\, dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\frac{T - L_d + L_s - x}{T} \frac{S}{\lambda} g^{S+1}(x) + h(L_d - L_s)\frac{T - L_d + L_s - x}{T} g^S(x)
\end{aligned}$$

$$
\begin{aligned}
&+ (h+w)\frac{(T-L_d+L_s-x)^2}{2T} g^S(x)\,dx \\
= &\int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{2(L_d-L_s)(T-L_d+L_s)}{2T} + \frac{(T-L_d+L_s)^2}{2T}\Big) g^S(x) \\
&+ h\Big(\frac{T-L_d+L_s}{T} - h\frac{L_d-L_s}{T} - \frac{T-L_d+L_s}{T}\Big)\frac{S}{\lambda} g^{S+1}(x) - h\frac{S(S+1)}{2T\lambda^2} g^{S+2}(x) \\
&+ w\frac{(T-L_d+L_s)^2}{2T} g^S(x) - w\frac{(T-L_d+L_s)S}{T\lambda} g^{S+1}(x) + w\frac{S(S+1)}{2T\lambda^2} g^{S+2}(x)\,dx \\
= &\int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{2TL_d - 2L_d^2 + 2L_sL_d - 2TL_s + 2L_sL_d - 2L_s^2 + T^2 + L_d^2 + L_s^2}{2T} \\
&+ \frac{-2TL_d + 2TL_s - 2L_sL_d}{2T}\Big) g^S(x) + h\frac{(L_s-L_d)S}{T\lambda} g^{S+1}(x) - h\frac{S(S+1)}{2T\lambda^2} g^{S+2}(x) \\
&+ w\frac{(T-L_d+L_s)^2}{2T} g^S(x) - w\frac{(T-L_d+L_s)S}{T\lambda} g^{S+1}(x + w\frac{S(S+1)}{2T\lambda^2} g^{S+2}(x)\,dx \\
= &\int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{T^2-(L_s-L_d)^2}{2T} g^S(x) + \frac{(L_s-L_d)S}{T\lambda} g^{S+1}(x) \\
&- \frac{S(S+1)}{2T\lambda^2} g^{S+2}(x)\Big) + w\Big(\frac{(T-L_d+L_s)^2}{2T} g^S(x) - \frac{(T-L_d+L_s)S}{T\lambda} g^{+1}S(x) \\
&+ \frac{S(S+1)}{2T\lambda^2} g^{S+2}(x)\Big)\,dx \\
= &\, h\Big(\frac{T^2-(L_s-L_d)^2}{2T}\Big(G^S(L_s-(L_d-T)^+) - G^S(L_s-L_d)\Big) \\
&+ \frac{(L_s-L_d)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+) - G^{S+1}(L_s-L_d)\Big) \\
&- \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+) - G^{S+2}(L_s-L_d)\Big)\Big) \\
&+ w\Big(\frac{(T-L_d+L_s)^2}{2T}\Big(G^S(L_s-(L_d-T)^+) - G^S(L_s-L_d)\Big) \\
&- \frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+) - G^{S+1}(L_s-L_d)\Big) \\
&+ \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+) - G^{S+2}(L_s-L_d)\Big)\Big) \qquad (32)
\end{aligned}
$$

Summarizing, we get

$$
\begin{aligned}
&E[C_D(\Omega(S),V)] \qquad (33)\\
&= \begin{cases}
h\Big(\frac{T^2-(L_s-L_d)^2}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\
+\frac{(L_s-L_d)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\
-\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big) \\
+w\Big(\frac{(T-L_d+L_s)^2}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\
-\frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big), & \text{if } L_d \le T \\
\\
h\Big(\frac{T^2-(L_s-L_d)^2}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\
+\frac{(L_s-L_d)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\
-\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big)\Big) \\
+w\Big(\frac{(T-L_d+L_s)^2}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\
-\frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big)\Big), & \text{otherwise.}
\end{cases}
\end{aligned}
$$

Situation E

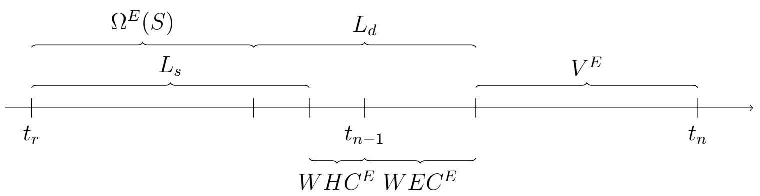

Fig. 14 Situation E

Now, t_o and t_a occur in different shipment intervals why flexible deliveries are possible if enough capacity is available.

