

Appendix 1 Derivation of constraints for force distribution with designed $(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$

Appendix 1.1 Derivation of Eqs. 13 and 14

Because the kinematic motion $((\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}))$ is given at each time step, Eq. 12 can be represented as follows:

$$\boldsymbol{\tau}_o - J_L \mathbf{f} = \boldsymbol{\tau}, \quad (\text{A1.1})$$

where $\boldsymbol{\tau}_o = M_L(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}_L(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_L(\mathbf{q})$. Then, we can derive Eq. 14.

By substituting Eq. 14 for Eq. 11, we can get Eq. 13 as follows:

$$\mathbf{b} = A\boldsymbol{\tau}, \quad (\text{A1.2})$$

where $A = J_B J_L^{-1}$, $\mathbf{b} = A\boldsymbol{\tau}_o + M_B(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}_B(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_B(\mathbf{q})$.

Appendix 1.2 Derivation of Eq. 20

The QP formulation is represented as follows for each Step i ($i = \text{A, B, C and D}$) of leg-grope-walk.

$$\hat{\boldsymbol{\tau}} = \begin{bmatrix} \boldsymbol{\tau} \\ s \end{bmatrix}_{13 \times 1}, \quad \hat{\mathbf{b}}_i = \hat{A}_i \hat{\boldsymbol{\tau}}, \quad \hat{G}_i \hat{\boldsymbol{\tau}} \leq \hat{\mathbf{d}}_i, \quad (\text{A1.3})$$

where \hat{A}_i and $\hat{\mathbf{b}}_i$ represent the equality constraints of Eq. 13 and leg-grope, \hat{G}_i and $\hat{\mathbf{d}}_i$ represent the inequality constraints. Here, we derive \hat{A}_i , $\hat{\mathbf{b}}_i$, \hat{G}_i and $\hat{\mathbf{d}}_i$ at each Step i ($i = \text{A, B, C and D}$) of leg-grope-walk.

Preparation

First, we define the rotation matrix ${}^{is}R_G$ which transforms the position vector from Σ_G to Σ_{is} . If we define ${}^s\mathbf{f} = [{}^s\mathbf{f}_1^T \ {}^s\mathbf{f}_2^T \ {}^s\mathbf{f}_3^T \ {}^s\mathbf{f}_4^T]^T \in R^{12 \times 1}$, the relation with \mathbf{f} can be written as follows:

$${}^s\mathbf{f} = {}^sC_G \mathbf{f} \quad (\text{A1.4})$$

$${}^sC_G = \begin{bmatrix} {}^{1s}R_G & 0 & 0 & 0 \\ 0 & {}^{2s}R_G & 0 & 0 \\ 0 & 0 & {}^{3s}R_G & 0 \\ 0 & 0 & 0 & {}^{4s}R_G \end{bmatrix}_{12 \times 12} \quad (\text{A1.5})$$

where, ${}^{is}\mathbf{f}_i$ represents the force vector \mathbf{f}_i on the contact coordinate frame Σ_{is}

Step A

At step A, the robot moves the COG while standing on four legs. Thus, the robot needs to fulfil the constraints (Eqs. 15, 17, 18 and 19) for all four legs.

First, the constraints for foot contact (Eq. 15) for four legs can be written by using $\boldsymbol{\tau}$ as follows:

$$B_{fz}\boldsymbol{\tau} \leq B_{fz}\boldsymbol{\tau}_o \in R^{4 \times 1}, \quad (\text{A1.6})$$

where,

$$\left\{ \begin{array}{l} B_{fz} = \begin{bmatrix} b_{fz} & 0 & 0 & 0 \\ 0 & b_{fz} & 0 & 0 \\ 0 & 0 & b_{fz} & 0 \\ 0 & 0 & 0 & b_{fz} \end{bmatrix} {}^s C_G J_L^{-1} \in R^{4 \times 12} \\ b_{fz} = [0 \ 0 \ 1] \end{array} \right. \quad (\text{A1.7})$$

