Supplementary Information for A loop enhancement strategy for network robustness

M. Chujyo[*] and Y. Hayashi[†]

Appendix

For 10 real networks shown in Table S1, we apply edge-rewiring methods: our proposed methods (BP), a method with smaller degrees instead of smaller q_i^0 of BP (Degree), SP [1], and WuHolme [2]. As a preprocessing, we transform the network data into undirected, unweighted, and no self-loop and no multiple edges, and extract the giant component. In the Figures, we compare the effects on the robustness index R_{hub} [3], the approximate size of FVS by Ref [4], and the degree-degree correlations r [5] versus the number of rewiring. In addition, to show the modifications in the degrees by rewiring in Non-Preserving, we show the gap between the maximum and minimum degrees versus the number of rewiring in original and after rewiring networks.

The robustness and the size of FVS are more strongly related to each other than the degree-degree correlations. There is an exception for Power Grid shown in Figs. S5ac. BP Preserving (denoted by the green line at the left) increases the robustness over the baseline but decreases the size of FVS. We consider that it is caused by a special property of Power Grid with a smaller average degree and maximum degree, but a larger diameter shown in Table S1.

*mchujyo@jaist.ac.jp
†yhayashi@jaist.ac.jp

Network	N	M	r	Min k	$< k >$	Max k	D	Figs.	Refs.	URL
AirTraffic	1226	2408	-0.015	1	3.9	34	17	S1, S2	[6]	url
E-mail	1133	5451	0.078	1	9.6	71	8	S3, S4	[7]	url
PowerGrid	4941	6594	0.003	1	2.7	19	46	S5, S6	[8]	url
Yeast	2224	6609	-0.105	1	5.9	64	11	S7, S8	[9]	url
Japanese	2698	7995	-0.259	1	5.9	725	8	S9, S10	[10]	url
Hamster	1788	12476	-0.089	1	14.0	272	14	S11, S12	[6]	url
GRQC	4158	13422	0.639	1	6.5	81	17	S13, S14	[11]	url
UCIrvine	1893	13835	-0.188	1	14.6	255	8	S15, S16	[6, 12]	url
OpenFlights	2905	15645	0.049	1	10.8	242	14	S17, S18	[6, 13]	url
Polblogs	1222	16714	-0.221	1	27.4	351	8	S19, S20	[14]	url

Table S1: Basic properties for real networks after preprocessing. From the left, we note the name of the network, the number of nodes, the number of edges, the degree-degree correlations, the minimum degree, the average degree, the maximum degree, the diameter, figures, references, and available URL to download the data.