$$
\begin{aligned}
F_E(x,y) &= P(\Omega(S) \le x, V \le y, t_o \le t_a, t_a < t_{n-1} < t_d < t_n, t_s = t_{n-1}) \\
&= P(\Omega(S) \le x, V \le y, t_r + \Omega(S) \le t_r + L_s, \\
&\quad t_r + L_s < t_{n-1} < t_r + \Omega(S) + L_d < t_n) \\
&= P(\Omega(S) \le x, V \le y, \Omega(S) \le L_s, \\
&\quad L_s < t_{n-1} - t_r < \Omega(S) + L_d < t_n - t_r)
\end{aligned}
$$

Since $T = t_r + \Omega(S) + L_d - t_{n-1} + V$ we get $t_{n-1} - t_r = \Omega(S) + L_d - T + V$

$$\begin{aligned} F_E(x,y) &= P(\Omega(S) \leq \min\{x, L_s\}, V \leq y, L_s < \Omega(S) + L_d - T + V \\ &\quad < \Omega(S) + L_d < t_n - t_{n-1} + t_{n-1} - t_r, M(t_n) < Cap) \\ &= P(\Omega(S) \leq \min\{x, L_s\}, V \leq y, L_s < \Omega(S) + L_d - T + \\ &\quad V < \Omega(S) + L_d < \Omega(S) + L_d + V, M(t_n) < Cap) \\ &= P(L_s - L_d < \Omega(S) \leq \min\{x, L_s\}, T + L_s - \Omega(S) - L_d < V \leq y) \end{aligned}$$

For $L_s - L_d \leq x \leq L_s$ we get

$$F_E(x,y) = \int_{L_s - L_d}^{x} P\Big(T + L_s - t - L_d < V \leq y\Big) g^S(t)\; dt$$

For $T + L_s - L_d - x \geq y$ this is zero. Now $T + L_s - L_d - x < y$

$$\begin{aligned} F_E(x,y) &= \int_{T+L_s-L_d-y}^{x} \Big(P(V \leq y) - P(V \leq T + L_s - t - L_d)\Big) g^S(t)\; dt \\ &= \int_{T+L_s-L_d-y}^{x} \Big(\frac{y}{T} - \frac{T + L_s - t - L_d}{T}\Big) g^S(t)\; dt \\ &= \frac{y + L_d - T - L_s}{T}\Big(G^S(x) - G^S(T + L_s - L_d - y)\Big) \\ &\quad + \frac{1}{T}\int_{T+L_s-L_d-y}^{x} t g^S(t)\; dt \\ &= \frac{y + L_d - T - L_s}{T}\Big(G^S(x) - G^S(T + L_s - L_d - y)\Big) \\ &\quad + \frac{1}{T}\frac{S}{\lambda}\int_{T+L_s-L_d-y}^{x} g^{S+1}(t)\; dt \\ &= \frac{y + L_d - T - L_s}{T}\Big(G^S(x) - G^S(T + L_s - L_d - y)\Big) \\ &\quad + \frac{1}{T}\frac{S}{\lambda}\Big(G^{S+1}(x) - G^{S+1}(T + L_s - L_d - y)\Big). \end{aligned} \tag{34}$$

To obtain $f_E(x,y)$ we have to compute partial derivative with respect to x and y For $L_s - L_d \leq x < L_s$ and $y > T - L_d + L_s - x$ we get

$$\begin{aligned} f_E(x,y) &= \frac{\partial}{\partial x \partial y} F_E(x,y) \\ &= \frac{\partial}{\partial x \partial y}\Big\{\frac{y + L_d - T - L_s}{T}\Big(G^S(x) - G^S(T + L_s - L_d - y)\Big) \\ &\quad + \frac{1}{T}\frac{S}{\lambda}\Big(G^{S+1}(x) - G^{S+1}(T + L_s - L_d - y)\Big)\Big\} \\ &= \frac{\partial}{\partial y}\Big\{\frac{y + L_d - T - L_s}{T} g^S(x) + \frac{1}{T}\frac{S}{\lambda} g^{S+1}(x)\Big\} \\ &= \frac{1}{T} g^S(x) = g^S(x) u(y). \end{aligned} \tag{35}$$

