Supplementary Materials
Modeling Algorithmic Bias: Simplicial Complexes and Evolving Network Topologies

In the Supplementary Material, we show additional figures for the average number of clusters and the average number of iterations at convergence with standard deviation values for the two models introduced in the present work.

1 Average number of clusters

1.1 AAB model

We can see from fig. 1 and fig. 2 the average number of clusters in the steady-state for the AAB model for both the Erdos-Renyii and Barabasi-Albert initial configuration. Conclusions remarked in the text are still valid for $\epsilon = 0.4$ for which the population always converges to consensus until $\gamma \geq 1.6$. From plots in 1 and 2, it is clear how the rewiring process does not determine the dynamics since the number of clusters is stable as p_r grows. From fig. 1 (d)-(f) and fig. 2 (d)-(f) we can see how, in the Barabasi-Albert network, the average number of clusters in the final state is more variable with respect to the Erdos-Renyii network and that such variability increases with the probability of rewiring, as well as the overall average number of clusters. In the Erdos-Renyii network, the dynamics and the average number of clusters are more stable.

1.2 AABSC model

In fig. 3 (a)-(c) and 4 (a)-(c) we can see the average number of clusters in the AABSC model in both Erdos-Renyii and Barabasi-Albert initial configuration. We can observe from fig. 3 that, starting from the Erdos-Renyii configuration, a consensus is always reached with the AABSC model, except for a few parameter combinations. We can observe from (a) that in the case of $\epsilon = 0.2$, $\gamma \geq 1.2$ (in the absence of rewiring), the population in the steady-state results fragmented into two clusters, despite one being more populated than the other. For the same value of ϵ polarization is also present when a higher probability of link rewiring is combined with a strong bias. This effect is still slightly visible when $\epsilon = 0.3$. It is absent for $\epsilon \geq 0.4$ where the population always reaches consensus. From fig. 4 we can observe that in the case of an initial scale-free topology, results are different from the previous network configuration in the AABSC model. In this case, we can see from (a) that for $\epsilon = 0.2$ in the absence of rewiring, the number of clusters obtained is similar to the one obtained in the AB model, despite having a lower level of fragmentation for high values of the algorithmic bias. In (b) and (c), we can see a lower fragmentation value due to a higher confidence bound, but the qualitative behavior is the same.

In this case, a higher probability of rewiring seems to increase fragmentation in the final opinion distribution but does not change the final state, which is always consensus on average. Conclusions reported in the main text are still valid for $\epsilon = 0.4$ as shown in fig. 3 and 4.