AirTraffic

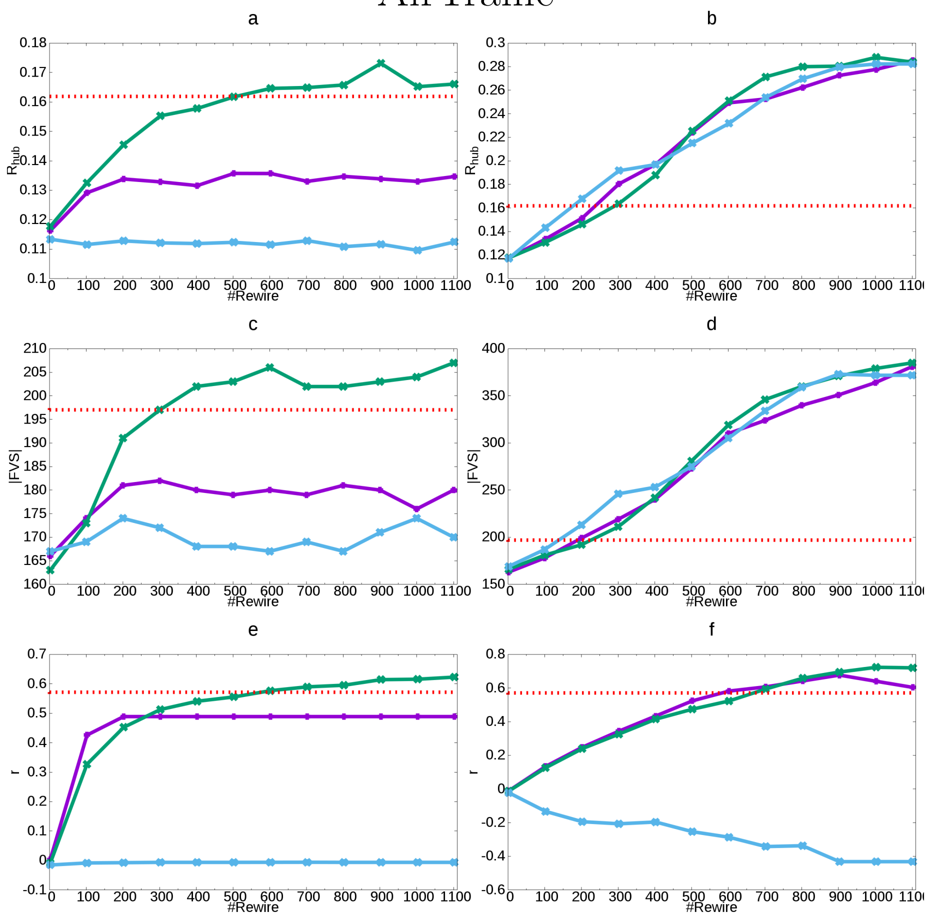

Figure S1: AirTraffic [6]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

AirTraffic

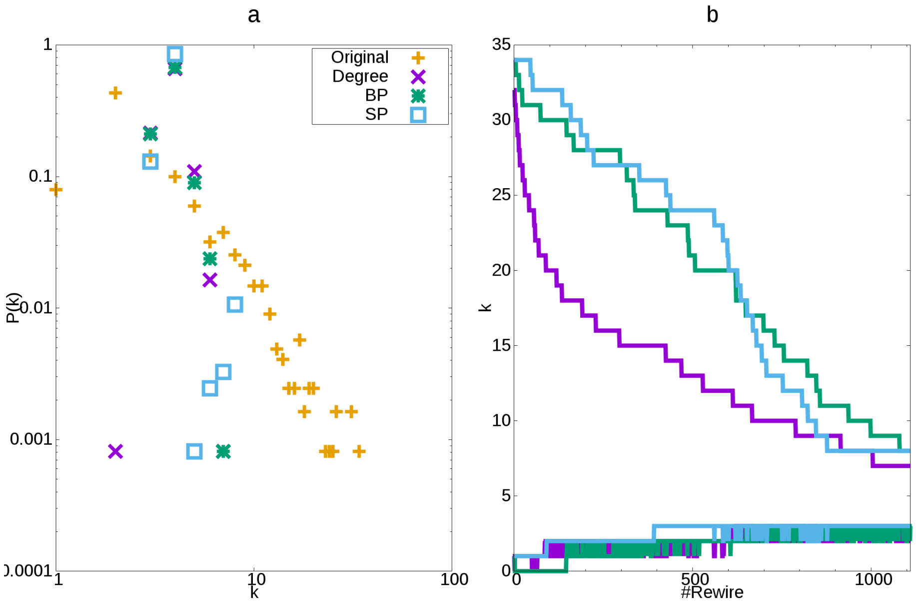

Figure S2: AirTraffic [6]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

E-mail

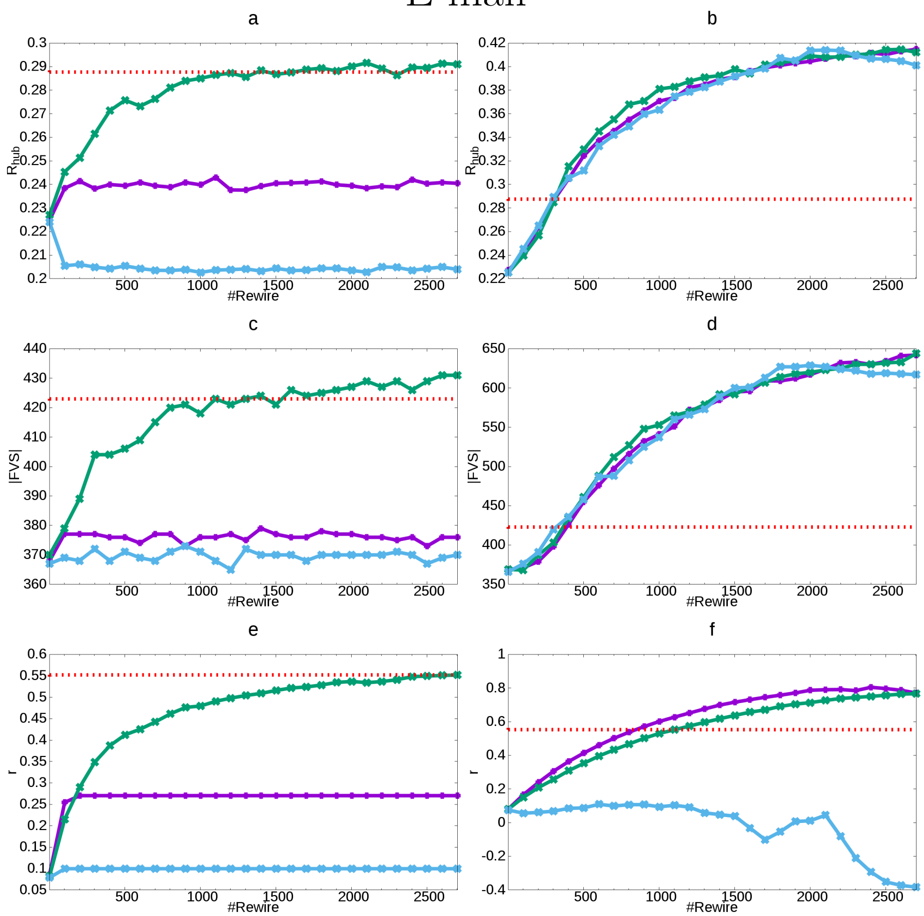

Figure S3: E-mail [7]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

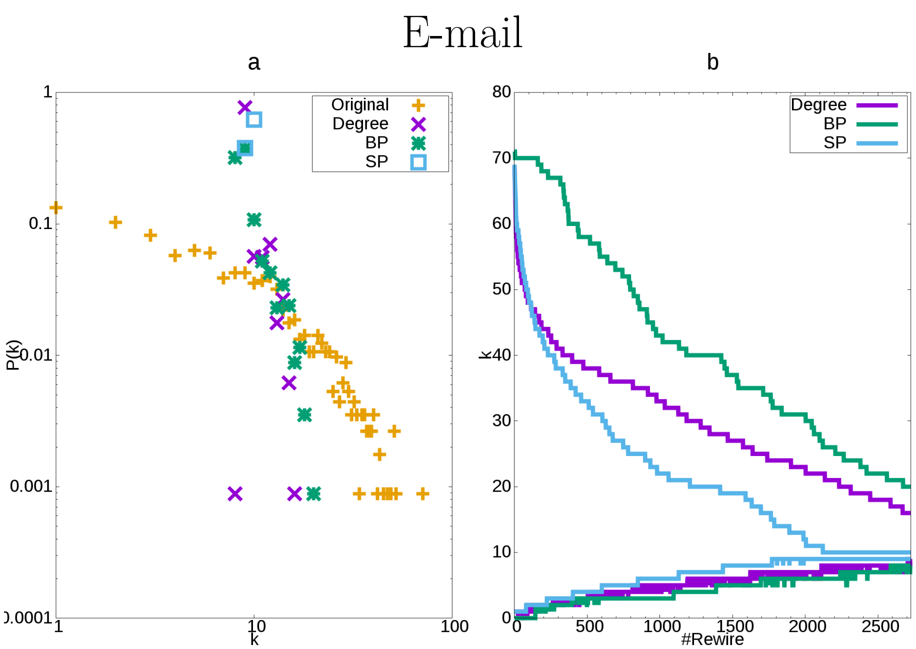

Figure S4: E-mail [7]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

Power Grid

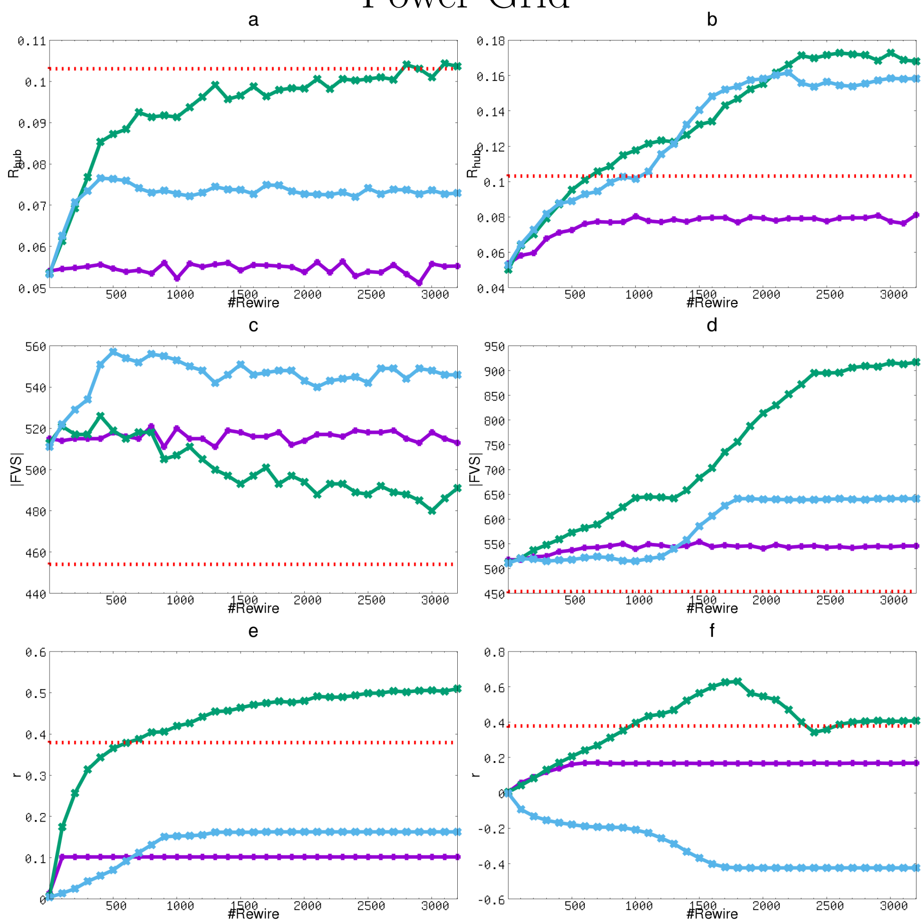

Figure S5: Power Grid [8]. Comparison of the robustness index $R_{\rm hub}$, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