Second, the constraints for slippage avoidance (Eqs. 17 and 18) for four legs can be written by using $\boldsymbol{\tau}$ as follows:

$$B_{fr} \boldsymbol{\tau} \leq B_{fr} \boldsymbol{\tau}_0 - s [1 \ 1 \ \dots \ 1]_{1 \times 16}^T \in R^{16 \times 1}, \quad (\text{A1.8})$$

where,

$$\left\{ \begin{array}{l} B_{fr} = \begin{bmatrix} b_{fr} & 0 & 0 & 0 \\ 0 & b_{fr} & 0 & 0 \\ 0 & 0 & b_{fr} & 0 \\ 0 & 0 & 0 & b_{fr} \end{bmatrix} {}^s C_G J_L^{-1} \in R^{16 \times 12} \\ b_{fr} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\mu}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\mu}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\mu}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\mu}{\sqrt{2}} \end{bmatrix} \end{array} \right. \quad (\text{A1.9})$$

Finally, the constraints for leg-grope (Eq. 19) for four legs can be written by using $\boldsymbol{\tau}$ as follows:

$$-B_{fz} \boldsymbol{\tau} \leq R_{\text{ref}} [1 \ 1 \ 1 \ 1]^T - B_{fz} \boldsymbol{\tau}_0 \in R^{4 \times 1} \quad (\text{A1.10})$$

Thus, these inequality constraints (Eqs. A1.6 , A1.8 , A1.10) and $s \geq 0$ can be written as follows by using $\hat{\boldsymbol{\tau}}$:

$$\hat{G}_A \hat{\boldsymbol{\tau}} \leq \hat{\boldsymbol{d}}_A \quad (\text{A1.11})$$

where,

$$\left\{ \begin{array}{l} \hat{G}_A = \begin{bmatrix} \hat{B}_{fz} \\ \hat{B}_{fr} \\ -\hat{B}_{fz} \\ [\mathbf{0}_{1 \times 12} \mid -1] \end{bmatrix}_{25 \times 13} \\ \hat{B}_{fz} = [B_{fz} \mid \mathbf{0}_{4 \times 1}]_{4 \times 13} \\ \hat{B}_{fr} = [B_{fr} \mid \mathbf{1}_{16 \times 1}]_{16 \times 13} \\ \hat{\boldsymbol{d}}_A = \begin{bmatrix} B_{fz} \boldsymbol{\tau}_0 \\ B_{fr} \boldsymbol{\tau}_0 \\ R_{\text{ref}} [1 \ 1 \ 1 \ 1]^T - B_{fz} \boldsymbol{\tau}_0 \\ 0 \end{bmatrix}_{25 \times 1} \end{array} \right. \quad (\text{A1.12})$$

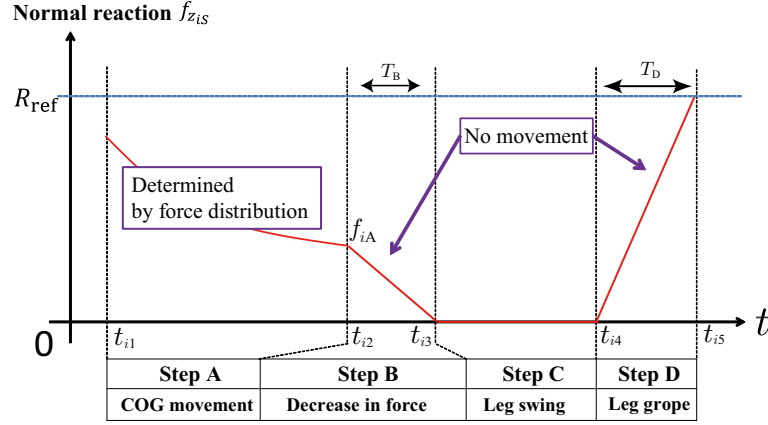


Figure A1.1: Time response of normal reaction of grope leg i . Step A: Robot moves COG standing on four legs. Step B: the robot reduces the force of the groping leg without any movement. Step C: the robot swings the groping leg to the point of the leg-grope. Step D: the robot applies the force to the ground gradually up to R_{ref} .