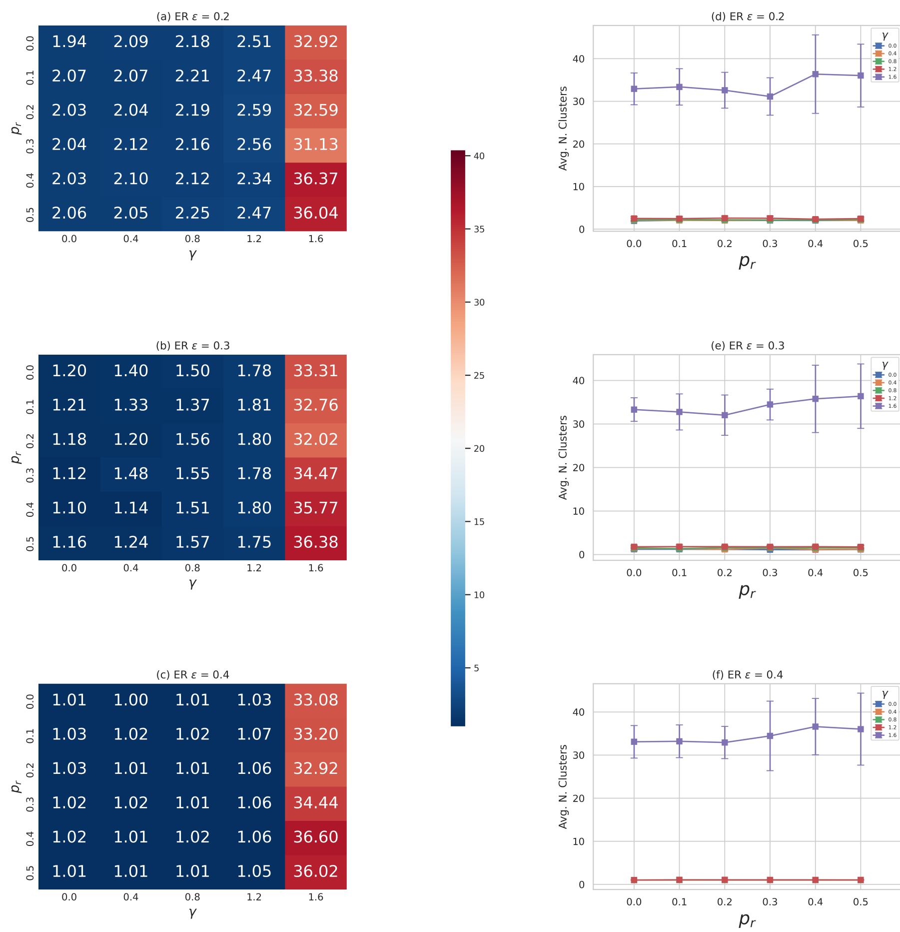


Figure 1: **Average number of clusters in the AAB model in the Erdős–Rényi network.** Average number of clusters in the AAB model in the Erdős–Rényi network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

2 Average time to convergence

In fig. 5 we simulated both models presented in this work starting from an Erdős–Rényi network of 100 nodes for computation time reasons for a fixed value of the confidence bound $\epsilon = 0.35$ for which we have convergence to consensus in both networks. Comparing the two models, we can confirm what we already stated in the text, i.e., in the AABSC model, the peer pressure mechanism sensibly enhances convergence. The algorithmic bias γ still maintains a slowing effect on convergence,

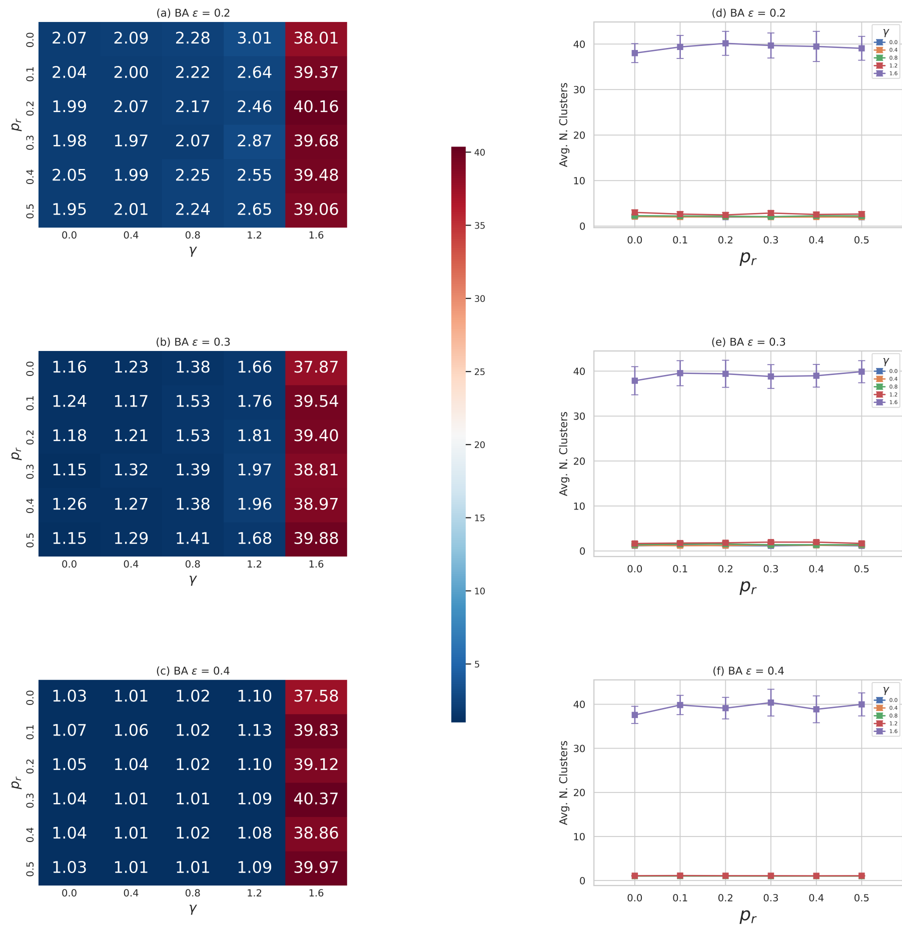


Figure 2: **Average number of clusters in the AAB model in the Barabasi-Albert network.** Average number of clusters in the AAB model in the Barabasi-Albert network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

i.e. while consensus can be reached, multiple opinion clusters remain present in the population for a more extended period of time in both models. Homophilic rewiring does not significantly impact the time to convergence in neither models on such initial network configurations.