Power Grid

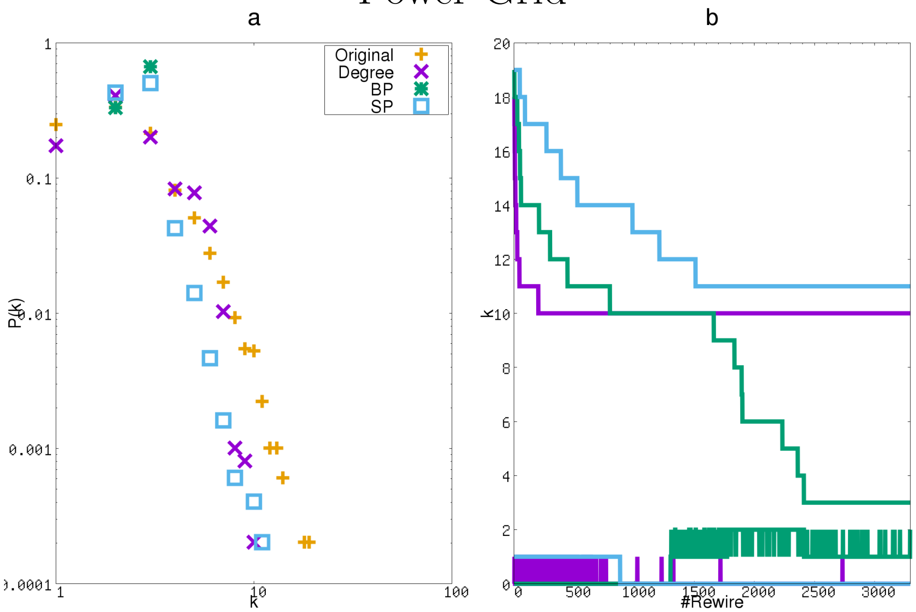

Figure S6: Power Grid [8]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

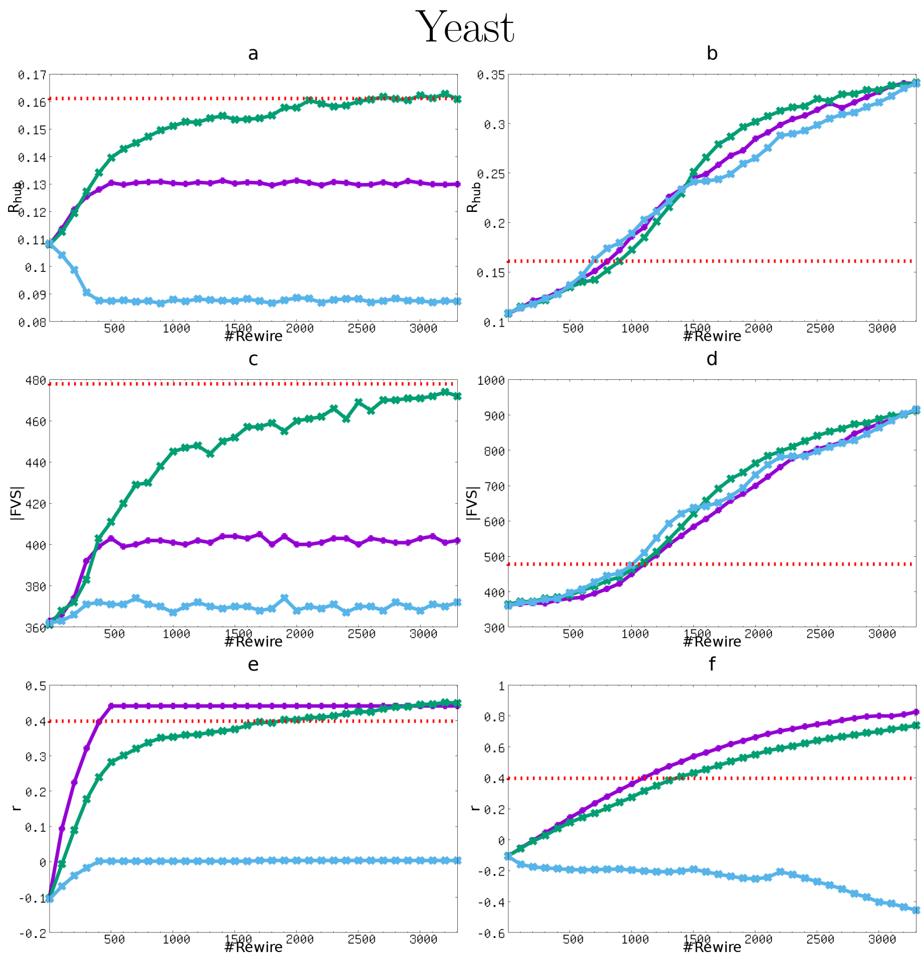

Figure S7: Yeast [9]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

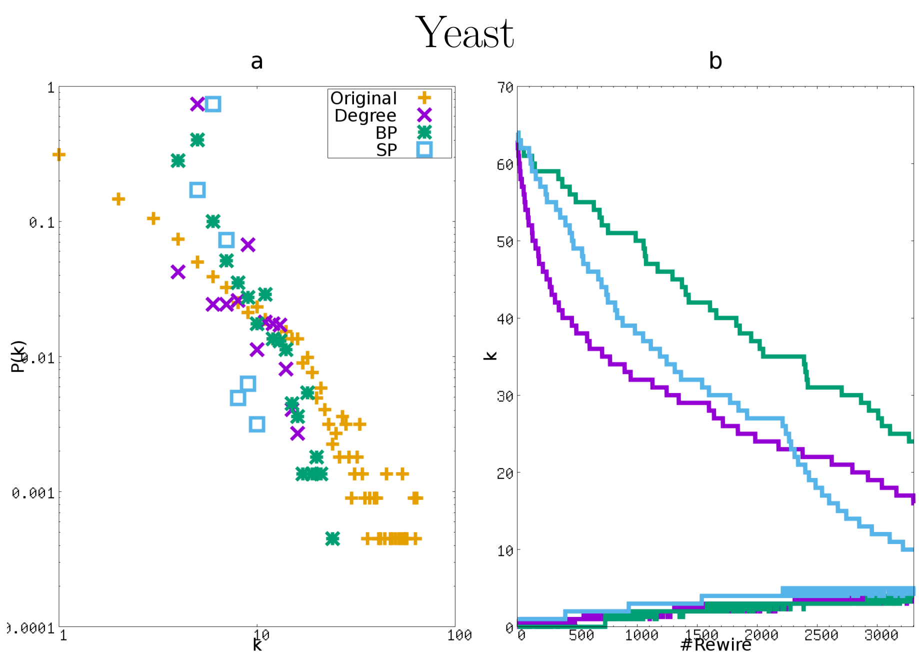

Figure S8: Yeast [9]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