Thus, we obtain

$$f_E(x,y) = \begin{cases} g^S(x)u(y) & , \quad L_s - L_d < x \leq L_s, T - L_d + L_s - x < y \leq T \\ 0 & , \quad \text{otherwise.} \end{cases} \tag{36}$$

Now, we can derive the inventory cost as follows.

$$E[C_E(\Omega(S),V)]$$
$$= \int_0^\infty \int_0^T C_E(x,y) f_E(x,y)\, dy\, dx$$
$$= \int_{L_s-L_d}^{L_s} \int_{(T-L_d+L_s-x)^+}^{T} \Big(h(x-T-L_s+L_d+y)+e(T-y)\Big) g^S(x)u(y)\, dy\, dx$$
$$= \int_{L_s-L_d}^{L_s} \int_{(T-L_d+L_s-x)^+}^{T} \Big(hx+h(L_d-T-L_s)+eT+(h-e)y\Big) g^S(x)u(y)\, dy\, dx$$

Again, we have to separate the interval $L_s-L_d \le \Omega(S) < L_s$ in $L_s-L_d \le \Omega(S) \le y \le L_s-(L_d-T)^+$ and $L_s-(L_d-T)^+ < \Omega(S) < L_s$ to divide $(T-L_d+L_s-x)^+ < y \le T$ into $T-L_d+L_s-x < y \le T$ and $0 \le y \le T$.

$$E[C_E(\Omega(S),V)]$$
$$= \int_{L_s-L_d}^{L_s-(L_d-T)^+} \int_{T-L_d+L_s-x}^{T} \Big(hx+h(L_d-T-L_s)+eT+(h-e)y\Big) g^S(x)u(y)\, dy\, dx$$
$$+ \int_{L_s-(L_d-T)^+}^{L_s} \int_0^T \Big(hx+h(L_d-T-L_s)+eT+(h-e)y\Big) g^S(x)u(y)\, dy\, dx$$
$$= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\frac{L_d-L_s+x}{T}\frac{S}{\lambda}g^{S+1}(x) + h(L_d-T-L_s)\frac{L_d-L_s+x}{T}g^S(x)$$
$$+ eT\frac{L_d-L_s+x}{T}g^S(x) + (h-e)\frac{T^2-(T-L_d+L_s-x)^2}{2T}g^S(x)\, dx$$
$$+ \int_{L_s-(L_d-T)^+}^{L_s} h\frac{S}{\lambda}g^{S+1}(x) + h(L_d-T-L_s)g^S(x) + eTg^S(x) + (h-e)\frac{T}{2}g^S(x)\, dx$$
$$= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\frac{L_d-L_s}{T}\frac{S}{\lambda}g^{S+1}(x) + h\frac{S(S+1)}{T\lambda^2}g^{S+2}(x)$$
$$+ h\frac{(L_d-T-L_s)(L_d-L_s)}{T}g^S(x) + h\frac{(L_d-T-L_s)}{T}\frac{S}{\lambda}g^{S+1}(x)$$
$$+ e\frac{T(L_d-L_s)}{T}g^S(x) + e\frac{TS}{T\lambda}gS+1(x) + (h-e)\frac{T^2-(T-L_d+L_s)^2}{2T}g^S(x)\, dx$$
$$+ (h-e)\frac{2(T-L_d+L_s)}{2T}\frac{S}{\lambda}g^{S+1}(x) - (h-e)\frac{S(S+1)}{T\lambda^2}g^{S+2}(x) +$$
$$\int_{L_s-(L_d-T)^+}^{L_s} h(L_d-L_s-\frac{T}{2})g^S(x) + h\frac{S}{\lambda}g^{S+1}(x) + e\frac{T}{2}g^S(x)\, dx$$
$$= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{2(L_d-T-L_s)(L_d-L_s)}{2T} + \frac{T^2-(T-L_d+L_s)^2}{2T}\Big)g^S(x)$$
$$+ h\Big(\frac{L_d-L_s}{T} + \frac{L_d-T-L_s}{T} + \frac{T-L_d+L_s}{T}\Big)\frac{S}{\lambda}g^{S+1}(x) + h\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)$$
$$+ e\Big(\frac{2T(L_d-L_s)}{2T} - \frac{T^2-(T-L_d+L_s)^2}{2T}\Big)g^S(x) + e\frac{S(L_d-L_s)}{T\lambda}g^{S+1}(x)$$
$$+ e\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\, dx + \int_{L_s-(L_d-T)^+}^{L_s} h(L_d-L_s-\frac{T}{2})g^S(x) + h\frac{S}{\lambda}g^{S+1}(x)$$

