1 Supplementary File

Proof of Property 1: Obviously established.

Proof of Property 2:

(1) Because $A1 \subseteq A2 \subseteq CA$, we have $NT^{\lambda}_{A2}(D_k) \subseteq NT^{\lambda}_{A1}(D_k)$, thus we can obtain $shar_{A1}(D_k) \le shar_{A2}(D_k)$ from *Definition 7*.

(2) It is similar to (1).

Proof of Property 3: (1) Because $A1 \subseteq A2$, according to *Property 2(1)*, it follows that $shar_{A1}(D_k) \le shar_{A2}(D_k)$. Then, we can obtain $\frac{shar_{A1}(D_k)}{dec(D_k)} \le \frac{shar_{A2}(D_k)}{dec(D_k)}$. By *Definition 8* we have $\theta^{(1)}_{A1}(D_k) \le \theta^{(1)}_{A2}(D_k)$.

(2) It follow from *Property 3(1)* that $\theta^{(1)}_{A1}(D_k) \le \theta^{(1)}_{A2}(D_k)$ for any $A1 \subseteq A2$, thus it can be easily obtained that $1-\theta^{(1)}_{A1}(D_k) \ge 1-\theta^{(1)}_{A2}(D_k)$. From *Definition 8*, we have $\sigma^{(1)}_{A1}(D_k) \ge \sigma^{(1)}_{A2}(D_k)$.

Proof of Property 4: It follows From *Property 3* that $\theta^{(1)}_{A1}(D_k) \le \theta^{(1)}_{A2}(D_k)$ for any $A1 \subseteq A2$. Then, we can have that $0 \le -\ln\theta^{(1)}_{A2}(D_k) \le -\ln\theta^{(1)}_{A1}(D_k)$, $0 \le \sigma^{(1)}_{A2}(D_k) \le \sigma^{(1)}_{A1}(D_k) \le 1$. Thus, we can obtain $I^1_{A1}(D_k) \ge I^1_{A2}(D_k)$.

Proof of Property 5: (1) For $A1 \subseteq A2$, it follows from *Property 2(2)* that $blun_{A1}(D_k) \ge blun_{A2}(D_k)$, therefore, we have $\frac{dec(D_k)}{blun_{A1}(D_k)} \le \frac{dec(D_k)}{blun_{A2}(D_k)}$. From *Definition 11*, we have $\theta^{(2)}_{A1}(D_k) \le \theta^{(2)}_{A2}(D_k)$.

(2) From *Property 5(1)*, it can be follow that $\theta^{(2)}_{A1}(D_k) \le \theta^{(2)}_{A2}(D_k)$ for any $A1 \subseteq A2$. So we can know that $1-\theta^{(2)}_{A1}(D_k) \ge 1-\theta^{(2)}_{A2}(D_k)$. It can be easily proved that $\sigma^{(2)}_{A1}(D_k) \ge \sigma^{(2)}_{A2}(D_k)$.

Proof of Property 6: By *Property 5(1)*, we can know that $\theta^{(2)}_{A1}(D_k) \le \theta^{(2)}_{A2}(D_k)$. Therefore, we can have that $0 \le -\ln\theta^{(2)}_{A2}(D_k) \le -\ln\theta^{(2)}_{A1}(D_k)$, $0 \le \sigma^{(2)}_{A2}(D_k) \le \sigma^{(2)}_{A1}(D_k) \le 1$. Then, we can obtain $I^2_{A1}(D_k) \ge I^2_{A2}(D_k)$.

Proof of Property 7: From *Properties 4* and *6*, we can obtain that $I^1_{A1}(D_k) \ge I^1_{A2}(D_k)$ and $I^2_{A1}(D_k) \ge I^2_{A2}(D_k)$ for any $A1 \subseteq A2$. Therefore, we can know that $I^1_{A1}(D_k) + I^2_{A1}(D_k) \ge I^1_{A2}(D_k) + I^2_{A2}(D_k)$. Hence, $I^3_{A1}(D_k) \ge I^3_{A2}(D_k)$.

Proof of Property 8: (1) For any $A1 \subseteq A2$, it follow $shar_{A1}(D_k) \le shar_{A2}(D_k)$ and $blun_{A1}(D_k) \ge blun_{A2}(D_k)$. Therefore, it can be obtained $\frac{shar_{A1}(D_k)}{blun_{A1}(D_k)} \le \frac{shar_{A2}(D_k)}{blun_{A2}(D_k)}$. According to *Definition 16*, we can know that $\theta^{(3)}_{A1}(D_k) \le \theta^{(3)}_{A2}(D_k)$. Then, $1-\theta^{(3)}_{A1}(D_k) \ge 1-\theta^{(3)}_{A2}(D_k)$. Hence, it can be obtained that $\sigma^{(3)}_{A1}(D_k) \ge \sigma^{(3)}_{A2}(D_k)$.