Japanese

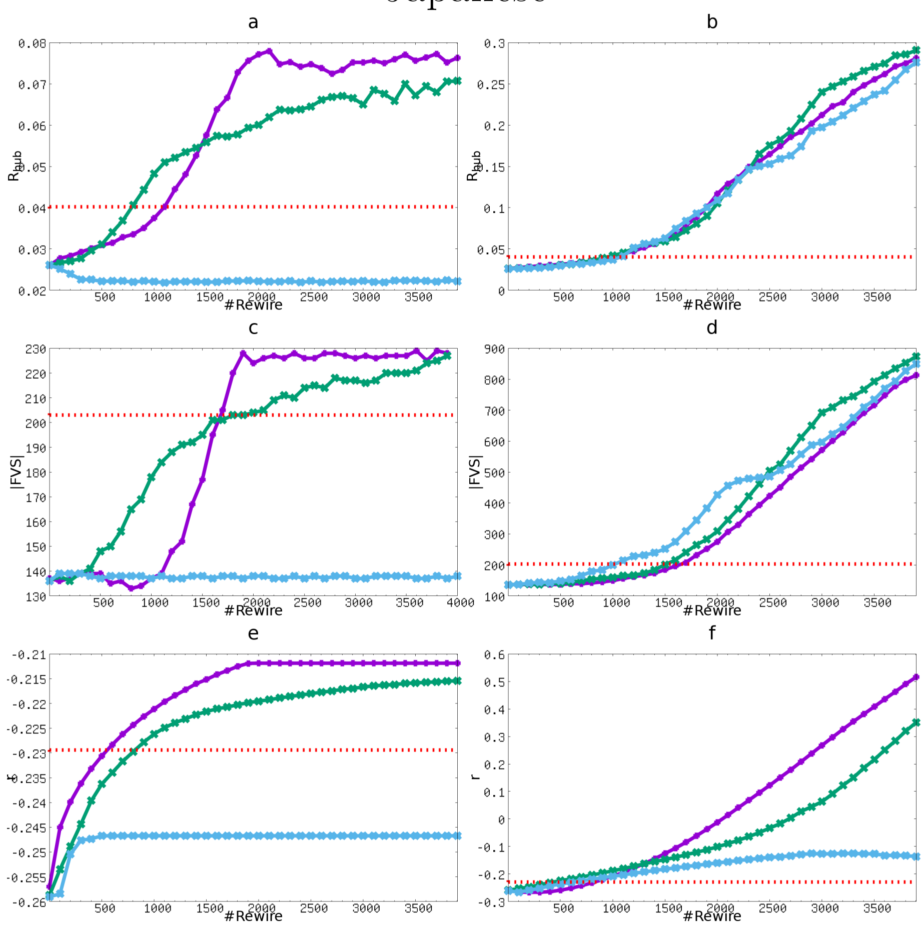

Figure S9: Japanese [10]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

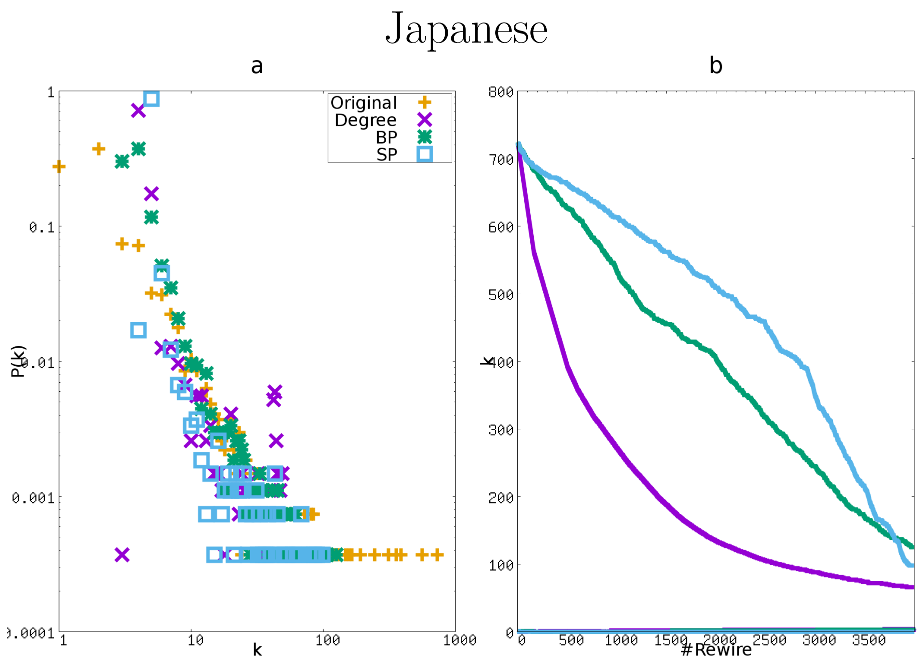

Figure S10: Japanese [10]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

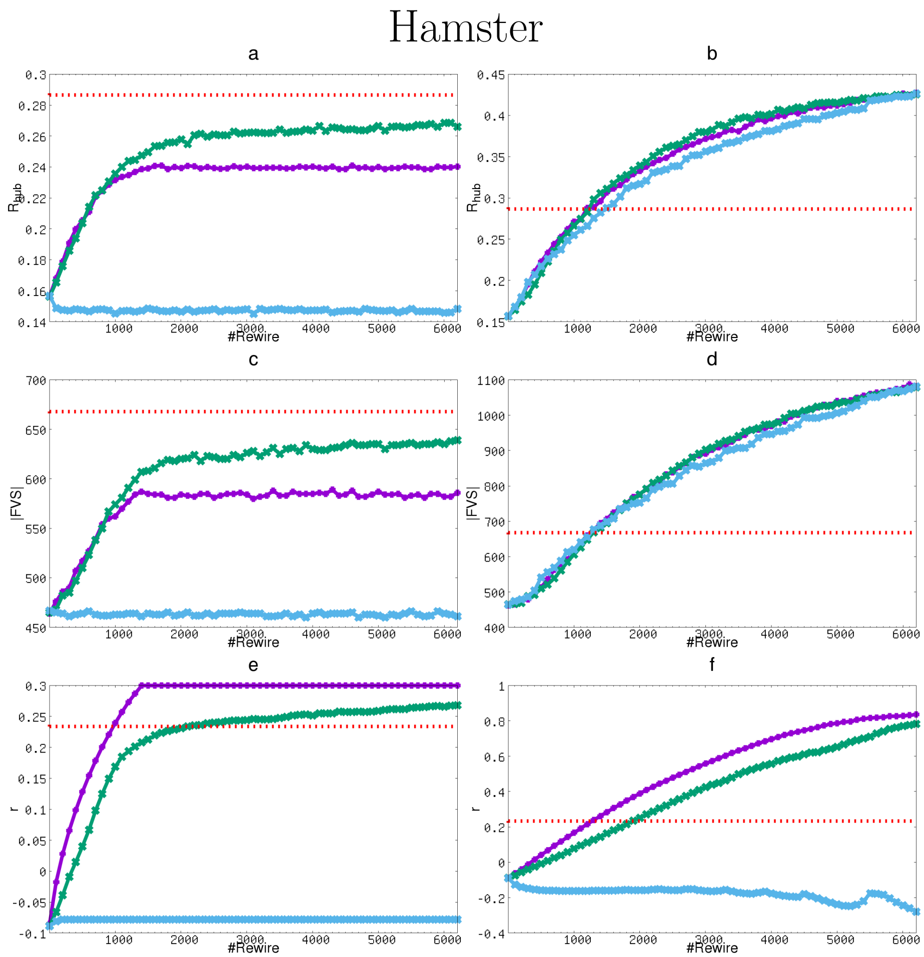