$$
\begin{aligned}
&+ e\frac{T}{2}g^S(x)\,dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\Big(\frac{2L_d^2 - 2TL_d - 2L_sL_d - 2L_sL_d + 2TL_s + 2L_s^2 + T^2 - T^2 - L_d^2}{2T} \\
&+ \frac{-L_s^2 + 2TL_d - 2TL_s + 2L_sL_d}{2T}\Big)g^S(x) + \frac{(L_s-L_d)S}{T\lambda}g^{S+1}(x) \\
&- \frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big) + e\Big(\Big(\frac{2TL_d - 2TL_s - T^2 + T^2 + L_d^2 + L_s^2 - 2TL_d + 2TL_s}{2T} \\
&+ \frac{-2L_sL_d}{2T}\Big)g^S(x) + \frac{(L_d-L_s)S}{T\lambda}g^{S+1}(x) + \frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)\,dx \\
&+ \int_{L_s-(L_d-T)^+}^{L_s} h\Big((L_d - L_s - \frac{T}{2})g^S(x) + \frac{S}{\lambda}g^{S+1}(x)\Big) + e\frac{T}{2}g^S(x)\,dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{(L_d-L_s)^2}{2T}g^S(x) + \frac{(L_d-L_s)S}{T\lambda}g^{S+1}(x) + \frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big) \\
&+ e\Big(\frac{(L_d-L_s)^2}{2T}g^S(x) + \frac{(L_d-L_s)S}{T\lambda}g^{S+1}(x) + \frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)\,dx \\
&+ \int_{L_s-(L_d-T)^+}^{L_s} h\Big((L_d - L_s - \frac{T}{2})g^S(x) + \frac{S}{\lambda}g^{S+1}(x)\Big) + e\frac{T}{2}g^S(x)\,dx \\
&= h\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s-(L_d-T)^+) - G^S(L_s-L_d)\Big) \\
&+ \frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+) - G^{S+1}(L_s-L_d)\Big) \\
&+ \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+) - G^{S+2}(L_s-L_d)\Big)\Big) \\
&+ e\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s-(L_d-T)^+) - G^S(L_s-L_d)\Big) \\
&+ \frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+) - G^{S+1}(L_s-L_d)\Big) \\
&+ \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+) - G^{S+2}(L_s-L_d)\Big)\Big) \\
&+ h\Big((L_d - L_s - \frac{T}{2})\Big(G^S(L_s) - G^S(L_s-(L_d-T)^+)\Big) \\
&+ \frac{S}{\lambda}\Big(G^{S+1}(L_s) - G^{S+1}(L_s-(L_d-T)^+)\Big)\Big) \\
&+ e\frac{T}{2}\Big(G^S(L_s) - G^S(L_s-(L_d-T)^+))\Big) \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
&E[C_E(\Omega(S),V)] \qquad (38)\\
&= \begin{cases}
h\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\
+\frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big) \\
+e\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\
+\frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big), & \text{if } L_d \leq T \\
\\
h\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\
+\frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big) \\
+(L_d-L_s-\frac{T}{2})\Big(G^S(L_s)-G^S(L_s-L_d+T)\Big) \\
+\frac{S}{\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d+T)\Big)\Big) \\
+e\Big(\frac{(L_d-L_s)^2}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\
+\frac{(L_d-L_s)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\
+\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big) \\
+\frac{T}{2}\Big(G^S(L_s)-G^S(L_s-L_d+T))\Big)\Big), & \text{otherwise}
\end{cases}
\end{aligned}
$$