(2) From *Definition 8* and *Definition 11*, we have $\theta^{(1)}_{A}(D_k) = \frac{shar_A(D_k)}{dec(D_k)}$ and $\theta^{(2)}_{A}(D_k) = \frac{dec(D_k)}{blun_A(D_k)}$. Then, $\theta^{(1)}_{A}(D_k)\cdot\theta^{(2)}_{A}(D_k) = \frac{shar_A(D_k)}{dec(D_k)}\cdot\frac{dec(D_k)}{blun_A(D_k)} = \frac{shar_A(D_k)}{blun_A(D_k)}$. From *Definition 16*, we can obtain $\theta^{(3)}_{A}(D_k) = \frac{shar_A(D_k)}{blun_A(D_k)}$, thus, $\theta^{(3)}_{A}(D_k) = \theta^{(1)}_{A}(D_k)\cdot\theta^{(2)}_{A}(D_k)$.

(3)

$$\begin{aligned}
\sigma^{(3)}_A(D_k) &= 1-\theta^{(3)}_A(D_k)\\
&= 1-\theta^{(1)}_A(D_k)\cdot\theta^{(2)}_A(D_k)\\
&= 1-\left[1-\sigma^{(1)}_A(D_k)\right]\cdot\left[1-\sigma^{(2)}_A(D_k)\right]\\
&= \sigma^{(1)}_A(D_k)+\sigma^{(2)}_A(D_k)-\sigma^{(1)}_A(D_k)\cdot\sigma^{(2)}_A(D_k
\end{aligned}$$

Proof of Property 9: Suppose any $A1 \subseteq A2$, according to *Property 8(1)*, it follows that $\theta^{(3)}_{A1}(D_k) \le \theta^{(3)}_{A2}(D_k)$. Therefore, one has $0 \le -\ln\theta^{(3)}_{A2}(D_k) \le -\ln\theta^{(3)}_{A1}(D_k)$, $0 \le \sigma^{(3)}_{A2}(D_k) \le \sigma^{(3)}_{A1}(D_k) \le 1$. From *Definition 17*, $I^4_{A1}(D_k) \ge I^4_{A2}(D_k)$ can be hold.

Proof of Property 10:

$$\begin{aligned}
I^{(4)}_A(D_k) &= -\sigma^{(3)}_A(D_k)\ln\theta^{(3)}_A(D_k)\\
&= -\left[\sigma^{(1)}_A(D_k)+\sigma^{(2)}_A(D_k)-\sigma^{(1)}_A(D_k)\cdot\sigma^{(2)}_A(D_k)\right]\\
&\cdot\ln\left[\theta^{(1)}_A(D_k)+\theta^{(2)}_A(D_k)\right]\\
&= -\left[\sigma^{(1)}_A(D_k)+\sigma^{(2)}_A(D_k)-\sigma^{(1)}_A(D_k)\cdot\sigma^{(2)}_A(D_k)\right]\\
&\cdot\left[\ln\theta^{(1)}_A(D_k)+\ln\theta^{(2)}_A(D_k)\right]\\
&= -\sigma^{(1)}_A(D_k)\cdot\ln\theta^{(1)}_A(D_k)-\sigma^{(2)}_A(D_k)\cdot\ln\theta^{(2)}_A(D_k)\\
&+\left[\sigma^{(1)}_A(D_k)-1\right]\cdot\sigma^{(2)}_A(D_k)\cdot\ln\theta^{(1)}_A(D_k)\\
&+\left[\sigma^{(2)}_A(D_k)-1\right]\cdot\sigma^{(1)}_A(D_k)\cdot\ln\theta^{(2)}_A(D_k)\\
&= I^1_A(D_k)+I^2_A(D_k)+\left[\sigma^{(1)}_A(D_k)-1\right]\cdot\sigma^{(2)}_A(D_k)\\
&\cdot\ln\theta^{(1)}_A(D_k)+\left[\sigma^{(2)}_A(D_k)-1\right]\cdot\sigma^{(1)}_A(D_k)\cdot\ln\theta^{(2)}_A(D_k)
\end{aligned}$$

From *Definition 8*, it follows that $0 \le \theta^{(1)}_A(D_k), \sigma^{(1)}_A(D_k) \le 1$, then, we have $\ln\theta^{(1)}_A(D_k) \le 0$, $\sigma^{(1)}_A(D_k)-1 \le 0$. In the same way, $0 \le \theta^{(2)}_A(D_k) \le 1$ and $0 \le \sigma^{(2)}_A(D_k) \le 1$ can be obtained from *Definition 11*, then it follows that $\ln\theta^{(1)}_A(D_k) \le 0$, $\sigma^{(2)}_A(D_k)-1 \le 0$. Hence, it can be proved that:

$$\begin{cases}
\left[\sigma^{(1)}_A(D_k)-1\right]\cdot\sigma^{(2)}_A(D_k)\cdot\ln\theta^{(1)}_A(D_k) \ge 0 & \text{①}\\
\left[\sigma^{(2)}_A(D_k)-1\right]\cdot\sigma^{(1)}_A(D_k)\cdot\ln\theta^{(2)}_A(D_k) \ge 0 & \text{②}
\end{cases}$$

Therefore, $\varphi = \text{①} + \text{②} \ge 0$.

$$\begin{aligned}
I^4_A(D_k) &= I^1_A(D_k)+I^2_A(D_k)+\varphi\\
&= I^3_A(D_k)+\varphi\\
&\ge I^3_A(D_k)
\end{aligned}$$

Proof of Property 11: It is straightforward from *Property 9*.