Figure S11: Hamster [6]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

Hamster

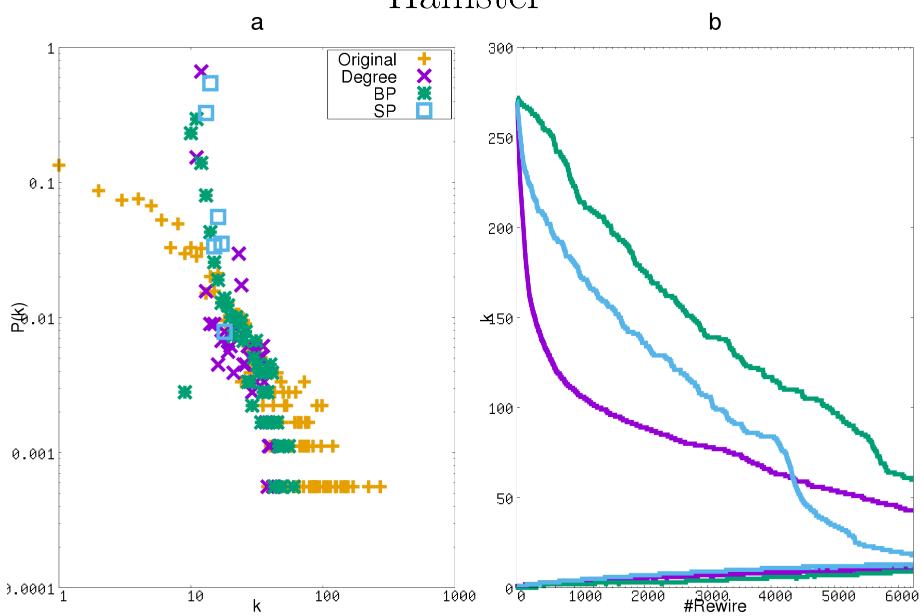

Figure S12: Hamster [6]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

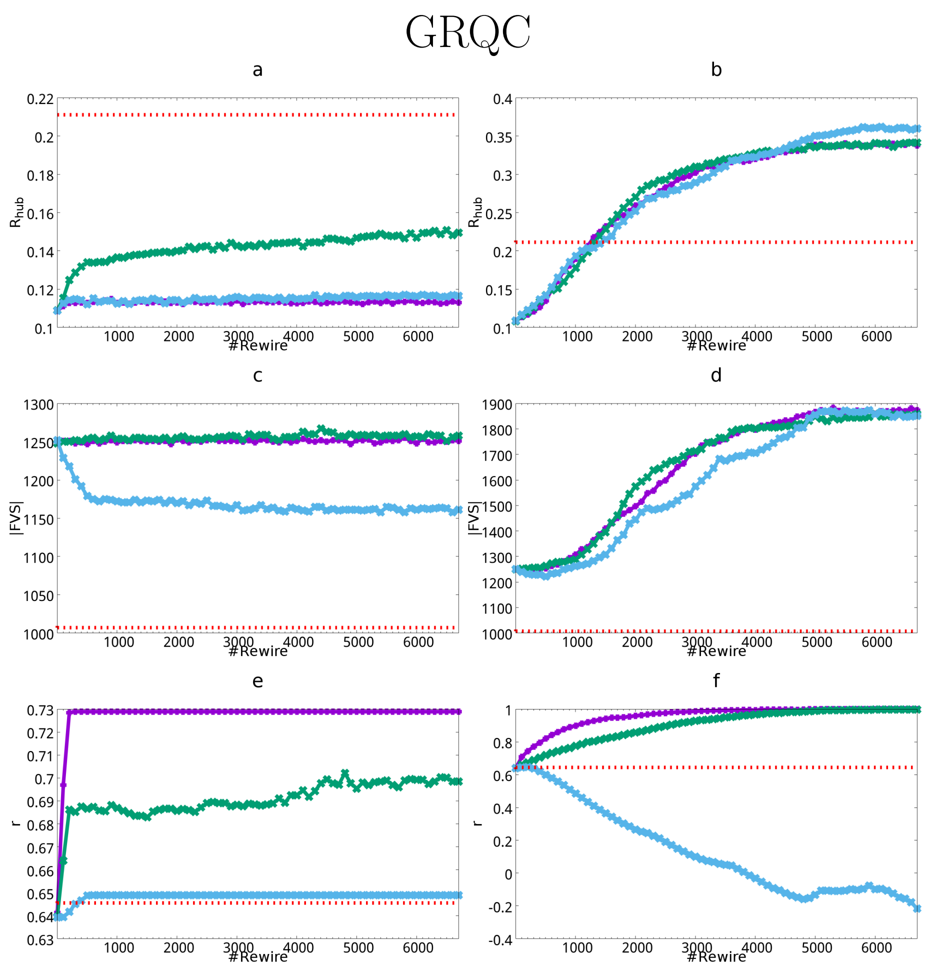

Figure S13: GRQC [11]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

GRQC

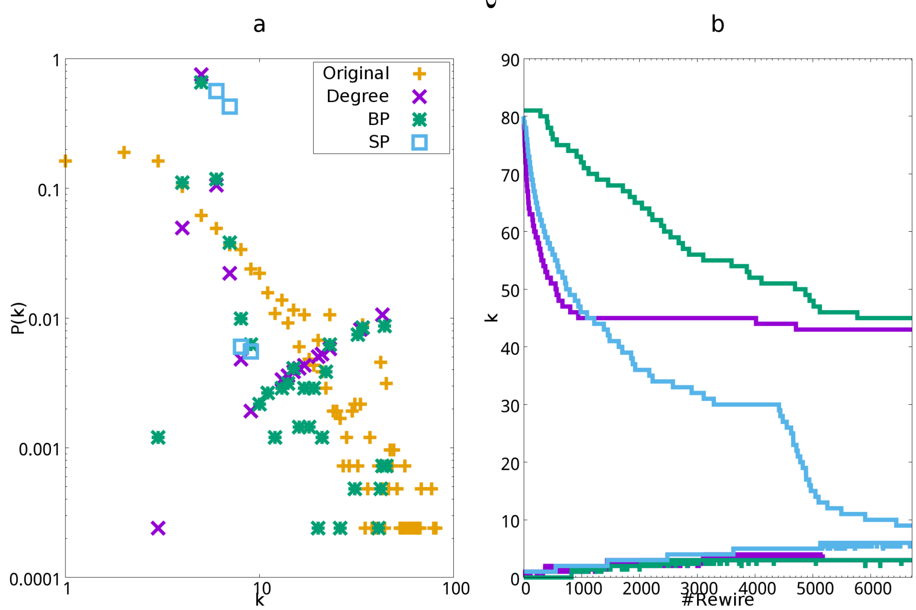