The equality constraints can be derived from Eq. 13 as follows:

$$\hat{\mathbf{b}}_A = \hat{A}_A \hat{\boldsymbol{\tau}}, \quad (\text{A1.13})$$

where,

$$\begin{cases} \hat{\mathbf{b}}_A = \mathbf{b} \\ \hat{A}_A = [A \mid \mathbf{0}_{6 \times 1}]_{6 \times 13} \end{cases} \quad (\text{A1.14})$$

Step B

At Step B, the robot reduces the force of the grope leg to 0 gradually standing on four legs. Thus, in addition to the equality constraints of Step A, we constrain the time response of the force on grope leg as shown in Step B of Fig. A1.1. The inequality constraint is the same as that of Step A.

We denote the time and the normal reaction of Leg i when Step A finishes as t_{i2} and f_{iA} , respectively. In addition, we set the duration of Step B as $T_B = 1\text{s}$. Then, the time response of the normal reaction on grope leg is set as following linear function.

$${}^{is}f_{iz} = f_B(t) \equiv f_{iA} - \frac{f_{iA}}{T_B}(t - t_{i2}). \quad (\text{A1.15})$$

Thus, we add the Eq.A1.15 to the equality constraints of Step A and can derive the following inequality constraint and equality constraint.

$$\hat{G}_B \hat{\boldsymbol{\tau}} \leq \hat{\mathbf{d}}_B, \quad (\text{A1.16})$$

where,

$$\begin{cases} \hat{G}_B = \hat{G}_A \\ \hat{\mathbf{d}}_B = \hat{\mathbf{d}}_A \end{cases} \quad (\text{A1.17})$$

and,

$$\hat{\mathbf{b}}_B = \hat{A}_B \hat{\boldsymbol{\tau}}, \quad (\text{A1.18})$$

where,

$$\begin{cases} \hat{\mathbf{b}}_B = \begin{bmatrix} \hat{\mathbf{b}}_A \\ f_B(t) - \mathbf{e}_{iz}^T C_G J_L^{-1} \boldsymbol{\tau}_o \end{bmatrix}_{7 \times 1} \\ \hat{A}_B = \begin{bmatrix} \hat{A}_A \\ [-\mathbf{e}_{iz}^T C_G J_L^{-1} \mid 0]_{1 \times 13} \end{bmatrix}_{7 \times 13} \end{cases} \quad (\text{A1.19})$$

where, \mathbf{e}_{iz} is defined as follows by using $\mathbf{e}_z = [001]^T$:

$$\mathbf{e}_{iz} = \begin{cases} \begin{bmatrix} \mathbf{e}_z^T \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \end{bmatrix}^T & (i = 1) \\ \begin{bmatrix} \mathbf{0}_{1 \times 3} \mathbf{e}_z^T \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \end{bmatrix}^T & (i = 2) \\ \begin{bmatrix} \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \mathbf{e}_z^T \mathbf{0}_{1 \times 3} \end{bmatrix}^T & (i = 3) \\ \begin{bmatrix} \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \mathbf{0}_{1 \times 3} \mathbf{e}_z^T \end{bmatrix}^T & (i = 4) \end{cases} \in R^{12 \times 1} \quad (\text{A1.20})$$

Step C

At Step C, the robot swings the grope leg to the point of the leg-grope. Thus, in addition to the equality constraints of Step A, we set the force on the grope leg to be 0 as shown in Step C of Fig. A1.1. The inequality constraint is the same as that of Step A. Derived inequality constraint and equality constraint are shown as follows:

$$\hat{G}_C \hat{\boldsymbol{\tau}} \leq \hat{\mathbf{d}}_C, \quad (\text{A1.21})$$

where,

$$\begin{cases} \hat{G}_C = \hat{G}_A \\ \hat{\mathbf{d}}_C = \hat{\mathbf{d}}_A \end{cases} \quad (\text{A1.22})$$

and,

$$\hat{\mathbf{b}}_C = \hat{A}_C \hat{\boldsymbol{\tau}}, \quad (\text{A1.23})$$

where,

$$\begin{cases} \hat{\mathbf{b}}_C = \begin{bmatrix} \hat{\mathbf{b}}_A \\ \boldsymbol{\tau}_{oi} \end{bmatrix}_{9 \times 1} \\ \hat{A}_C = \begin{bmatrix} \hat{A}_A \\ \hat{B}_{Ci} \end{bmatrix}_{9 \times 13} \end{cases} \quad (\text{A1.24})$$

where, \hat{B}_{Ci} is defined as follows:

$$\hat{B}_{Ci} = \begin{cases} \begin{bmatrix} I_3 \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 1} \end{bmatrix}^T & (i = 1) \\ \begin{bmatrix} \mathbf{0}_{3 \times 3} I_3 \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 1} \end{bmatrix}^T & (i = 2) \\ \begin{bmatrix} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} I_3 \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 1} \end{bmatrix}^T & (i = 3) \\ \begin{bmatrix} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} \mathbf{0}_{3 \times 3} I_3 \mathbf{0}_{3 \times 1} \end{bmatrix}^T & (i = 4) \end{cases} \in R^{3 \times 13} \quad (\text{A1.25})$$

Step D

At Step D, the robot increases the force of the grope leg to R_{ref} gradually while standing on four legs. Thus, in addition to the equality constraints of Step A, we constrain the time response of the force on grope leg as shown in Step D of Fig. A1.1. The inequality constraint is the same as that of Step A.

We denote the time when Step C finishes as t_{i4} . In addition, we set the duration of Step D as $T_D = 3\text{s}$. Then, the time response of the normal reaction on grope leg is set as following linear function.

$${}^{is}f_{iz} = f_D(t) \equiv \frac{R_{\text{ref}}}{T_D}(t - t_{i4}). \quad (\text{A1.26})$$

Thus, we add the Eq.A1.26 to the equality constraints of Step A and can derive the following inequality constraint and equality constraint.

$$\hat{G}_D \hat{\tau} \leq \hat{\mathbf{d}}_D, \quad (\text{A1.27})$$

where,

$$\begin{cases} \hat{G}_D = \hat{G}_A \\ \hat{\mathbf{d}}_D = \hat{\mathbf{d}}_A \end{cases} \quad (\text{A1.28})$$

and,

$$\hat{\mathbf{b}}_D = \hat{A}_D \hat{\tau}, \quad (\text{A1.29})$$

where,

$$\begin{cases} \hat{\mathbf{b}}_D = \begin{bmatrix} \hat{\mathbf{b}}_A \\ f_D(t) - \mathbf{e}_{iz}^{T_S} C_G J_L^{-1} \boldsymbol{\tau}_o \end{bmatrix}_{7 \times 1} \\ \hat{A}_D = \begin{bmatrix} \hat{A}_A \\ [-\mathbf{e}_{iz}^{T_S} C_G J_L^{-1} \mid 0]_{1 \times 13} \end{bmatrix}_{7 \times 13} \end{cases} \quad (\text{A1.30})$$

Appendix 2 Explanation of how to design kinematic motion for the simulation

In this appendix, we briefly explain how to design kinematic motion of the simulation. Because the robot walks on the slopes in the simulation, we assumed that the robot keeps its body parallel to the surface (Height of the body center from the surface is set as $h_b = 0.12$ [m] in this simulation). In addition, we also assumed that the body always faces toward the desired direction. Hence, the attitude of the robot (roll, pitch, yaw angles) is determined based on the inclination of the slope and the walking direction of the robot (θ and ψ). In addition, because a leg is not heavy, we assumed that the COG of the robot locates on constant position on robot coordinate frame Σ_R (We ignore the COG change induced by relative leg movements).

Appendix 2.1 Designing contact points of groping legs and COG positions

As explained in the Results and Discussion section, the grope reaction is set as $R_{\text{ref}} = \frac{1}{2}Mg \cos \theta$ depending on slope inclination θ . The robot swings its four legs L_2 , L_1 , L_3 and L_4 in sequence using the explained leg-grope walk method. Admissible regions for a groping leg and the COG are represented on $O_G - x_G y_G$ in Fig. 6 for each one-leg cycle walking. We note that these admissible regions are modified to compensate the assumption about COG based on the proposed method. We omit the explanation of this modification because this is not our main target.

To determine the contact points of groping legs and the COG positions, we also need to consider leg workspaces and conflict among the legs and bodies. In this simulation, we manually tuned the contact points and COG positions by considering these matters, because our aim is not to propose a path planing algorithm. Concretely, when the initial foot and COG positions are given as Table A2.1 and the robot walks to x axis direction, the contact points of legs and COG positions at groping are set as Table A2.1 on $O_G - x_G y_G$ plane (Fig. 6). Because the geometrical information of environment, the body attitude and the height of body are known, these positions on base coordinate frame can be transformed from these values.