Situation F

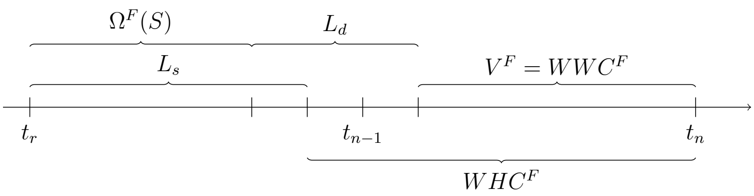

Fig. 15 Situation F

The inventory cost changes if no reserved transportation capacity is available for the considered ordered unit.

$$F_F(x,y) = P(\Omega(S) \leq x, V \leq y, t_o \leq t_a, t_a < t_{n-1} < t_d < t_n, t_s = t_n)$$

We get the same probabilities since only the shipment time is different compared to E. Thus, we obtain

$$f_F(x,y) = \begin{cases} g^S(x)u(y) & , \quad L_s - L_d < x \leq L_s, T - L_d + L_s - x < y \leq T \\ 0 & , \quad \text{otherwise} \end{cases} \tag{39}$$

$$\begin{aligned}
&E[C_F(\Omega(S),V)] \\
&= \int_0^\infty \int_0^T C_F(x,y) f_F(x,y)\, dy\, dx \\
&= \int_{L_s-L_d}^{L_s} \int_{(T-L_d+L_s-x)^+}^{T} \Big(h(x - L_s + L_d + y) + wy\Big) g^S(x)u(y)\, dy\, dx \\
&= \int_{L_s-L_d}^{L_s} \int_{(T-L_d+L_s-x)^+}^{T} \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x)u(y)\, dy\, dx
\end{aligned}$$

The interval $L_s - L_d \leq \Omega(S) < L_s$ has to be separated in $L_s - L_d \leq \Omega(S) \leq L_s - (L_d - T)^+$ and $L_s - (L_d - T)^+ < \Omega(S) < L_s$ to divide the integral $(T - L_d + L_s - x)^+ < y \leq T$ into $T - L_d + L_s - x < y \leq T$ and $0 \leq y \leq T$.

$$\begin{aligned}
&E[C_F(\Omega(S),V)] \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} \int_{T-L_d+L_s-x}^{T} \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x)u(y)\, dy\, dx \\
&\quad + \int_{L_s-(L_d-T)^+}^{L_s} \int_0^T \Big(hx + h(L_d - L_s) + (h+w)y\Big) g^S(x)u(y)\, dy\, dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\frac{L_d - L_s + x}{T}\frac{S}{\lambda} g^{S+1}(x) + h(L_d - L_s)\frac{L_d - L_s + x}{T} g^S(x) \\
&\quad + (h+w)\frac{T^2 - (T - L_d + L_s - x)^2}{2T} g^S(x)\, dx \\
&\quad + \int_{L_s-(L_d-T)^+}^{L_s} h\frac{S}{\lambda} g^{S+1}(x) + h(L_d - L_s) g^S(x) + (h+w)\frac{T}{2} g^S(x)\, dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\frac{L_d - L_s}{T}\frac{S}{\lambda} g^{S+1}(x) + h\frac{S(S+1)}{T\lambda^2} g^{S+2}(x) + h\frac{(L_d - L_s)^2}{T} g^S(x) \\
&\quad + h\frac{L_d - L_s}{T}\frac{S}{\lambda} g^{S+1}(x) + (h+w)\frac{T^2 - (T - L_d + L_s)^2}{2T} g^S(x)\, dx \\
&\quad + (h+w)\frac{2(T - L_d + L_s)}{2T}\frac{S}{\lambda} g^{S+1}(x) - (h+w)\frac{S(S+1)}{T\lambda^2} g^{S+2}(x) \\
&\quad + \int_{L_s-(L_d-T)^+}^{L_s} h\frac{S}{\lambda} g^{S+1}(x) + h(L_d - L_s) g^S(x) + (h+w)\frac{T}{2} g^S(x)\, dx \\
&= \int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{2(L_d - L_s)^2}{2T} + \frac{T^2 - (T - L_d + L_s)^2}{2T}\Big) g^S(x) \\
&\quad + h\Big(\frac{L_d - L_s}{T} + \frac{L_d - L_s}{T} + \frac{T - L_d + L_s}{T}\Big)\frac{S}{\lambda} g^{S+1}(x)
\end{aligned}$$