Figure S14: GRQC [11]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

UCIrvine

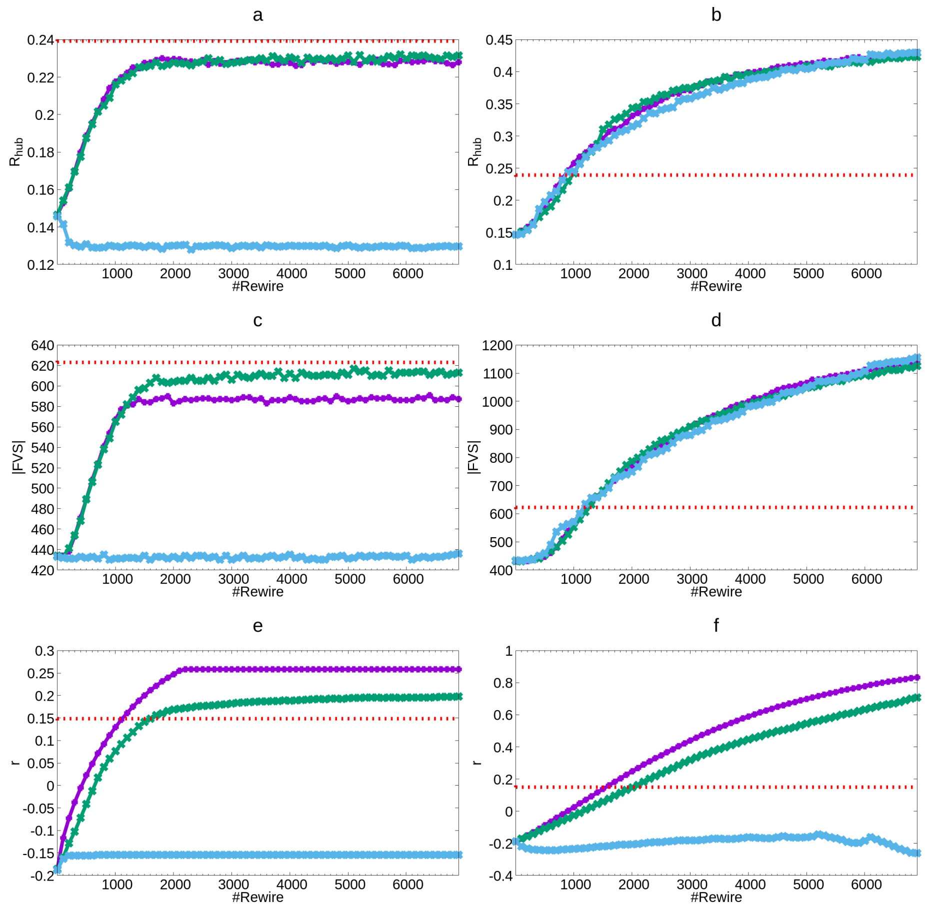

Figure S15: UCIrvine [6, 12]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

UCIrvine

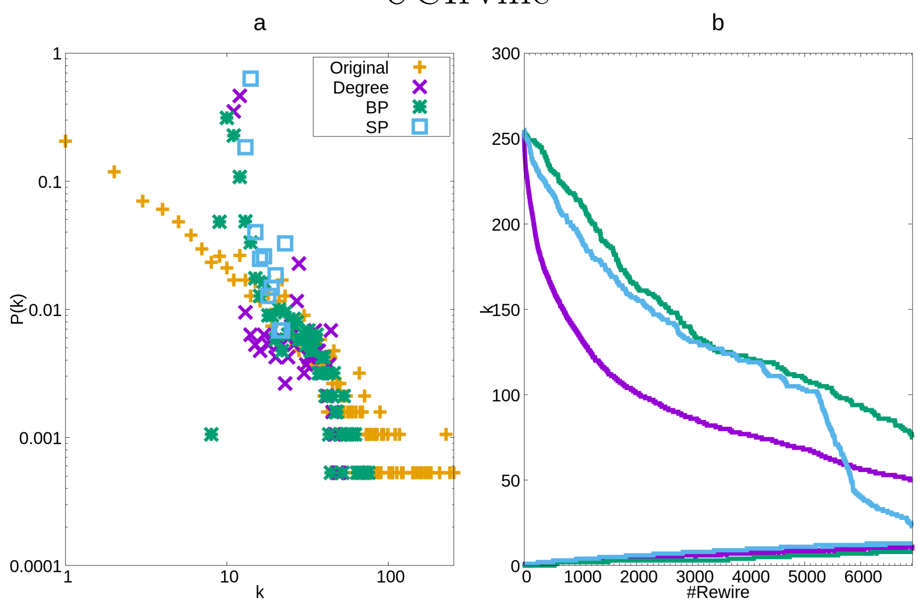

Figure S16: UCIrvine [6, 12]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