2.1 AAB model

Also, for the time at convergence, we can see from fig. 6 and 7 that results presented in the text are still valid for $\epsilon = 0.4$. However, in this case, since convergence is faster due to the higher confidence bound, rewiring has a smaller effect on the time to convergence because convergence happens before the rewiring can affect the topology enough the change the dynamics and the process stops. As was stated in the main text, a significant slow-down of the process happens for different values

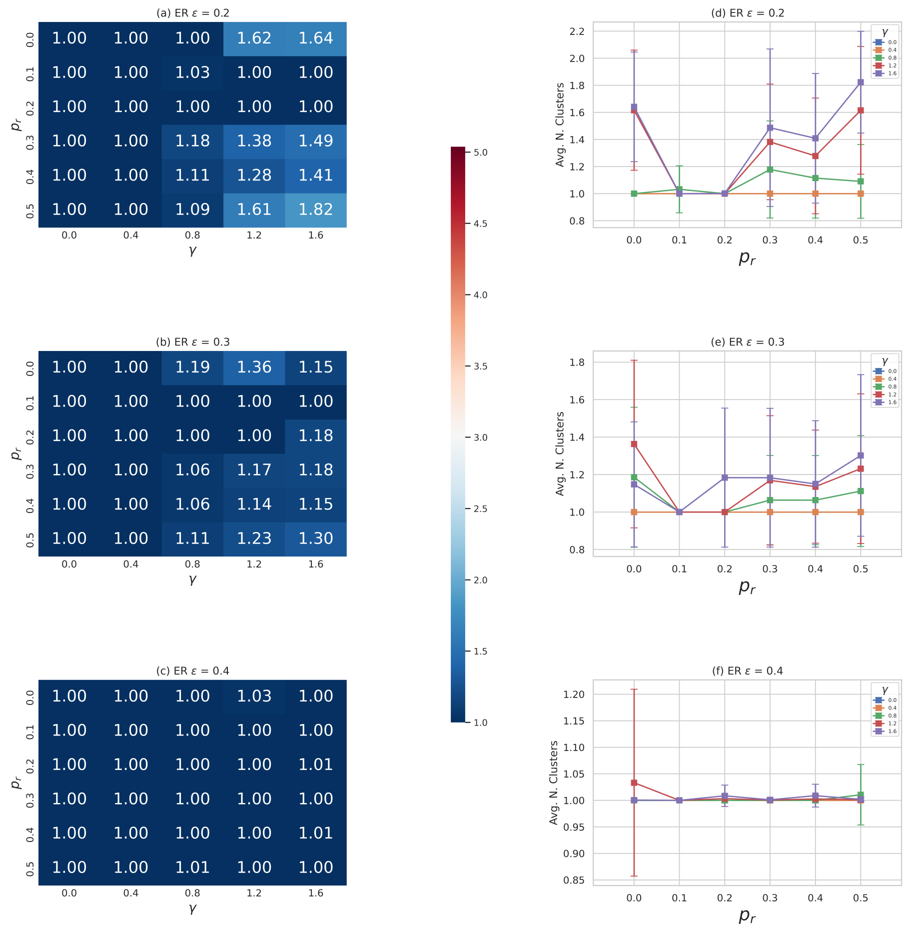


Figure 3: **Average number of clusters in the AABSC model in the Erdős–Rényi network.** Average number of clusters in the AAB model in the Erdős–Rényi network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

of the algorithmic bias in the two networks, and we can see from plots (d)-(f) in both figures that the highest standard deviation of the average number of iterations there is when the average time to convergence is slower: $\gamma = 1.6$ in the Erdos-Renyii network and $\gamma = 1.2$ in the Barabasi-Albert network.