$$
\begin{aligned}
&+ h\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x) + w\frac{T^2-(T-L_d+L_s)^2}{2T}g^S(x)\\
&+ w\frac{(T-L_d+L_s)S}{T\lambda}g^{+1}S(x) - w\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\,dx\\
&+ \int_{L_s-(L_d-T)^+}^{L_s} h\Big((L_d-L_s+\frac{T}{2})g^S(x)+\frac{S}{\lambda}g^{S+1}(x)\Big)+w\frac{T}{2}g^S(x)\,dx\\
= &\int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\Big(\frac{2L_d^2-4L_sL_d+2L_s^2+T^2-T^2-L_d^2-L_s^2+2TL_d-2TL_s}{2T}\\
&+ \frac{2L_sL_d}{2T}\Big)g^S(x)+\frac{(L_d-L_s+T)S}{T\lambda}g^{S+1}(x)+\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)\\
&+ w\Big(\frac{T^2-T^2-L_d^2-L_s^2+2TL_d-2TL_s+2L_sL_d}{2T}g^S(x)\\
&+ \frac{(T-L_d+L_s)S}{T\lambda}g^{S+1}(x)-\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)\,dx\\
&+ \int_{L_s-(L_d-T)^+}^{L_s} h\Big((L_d-L_s+\frac{T}{2})g^S(x)+\frac{S}{\lambda}g^{S+1}(x)\Big)+w\frac{T}{2}g^S(x)\,dx\\
= &\int_{L_s-L_d}^{L_s-(L_d-T)^+} h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}g^S(x)+\frac{(L_d-L_s+T)S}{T\lambda}g^{S+1}(x)\\
&+ \frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)+w\Big(\frac{-(L_d-L_s)^2+2T(L_d-L_s)}{2T}g^S(x)\\
&+ \frac{(T-L_d+L_s)S}{T\lambda}g^{S+1}(x)-\frac{S(S+1)}{2T\lambda^2}g^{S+2}(x)\Big)\,dx\\
&+ \int_{L_s-(L_d-T)^+}^{L_s} h\Big((L_d-L_s+\frac{T}{2})g^S(x)+\frac{S}{\lambda}g^{S+1}(x)\Big)+w\frac{T}{2}g^S(x)\,dx\\
= &\, h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}\Big(G^S(L_s-(L_d-T)^+)-G^S(L_s-L_d)\Big)\\
&+ \frac{(L_d-L_s+T)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+)-G^{S+1}(L_s-L_d)\Big)\\
&+ \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+)-G^{S+2}(L_s-L_d)\Big)\Big)\\
&+ w\Big(\frac{2T(L_d-L_s)-(L_d-L_s)^2}{2T}\Big(G^S(L_s-(L_d-T)^+)-G^S(L_s-L_d)\Big)\\
&+ \frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s-(L_d-T)^+)-G^{S+1}(L_s-L_d)\Big)\\
&- \frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-(L_d-T)^+)-G^{S+2}(L_s-L_d)\Big)\Big)\\
&+ h\Big((L_d-L_s+\frac{T}{2})\Big(G^S(L_s)-G^S(L_s-(L_d-T)^+)\Big)\\
&+ \frac{S}{\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-(L_d-T)^+)\Big)\Big)\\
&+ w\frac{T}{2}\Big(G^S(L_s)-G^S(L_s-(L_d-T)^+)\Big) \qquad (40)
\end{aligned}
$$

$$E[C_F(\Omega(S),V)]$$

$$= \begin{cases} h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\ +\frac{(L_d-L_s+T)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\ +\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big) \\ +w\Big(\frac{2T(L_d-L_s)-(L_d-L_s)^2}{2T}\Big(G^S(L_s)-G^S(L_s-L_d)\Big) \\ +\frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d)\Big) \\ -\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s)-G^{S+2}(L_s-L_d)\Big)\Big), & \text{if } L_d \leq T \\ \\ h\Big(\frac{(L_d-L_s)^2+2T(L_d-L_s)}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\ +\frac{(L_d-L_s+T)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\ +\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big) \\ +(L_d-L_s+\frac{T}{2})\Big(G^S(L_s)-G^S(L_s-L_d+T)\Big) \\ +\frac{S}{\lambda}\Big(G^{S+1}(L_s)-G^{S+1}(L_s-L_d+T)\Big)\Big) \\ +w\Big(\frac{2T(L_d-L_s)-(L_d-L_s)^2}{2T}\Big(G^S(L_s-L_d+T)-G^S(L_s-L_d)\Big) \\ +\frac{(T-L_d+L_s)S}{T\lambda}\Big(G^{S+1}(L_s-L_d+T)-G^{S+1}(L_s-L_d)\Big) \\ -\frac{S(S+1)}{2T\lambda^2}\Big(G^{S+2}(L_s-L_d+T)-G^{S+2}(L_s-L_d)\Big) \\ +\frac{T}{2}\Big(G^S(L_s)-G^S(L_s-L_d+T)\Big)\Big), & \text{otherwise} \end{cases} \tag{41}$$