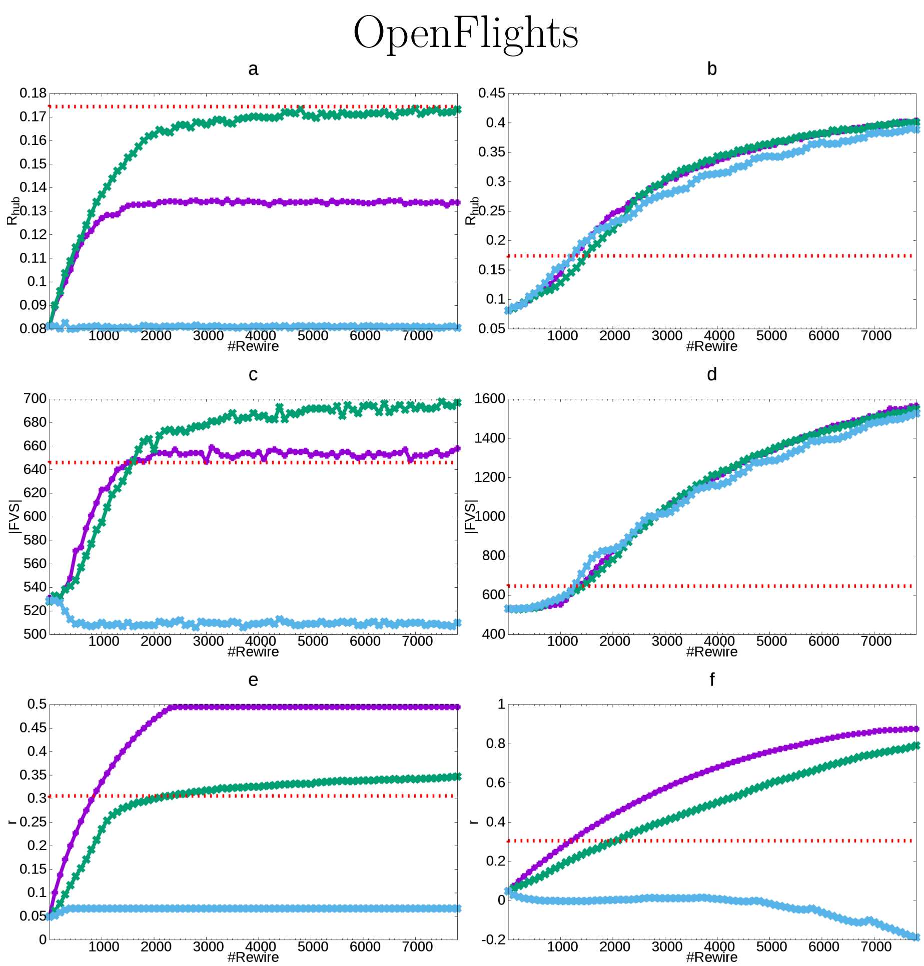

Figure S17: OpenFlights [6, 13]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

OpenFlights

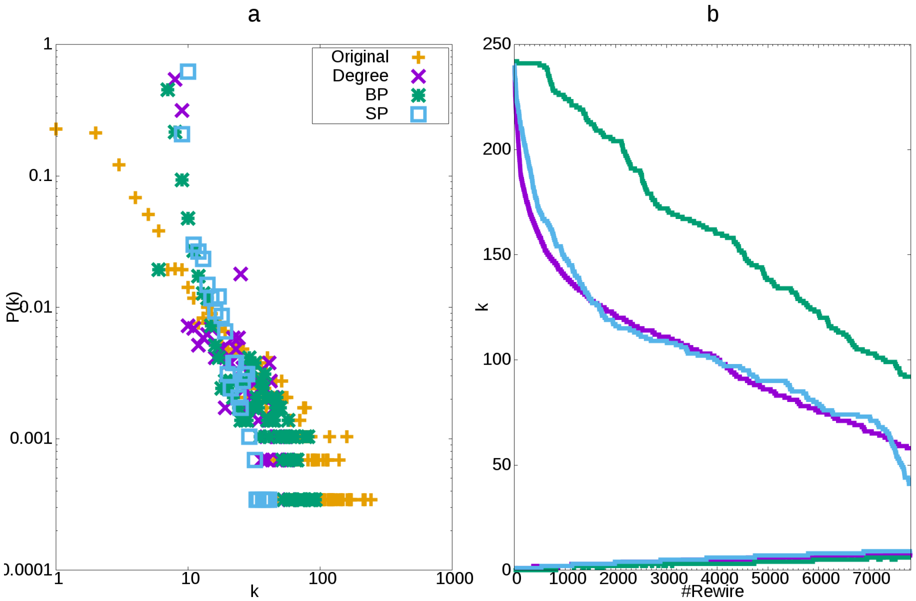

Figure S18: OpenFlights [6, 13]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