2.2 AABSC model

We can see from fig. 8 and 9 that results presented in the text are still valid for $\epsilon = 0.4$. However, from fig. 8 we can see that in the Erdos-Renyii initial configuration, $\epsilon = 0.4$ makes convergence a lot faster with respect to $\epsilon = 0.3$ and, therefore, both rewiring and bias have a negligible effect on the time to convergence. Also, we can observe that the standard deviation is always high. Similar conclusions can be drawn for the Barabasi-Albert network.

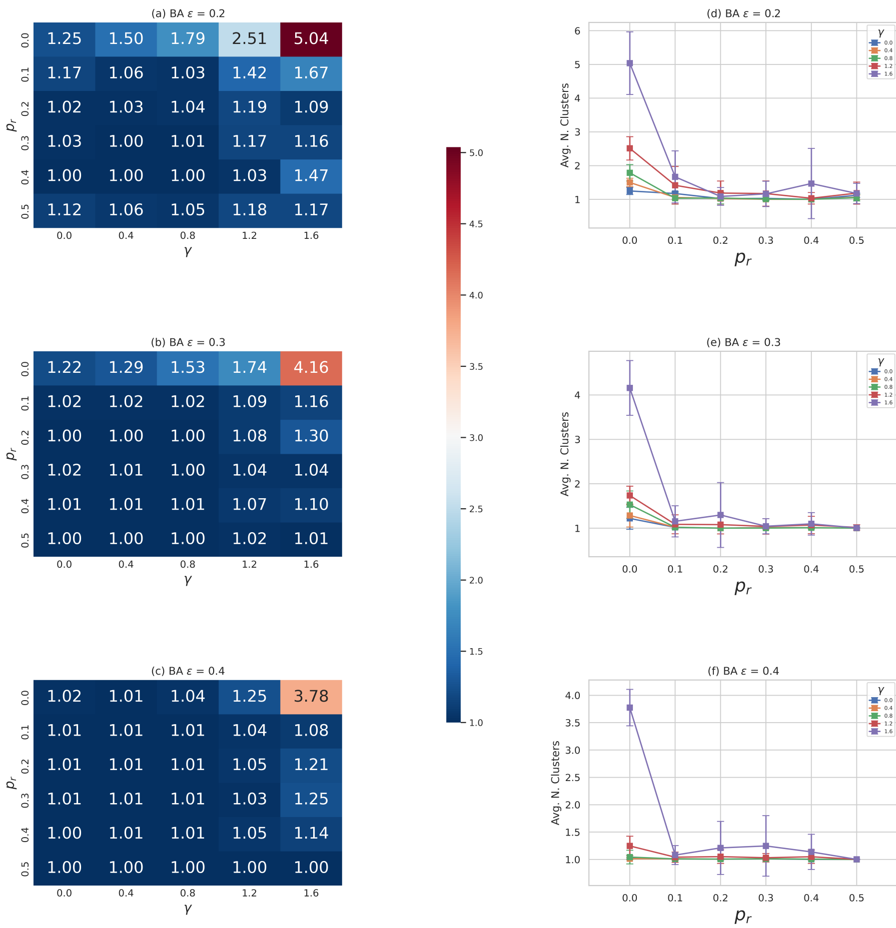
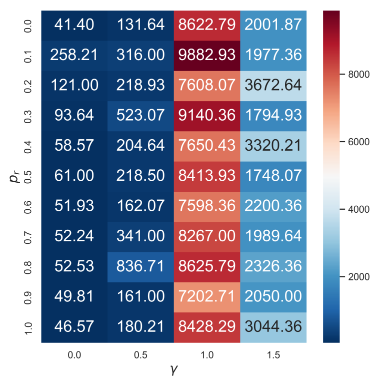
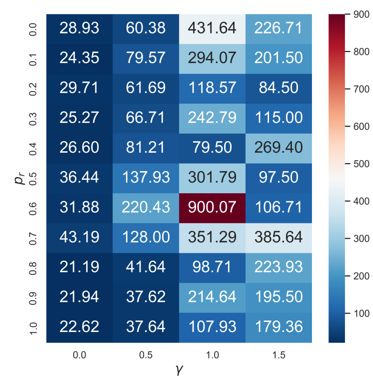


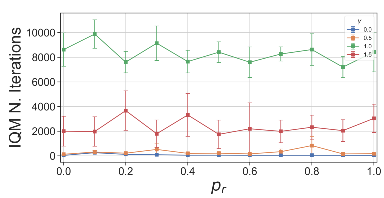
Figure 4: **Average number of clusters in the AABSC model in the Barabasi-Albert network.** Average number of clusters in the AAB model in the Barabasi-Albert network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).