Appendix 2.2 Designing kinematic motions

We briefly explain how to design the kinematic motions for an one-leg cycle walking. First, we explain how to design an one dimensional kinematic movement. Then we explain how to move legs at each step (step A-D) based on the designed one dimensional kinematic movement.

1) One dimensional kinematic movement

We design a kinematic movement ($x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$) of a object which moves from $x = 0$ to $x = L$ on one dimensional coordinate x . We assume that the object stops at $x = 0$ at $t = 0$, then it moves, and finally it stops at $x = L$. We set the maximum accerelation a_{max} and maximum velocity v_{max} as design parameters. In this case, we design the kinematic movement as Fig. A2.1 and following formulation:

1. $\pi v_{\text{max}}^2 / 2a_{\text{max}} < L$ (The case that there is a uniform motion)

Table A2.1: The Parameters for the leg-grope walking.

One-leg cycle walking	Position on $O_G - x_G y_G$ [m]				
	COG	Leg 1	Leg 2	Leg 3	Leg 4
Initial state	(0, 0)	(0.105, 0.270)	(-0.185, 0.270)	(-0.145, -0.270)	(0.145, -0.270)
Grope with L_2	(0.008, -0.010)	(0.105, 0.270)	(0.013, 0.230)	(-0.145, -0.270)	(0.145, -0.270)
Grope with L_1	(0.051, -0.030)	(0.206, 0.190)	(0.013, 0.230)	(-0.145, -0.270)	(0.145, -0.270)
Grope with L_3	(0.109, -0.016)	(0.206, 0.190)	(0.013, 0.230)	(0.063, -0.212)	(0.145, -0.270)
Grope with L_4	(0.109, 0.005)	(0.206, 0.190)	(0.013, 0.230)	(0.063, -0.212)	(0.172, -0.194)

We set the variables as follows (see Fig. A2.1)

$$t_o = \frac{\pi v_{\max}}{2a_{\max}} \quad (\text{A2.1})$$

$$T_u = \frac{L}{v_{\max}} - t_o \quad (\text{A2.2})$$

$$t_{\text{end}} = T_u + 2t_o \quad (\text{A2.3})$$

We design the kinematic movement as follows depending on time t .

(a) $0 \leq t \leq t_o$

$$\begin{cases} \ddot{x}(t) = a_{\max} \sin(t\pi/t_o) \\ \dot{x}(t) = -\frac{1}{2}v_{\max} \cos(t\pi/t_o) + \frac{1}{2}v_{\max} \\ x(t) = -\frac{v_{\max}^2}{4a_{\max}} \sin(t\pi/t_o) + \frac{1}{2}v_{\max}t \end{cases} \quad (\text{A2.4})$$

(b) $t_o < t \leq T_u + t_o$

$$\begin{cases} \ddot{x}(t) = 0 \\ \dot{x}(t) = v_{\max} \\ x(t) = -\frac{\pi v_{\max}^2}{4a_{\max}} + v_{\max}(t - t_o) \end{cases} \quad (\text{A2.5})$$

(c) $T_u + t_o < t \leq t_{\text{end}}$

$$\begin{cases} \ddot{x}(t) = a_{\max} \sin((t - T_u)\pi/t_o) \\ \dot{x}(t) = -\frac{1}{2}v_{\max} \cos((t - T_u)\pi/t_o) + \frac{1}{2}v_{\max} \\ x(t) = -\frac{v_{\max}^2}{4a_{\max}} \sin((t - T_u)\pi/t_o) + \frac{1}{2}(v_{\max}t + L) - \frac{\pi v_{\max}^2}{4a_{\max}} \end{cases} \quad (\text{A2.6})$$

2. $\pi v_{\max}^2/2a_{\max} \geq L$ (The case that there is no uniform motion)

We set the variable v_d which is the maximum velocity in this kinematic movement.