PolBlogs

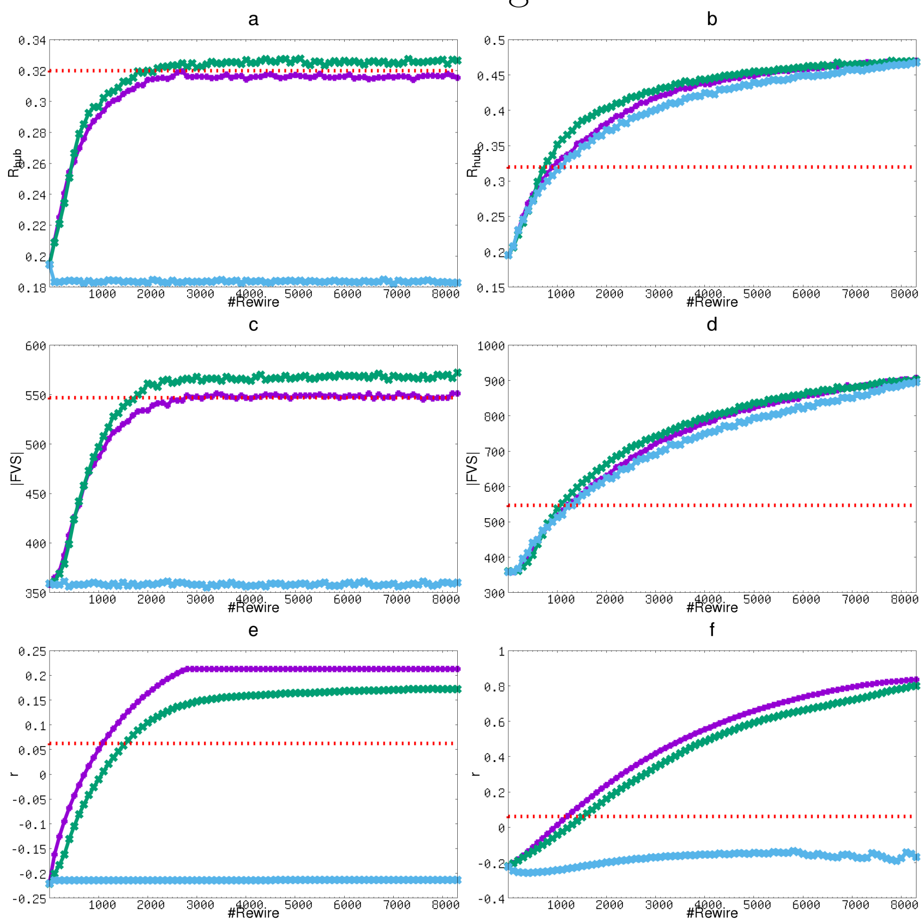

Figure S19: PolBlogs [14]. Comparison of the robustness index R_{hub}, the size approximate of FVS, and the degree-degree correlation coefficient r vs. the number of rewiring. (Left: a, c, e) Rewirings in Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Preserving, respectively. The red dot line indicates a baseline of the conventional best. (Right: b, d, f) Rewirings in Non-Preserving. Violet, green, and light blue solid lines denote the result by Degree, BP, and SP Non-Preserving, respectively.

PolBlogs

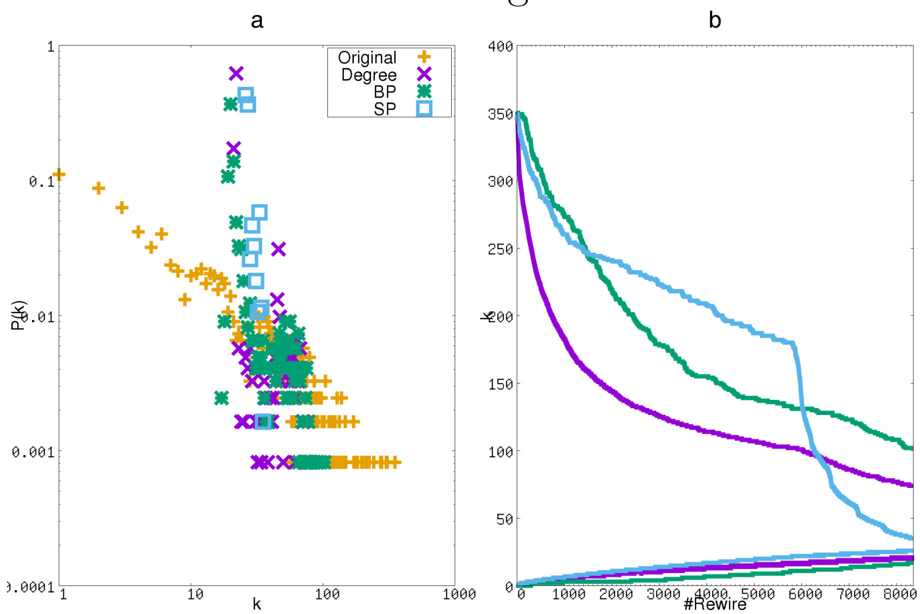

Figure S20: PolBlogs [14]. (a) Degree distributions in original and after rewiring networks, (b) Maximum and minimum degrees vs. the number of rewiring in Degree, BP, and SP Non-Preserving. The above three lines show the maximum degrees. The below three lines show the minimum degrees. Violet, green, and light blue denote Degree, BP, SP Non-Preserving. Orange denotes the original degree distribution.

References

[1] Chan, H., Akoglu, L.: Optimizing network robustness by edge rewiring: a general framework. Data Mining and Knowledge Discovery **30**(5), 1395–1425 (2016)

[2] Wu, Z.-X., Holme, P.: Onion structure and network robustness. Physical Review E **84**(2), 026106 (2011)

[3] Schneider, C.M., Moreira, A.A., Andrade, J.S., Havlin, S., Herrmann, H.J.: Mitigation of malicious attacks on networks. Proceedings of the National Academy of Sciences **108**(10), 3838–3841 (2011)

[4] Zhou, H.-J.: Spin glass approach to the feedback vertex set problem. European Physical Journal B **86**(11), 455 (2013)

[5] Newman, M.E.: Assortative mixing in networks. Physical Review Letters **89**(20), 208701 (2002)

[6] Kunegis, J.: Konect: the koblenz network collection. In: Proceedings of the 22nd International Conference on World Wide Web, pp. 1343–1350 (2013)

[7] Guimera, R., Danon, L., Diaz-Guilera, A., Giralt, F., Arenas, A.: Self-similar community structure in a network of human interactions. Physical Review E **68**(6), 065103 (2003)

[8] Watts, D.J., Strogatz, S.H.: Collective dynamics of small-worldnetworks. Nature **393**(6684), 440 (1998)

[9] Bu, D., Zhao, Y., Cai, L., Xue, H., Zhu, X., Lu, H., Zhang, J., Sun, S., Ling, L., Zhang, N., *et al.*: Topological structure analysis of the protein–protein interaction network in budding yeast. Nucleic Acids Research **31**(9), 2443–2450 (2003)

[10] Milo, R., Itzkovitz, S., Kashtan, N., Levitt, R., Shen-Orr, S., Ayzenshtat, I., Sheffer, M., Alon, U.: Superfamilies of evolved and designed networks. Science **303**(5663), 1538–1542 (2004)

[11] Leskovec, J., Kleinberg, J., Faloutsos, C.: Graph evolution: Densification and shrinking diameters. ACM Transactions on Knowledge Discovery from Data (TKDD) **1**(1), 2 (2007)

[12] Opsahl, T., Panzarasa, P.: Clustering in weighted networks. Social Networks **31**(2), 155–163 (2009)

[13] Opsahl, T., Agneessens, F., Skvoretz, J.: Node centrality in weighted networks: Generalizing degree and shortest paths. Social Networks **32**(3), 245–251 (2010)

[14] Adamic, L.A., Glance, N.: The political blogosphere and the 2004 us election: divided they blog. In: Proceedings of the 3rd International Workshop on Link Discovery, pp. 36–43 (2005)