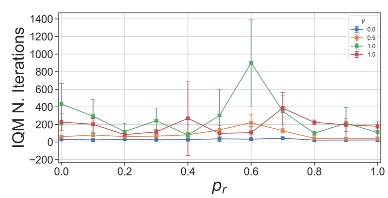
(a) AAB model



(b) AABSC model



(c) Average IQM number of iterations



(d) Average IQM number of iterations

Figure 5: Average interquartile mean number of iterations in the AAB and AABSC model in an Erdős–Rényi initial configuration of 100 nodes for $p_r \in [0.0, 1.0]$ and $\gamma \in \{0.0, 0.5, 1.0, 1.5\}$ with a fixed level of confidence bound of $\epsilon = 0.35$. Results are averaged over 30 runs. We used IQM to reduce the impact of occasional outlier values on the mean number of iterations shown.

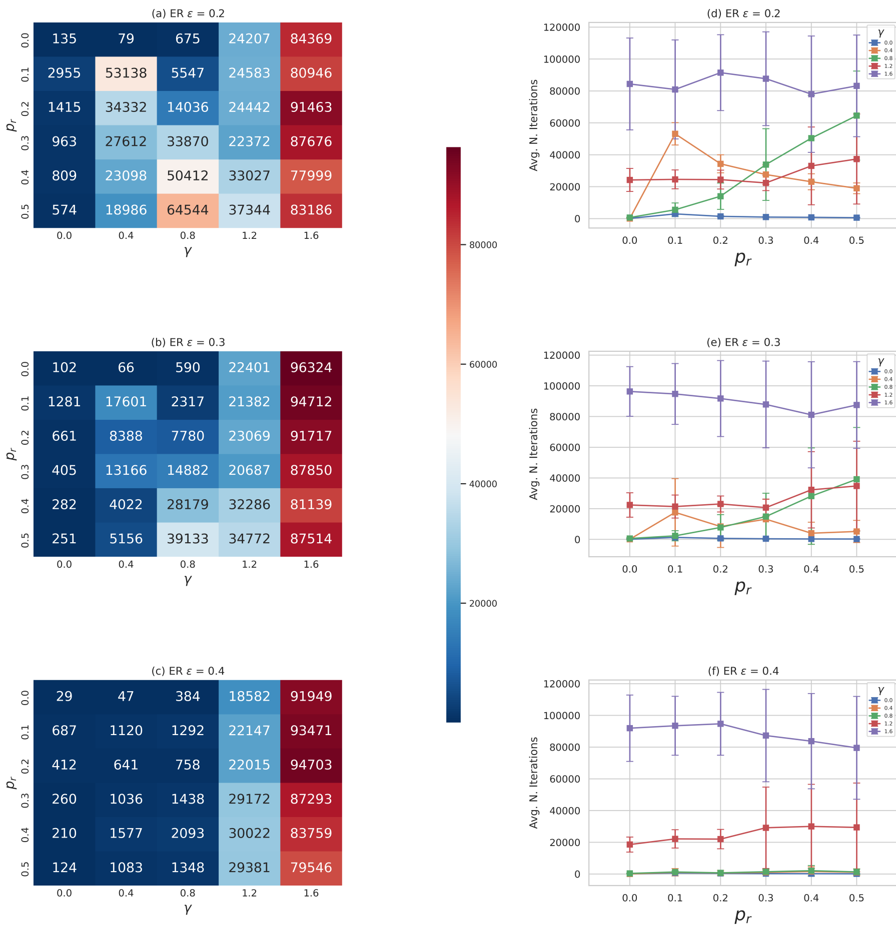


Figure 6: **Average number of iterations in the AAB model in the Erdős–Rényi network.** Average number of iterations in the AAB model in the Erdős–Rényi network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation is plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