$$v_d = \sqrt{\frac{2}{\pi} a_{\max} L} \quad (\text{A2.7})$$

$$t_o = \frac{\pi v_d}{2a_{\max}} \quad (\text{A2.8})$$

$$t_{\text{end}} = 2t_o \quad (\text{A2.9})$$

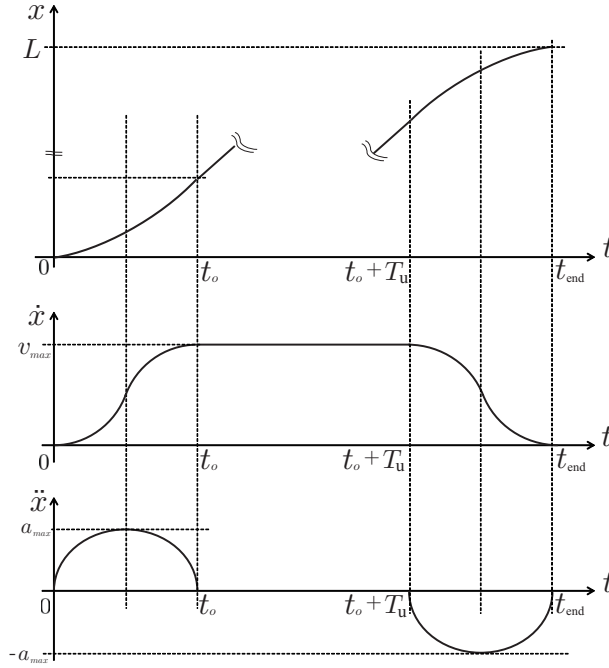


Figure A2.1: Example of one dimensional kinematic movement

$$\begin{cases} \ddot{x}(t) = a_{\max} \sin(t\pi/t_o) \\ \dot{x}(t) = -\frac{1}{2}v_d \cos(t\pi/t_o) + \frac{1}{2}v_d \\ x(t) = -\frac{L}{\pi} \sin(t\pi/t_o) + \frac{1}{2}v_d t \end{cases} \quad (\text{A2.10})$$

2) Kinematic motion of Step A

In Step A, the COG moves from the previous position to the position for groping with straight line while standing on four legs. A kinematic motion of COG is designed using the one dimensional kinematic movement (previous section), where we set the maximum accerelation $a_{\max} = 0.15$ [m/s²] and maximum velocity $v_{\max} = 0.1$ [m/s].

Because the COG position is constant on robot coordinate frame, we can transform the COG motion to the joint kinematic motion $((\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}))$ using inverse kinematics.

3) Kinematic motion of Step B

In Step B, the robot reduces the force on groping leg without movement. Thus, the robot does not move for $T_B = 1$ [s].

4) Kinematic motion of Step C

In Step C, the robot swings the groping leg to the groping position as Fig. A2.2. This step consists of three motions. First, the robot lifts up the foot of the groping leg vertically to height $h_{\text{sw}} = 0.05$ [m] using the one dimensional kinematic movement ($a_{\max} = 0.15$, $v_{\max} = 0.1$). Second, the robot moves the foot forward to the groping position while keeping the foot height constant using one dimensional kinematic movement ($a_{\max} = 0.15$, $v_{\max} = 0.1$). Finally, the robot puts down the foot vertically to the ground using one dimensional kinematic movement ($a_{\max} = 0.15$, $v_{\max} = 0.1$). We note that the robot keeps its body center position and attitude constant while these motions.

Then, the joint kinematic motions $((\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}))$ of these three motions are calculated using inverse kinematics.

5) Kinematic motion of Step D

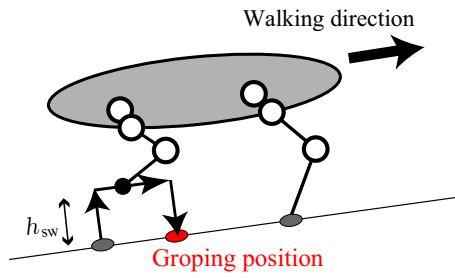


Figure A2.2: Example of the motion of Step C from side view. The robot lifts up the foot vertically to height h_{sw} , then moves foot forward parallel to the ground, finally places foot down to the groping position.

In Step D, the robot increases the force on groping leg without movement. Thus, the robot does not move for $T_D = 3$ [s].

6) Summary

A kinematic motion of the robot is calculated by repeating these kinematic movement designs for all groping legs.