Situation G

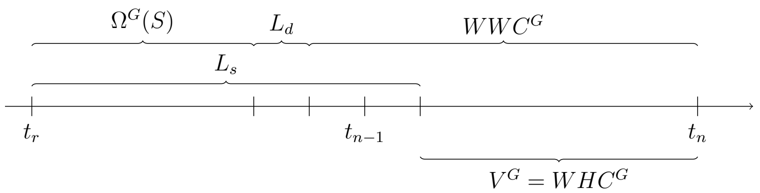

Fig. 16 Situation G

Since the unit is available after it is demanded, early-deliveries are not possible.

$$\begin{aligned} F_G(x,y) &= P(\Omega(S) \le x, V \le y, t_o \le t_a, t_o < t_d \le t_a < t_n, t_s = t_n) \\ &= P(\Omega(S) \le x, V \le y, t_r + \Omega(S) \le t_r + L_s, \\ &\quad t_r + \Omega(S) < t_r + \Omega(S) + L_d \le t_r + L_s < t_n, t_s = t_n) \\ &= P(\Omega(S) \le x, V \le y, \Omega(S) \le L_s, \Omega(S) < \Omega(S) + L_d \le L_s) \end{aligned} \tag{42}$$

For $x \le L_s - L_d$ we get

$$\begin{aligned} F_G(x,y) &= P(\Omega(S) \le x, V \le y, \Omega(S) \le L_s, \Omega(S) < \Omega(S) + L_d \le L_s) \\ &= P(\Omega(S) \le x, V \le y) \\ &= G^S(x)U(y) \end{aligned} \tag{43}$$

Thus, the joint density is given as

$$f_G(x,y) = \begin{cases} g^S(x)u(y) & , \quad x \le L_s - L_d, y \le T \\ 0 & , \quad \text{otherwise} \end{cases} \tag{44}$$

$$\begin{aligned} &E[C_G(\Omega(S), V)] \\ &= \int_0^\infty \int_0^T C_G(x,y) f_G(x,y)\, dy\, dx \\ &= \int_0^{L_s - L_d} \int_0^T \Big(hy + w(y + L_s - L_d - x)\Big) g^S(x)u(y)\, dy\, dx \\ &= \int_0^{L_s - L_d} \int_0^T \Big((h+w)y + w(L_s - L_d) - wx\Big) g^S(x)u(y)\, dy\, dx \\ &= \int_0^{L_s - L_d} (h+w)\frac{T}{2} g^S(x) + w(L_s - L_d) g^S(x) - w\frac{S}{\lambda} g^{S+1}(x)\, dy\, dx \\ &= h\frac{T}{2} G^S(L_s - L_d) + w\Big((L_s - L_d + \frac{T}{2}) G^S(L_s - L_d) - \frac{S}{\lambda} G^{S+1}(L_s - L_d)\Big) \end{aligned} \tag{45}$$

Part 3: Derivations of the expected inventory cost when $S \le 0$

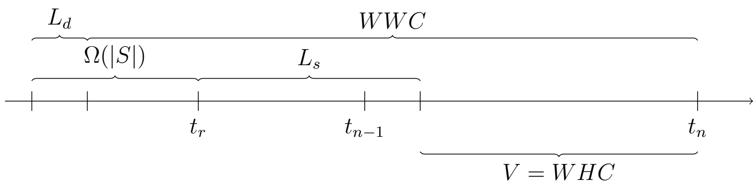

Fig. 17 Time points when $S \le 0$

The expected inventory cost per unit when $S \leq 0$ comply with the expected warehouse holding cost per unit in situation G when $S > 0$ because in both cases the unit will always be first demanded and then it is available. If $S = 0$, the unit is ordered at the same time point where the facility order arrives, whereas $S < 0$ means that we order the considered unit after the next $|S|$ facility orders are placed. The expected inventory cost per unit are derived below. Early-deliveries are not possible. The time points when the unit is ordered, demand or available can be described as: $t_o = t_r - \Omega(|S|) < t_d = t_r - \Omega(|S|) + L_d \leq t_a = t_r + L_s$.