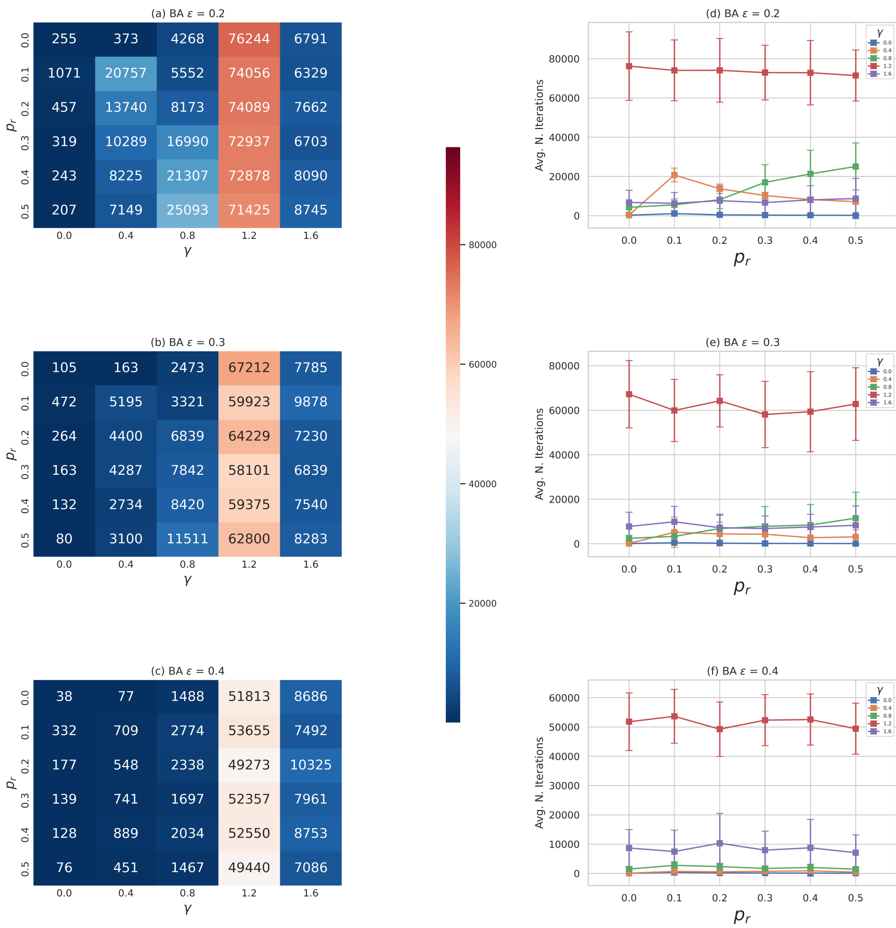


Figure 7: **Average number of iterations in the AAB model in the Barabasi-Albert network.** Average number of iterations in the AAB model in the Barabasi-Albert network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation is plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

