Measures of distance used for the comparison of estimated and true vectors of changes

(online appendix to "Structural decomposition analysis with disaggregate factors within the Leontief inverse")

Measure	Abbreviation	Formula	Reference
Mean absolute difference	MAD	$\frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{est} - x_{ij}^{true} \rvert$	Lahr (2001)
Mean absolute percentage error	MAPE	$\frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\lvert x_{ij}^{true} \rvert} \times 100$	Temurshoev et al. (2011)
Mean absolute scaled error	MASE	$\sum_{j=1}^{J} \sum_{i=1}^{I} \frac{\lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} - \overline{x^{true}} \rvert}$	Valderas-Jaramillo et al. (2019)
Mean squared deviation	MSD	$\frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{est} - x_{ij}^{true})^2$	Steen-Olsen et al. (2016)
Root mean squared error	RMSE	$\frac{1}{I \times J} \left(\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{est} - x_{ij}^{true})^2 \right)^{\frac{1}{2}}$	Lahr (2001)
Standardized weighted absolute difference	SWAD	$\frac{\sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} \rvert \times \lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{true})^2} \times 100$	Lahr (2001)
Symmetric absolute mean percentage error	SWAPE	$\sum_{j=1}^{J} \sum_{i=1}^{I} \left(\frac{\lvert x_{ij}^{true} \rvert}{\sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} \rvert} \right) \left\lvert \frac{x_{ij}^{est} + x_{ij}^{true}}{x_{ij}^{est} - x_{ij}^{true}} \right\rvert \times 100$	Valderas-Jaramillo et al. (2019)
Theil's U statistic (square root of Theil's U index of inequality)	Theil	$\left(\frac{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{est} - x_{ij}^{true})^2}{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{true})^2} \right)^{\frac{1}{2}} \times 100$	Lahr (2001)
Weighted absolute difference	WAD	$\frac{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{est} + x_{ij}^{true}) \times \lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{est} + x_{ij}^{true})}$	Lahr (2001)
Weighted average percentage error	WAPE	$\sum_{j=1}^{J} \sum_{i=1}^{I} \left(\frac{\lvert x_{ij}^{true} \rvert}{\sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} \rvert} \right) \frac{\lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\lvert x_{ij}^{true} \rvert} \times 100$	Temurshoev et al. (2011)
Weighted absolute scaled error	WASE	$\sum_{j=1}^{J} \sum_{i=1}^{I} \left(\frac{\lvert x_{ij}^{true} \rvert}{\sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} \rvert} \right) \frac{\lvert x_{ij}^{est} - x_{ij}^{true} \rvert}{\frac{1}{I \times J} \sum_{j=1}^{J} \sum_{i=1}^{I} \lvert x_{ij}^{true} - \overline{x^{true}} \rvert}$	Valderas-Jaramillo et al. (2019)
Weighted average square percentage error	WASPE	$\sum_{j=1}^{J} \sum_{i=1}^{I} \left(\frac{(x_{ij}^{true})^2}{\sum_{j=1}^{J} \sum_{i=1}^{I} (x_{ij}^{true})^2} \right) \frac{(x_{ij}^{est} - x_{ij}^{true})^2}{(x_{ij}^{true})^2} \times 100$	Rueda-Cantuche et al. (2018)

Note: x_{ij}^{est} is the i, j-th element in the estimated matrix and x_{ij}^{true} is the respective element in the true matrix. I and J are the matrix dimensions.

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