$$\begin{aligned}\tilde{F}(x,y) &= P(\Omega(|S|) \leq x, V \leq y, t_o < t_d \leq t_a)\\ &= P(\Omega(|S|) \leq x, V \leq y, t_r - \Omega(|S|) < t_r - \Omega(|S|) + L_d \leq t_r + L_s)\\ &= P(\Omega(|S|) \leq x, V \leq y, 0 < L_d \leq \Omega(|S|) + L_s)\end{aligned} \tag{46}$$

For $x \leq \infty$ we get

$$\begin{aligned}\tilde{F}(x,y) &= P(\Omega(|S|) \leq x, V \leq y, 0 < L_d \leq \Omega(|S|) + L_s)\\ &= P(\Omega(|S|) \leq x, V \leq y)\\ &= G^{|S|}(x)U(y)\end{aligned} \tag{47}$$

Thus, the joint density is given as

$$\tilde{f}(x,y) = \begin{cases} g^{|S|}(x)u(y) & , \quad x \leq \infty, y \leq T \\ 0 & , \quad \text{otherwise} \end{cases} \tag{48}$$

During V time units the warehouse has to keep stock on hand and during an expected time interval of $V + L_s - L_d + \Omega(|S|)$ waiting costs occur, why we get the following equations.

$$\begin{aligned}E[\tilde{C}(\Omega(S),V)] &= \int_0^\infty \int_0^T \tilde{C}(x,y)\tilde{f}(x,y)\; dy\; dx\\ &= \int_0^\infty \int_0^T \Big(hy + w(y + L_s - L_d + x)\Big) g^{|S|}(x)u(y)\; dy\; dx\\ &= \int_0^\infty \int_0^T \Big((h+w)y + w(L_s - L_d) + wx\Big) g^{|S|}(x)u(y)\; dy\; dx\\ &= \int_0^\infty (h+w)\frac{T}{2} g^{|S|}(x) + w(L_s - L_d)g^{|S|}(x) + w\frac{|S|}{\lambda} g^{S+1}(x)\; dy\; dx\\ &= h\frac{T}{2}G^{|S|}(\infty) + w\Big((L_s - L_d + \frac{T}{2})G^{|S|}(\infty) + \frac{|S|}{\lambda}G^{S+1}(\infty)\Big)\\ &= h\frac{T}{2} + w\Big(\frac{|S|}{\lambda} + L_s - L_d + \frac{T}{2}\Big)\end{aligned} \tag{49}$$

Part 4: Total cost of the system when flexible deliveries are not allowed

For our numerical study we need to calculate the shipment costs and inventory cost when ADI is available but flexible deliveries are not allowed. In that case, the shipment can be obtained in a similar way as shown in **?**. Consider that advance orders can only be shipped after they are due. When computing the expected inventory cost, there is no decision about early shipments. Therefore, we do not have to distinguish between all 7 situations as before, we only distinguish between situation A ($t_a \leq t_o$) and situation G ($t_o < t_a$). This yields inventory cost at the warehouse of

$$\begin{aligned} E[C_A(\Omega(S))] = h\Big(\Big(L_d - L_s + \frac{T}{2}\Big)\Big(1 - G^S(L_s - L_d)\Big) + \frac{S}{\lambda}\Big(1 - G^{S+1}(L_s - L_d)\Big)\Big) \\ + w\frac{T}{2}\Big(1 - G^S(L_s - L_d)\Big) \end{aligned} \tag{50}$$

$$\begin{aligned} E[C_G(\Omega(S))] = h\frac{T}{2}G^S(L_s - L_d) \\ + w\Big(\Big(L_s - L_d + \frac{T}{2}\Big)G^S(L_s - L_d) - \frac{S}{\lambda}G^{S+1}(L_s - L_d)\Big) \end{aligned} \tag{51}$$