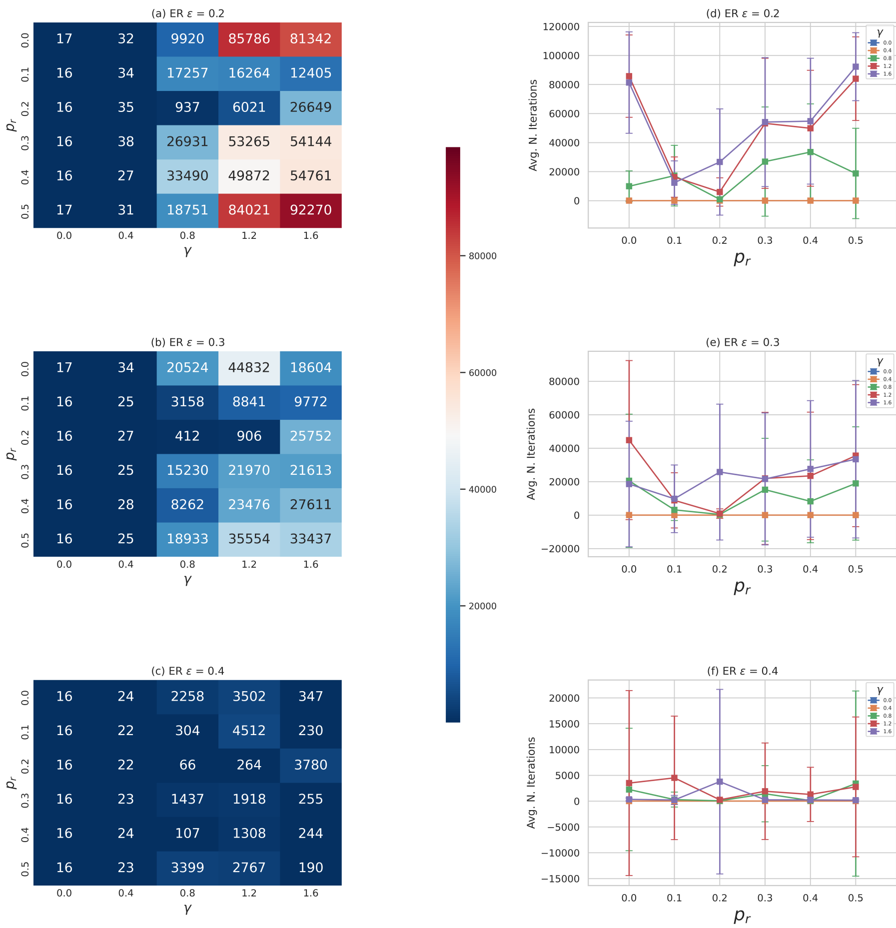


Figure 8: **Average number of iterations in the AABSC model in the Erdos-Renyi network.** Average number of clusters in the AAB model in the Erdos-Renyi network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of iterations and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).

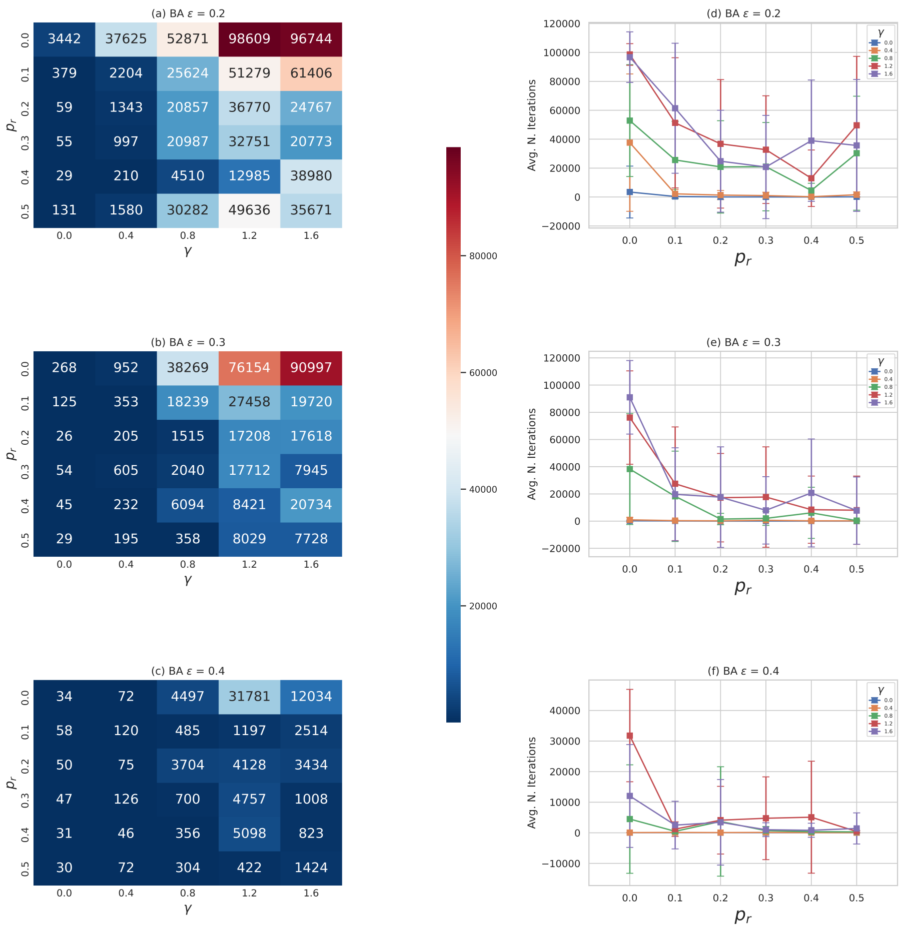


Figure 9: **Average number of iterations in the AABSC model in the Barabasi-Albert network.** Average number of clusters in the AAB model in the Barabasi-Albert network for $\epsilon = 0.2$ (a), $\epsilon = 0.3$ (b) e (c) $\epsilon = 0.4$ as a function of γ and p_r. In (d)-(f) the average number of clusters and the standard deviation are plotted as a function of p_r for different values of γ ($\gamma = 0.0$ in blue, 0.4 in orange, 0.8 in green, 1.2 in red and 1.6 in purple).